General Equilibrium Oligopoly and Ownership Structure*

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Abstract

We develop a tractable general equilibrium framework in which firms are large, have market power with respect to both products and labor, and in which ownership structure influences firms' decisions. We characterize the Cournot-Walras equilibrium of an economy where each firm maximizes a share-weighted average of shareholder utilities, which makes the equilibrium independent of price normalization. In a one-sector economy, if returns to scale are non-increasing, then an increase in "effective" market concentration (which accounts for common ownership) leads to declines in employment, real wages, and the labor share. Yet when there are multiple sectors, due to an intersectoral pecuniary externality, an increase in common ownership could stimulate the economy when the elasticity of labor supply is high relative to the elasticity of substitution in product markets. We characterize for which ownership structures the monopolistically competitive limit or an oligopolistic one are attained as the number of sectors in the economy increases.

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1 Introduction

Oligopoly is widespread and allegedly on the rise. Many industries are characterized by oligopolistic conditions—including, but not limited to, the digital ones dominated by GAFA: Google (now Alphabet), Apple, Facebook, and Amazon, to which Microsoft should be added. Those firms, as well as others, have influence in the aggregate economy.[1] Yet oligopoly is seldom considered by macroeconomic models, which focus on monopolistic competition because of its analytical tractability. A typical limitation of monopolistic competition models is that they have no role for market concentration to play in conditioning competition since the summary statistic for competition is the elasticity of substitution. In international trade a few papers consider oligopoly but with a continuum of sectors, and therefore with negligible firms in relation to the economy (Atkeson and Burstein 2008, Neary 2003a, 2003b, and 2010).

In this paper we build a tractable general equilibrium model of oligopoly allowing for ownership diversification, characterize its equilibrium and comparative statics properties, and then use it to analyze the effect of competition policies. Our contribution is mostly methodological although we have applied our model elsewhere to explain the evolution of macroeconomic magnitudes (Azar and Vives, 2018 and 2019a). We have done so at the light of the upward trends in the US economy in concentration in product and labor markets and in common ownership due to the increase in institutional investment (e.g. for close to 90 per cent of SP 500 firms the three big funds BlackRock, Vanguard, and State Street together constitute the largest shareholder). Those trends have raised concerns of increased market power and markups (e.g. De Loecker and Eeckhout, forth., Azar et al., 2018b, Azar, 2012), and calls for antitrust action and regulation of common ownership which are hotly debated (e.g. Elhauge, 2016, Posner et al., 2016).

The difficulties of incorporating oligopoly into a general equilibrium framework have hindered the modeling of market power in macroeconomics and international trade. The reason is that there is no simple objective for the firm when firms are not price takers.[2] In addition, in general equilibrium, a firm with pricing power will influence not only its own profits but also the wealth of consumers and therefore demand (these feedback effects are sometimes called Ford effects). Firms that are large relative to factor markets also have to take into account their impact on factor prices. Gabszewicz and Vial (1972) proposed the Cournot-Walras equilibrium concept assuming firms maximize profit in general equilibrium oligopoly but then equilibrium depends on the choice of numéraire.[3] The problem has been side-stepped by assuming that there is only one good (an outside good or numéraire) that owners of the firm care about (e.g., Mas-Colell, 1982); or that firms are small relative to the economy, be it in

[1] An extreme example is provided by Samsung and Hyundai, which are large relative to Korea's economy (Gabaix, 2011). Although even General Motors and Walmart have never employed more than 1% of the US workforce, those firms may weigh prominently in *local* labor markets.

[2] With price-taking firms, a firm's shareholders agree unanimously that the objective of the firm should be to maximize its own profits. This result is called the "Fisher Separation Theorem" (Ekern and Wilson, 1974; Radner, 1974; Leland, 1974; DeAngelo, 1981). Hart (1979) extends the result to incomplete markets.

[3] When firms have market power the outcome of their optimization depends on what price is taken as the numéraire since by changing the numéraire the profit function is generally not a monotone transformation of the original one (see Ginsburgh (1994)).

monopolistic competition (Hart, 1983) or sector oligopoly (Neary, 2003a).

Furthermore, a question arises as to what is the objective of the firm when there is overlapping ownership due to owners' diversification. If a firms' shareholders have holdings in competing firms, they would benefit from high prices through their effect not only on their own profits, but also on the profits of rival firms, as well as internalizing other externalities between firms (Gordon, 1990; Hansen and Lott, 1996). Rotemberg (1984) proposes a parsimonious model where the manager of a firm maximizes a weighted average of shareholders' utilities and therefore internalizing inter-firm externalities.[4] We follow this approach here.

We build a model of oligopoly under general equilibrium, allowing firms to be large in relation to the economy, and then examine the effect of oligopoly on macroeconomic performance. The ownership structure allows investors to diversify both intra- and inter-industry. We assume that firms maximize a weighted average of shareholder utilities in Cournot–Walras equilibrium. The weights in a firm's objective function are given by the influence or "control weight" of each shareholder. This solves the numéraire problem because indirect utilities depend only on relative prices and not on the choice of numéraire. Firms are assumed to make strategic decisions that account for the effect of their actions on prices and wages. When making decisions about hiring, for instance, a firm realizes that increasing employment could put upward pricing pressure on real wages—reducing not only its own profits but also the profits of all other firms in its shareholders' portfolios. The model is parsimonious and identifies the key parameters driving equilibrium: the elasticity of substitution across industries, the elasticity of labor supply, the market concentration of each industry, and the ownership structure (i.e., extent of diversification) of investors.

Our model may help illuminate some key questions. How do output, labor demand, prices, and wages depend on market concentration and the degree of common ownership? To what extent are markups in product markets, and markdowns in the labor market, affected by how much the firm internalizes other firms' profits? Can common ownership be pro-competitive in a general equilibrium framework? How do common ownership effects change when the number of industries increases? In the presence of ownership diversification, is the monopolistically competitive limit (as described by Dixit and Stiglitz, 1977) attained when firms become small relative to the market?—and, more generally, how does ownership structure affect this limit? Is traditional antitrust policy a complement or rather a substitute with respect to controlling common ownership when the aim is boosting employment?[5]

[4]The maximization of the objective function "weighted average of shareholder utilities" depends on the *cardinal* properties of the preferences of the shareholders. However, it can be microfounded using a purely ordinal model as long as the preferences of the shareholders are random from the point of view of the managers that run the firms (Azar, 2012, 2017; Brito et al., 2018). Azar (2012) and Brito et al. (2018) show that, a probabilistic voting setting in which two managers compete for shareholder votes by developing strategic reputations, leads to an objective of the firm of maximization of a weighted average of shareholder utilities. Moreover, Azar (2017) shows that the assumption of competition between two symmetric managers is not necessary, and this objective function of the firm arises as long as one assumes that dissenting votes by shareholders are costly for the incumbent management. Some available evidence is consistent with the assumption: Matvos and Ostrovsky (2008) show that shareholders take portfolio considerations into account in voting decisions (e.g. mergers); Fos and Tsoutsoura (2014) and Aggarwal et al. (2019) show that shareholder dissent hurts directors and that director elections matter because of career concerns. He et al. (2019) show that diversified managers are more likely to vote against management.

[5]In Azar and Vives (2019b) we look at the interaction of competition policy with other government policies to foster employment.

In the base model we develop here, there is one good in addition to leisure; also, the model assumes both oligopoly in the product market and oligopsony in the labor market. Firms compete by setting their labor demands à la Cournot and thus have market power. There is a continuum of risk-neutral owners, who have a proportion of their respective shares invested in one firm and have the balance invested in the market portfolio (say, an index fund). This formulation is numéraire-free and allows us to characterize the equilibrium. The extent to which firms internalize competing firms' profits depends on market concentration and investor diversification. We demonstrate the existence and uniqueness of equilibrium, and then characterize its comparative static properties, under the assumption that labor supply is upward sloping (while allowing for some economies of scale in production). Our results show that, in the one-sector model, the markdown of real wages with respect to the marginal product of labor is driven by the common ownership–modified Herfindahl–Hirschman index (H for short) for the labor market and also by labor supply elasticity (but *not* by product market power since ownership is proportional to consumption). We perform comparative statics on the equilibrium (employment and real wages) with respect to market concentration and degree of common ownership, and we develop an example featuring Cobb–Douglas firms and consumers with additively separable isoelastic preferences. We find that increased market concentration—due either to fewer firms or to more diversification (common ownership)—depresses the economy by *reducing* employment, output, real wages, and the labor share (if one assumes non-increasing returns to scale).

We extend our base model to allow for multiple sectors and differentiated products across sectors (with CES aggregators). The firms supplying each industry's product are finite in number and engage in Cournot competition. We allow here for investors to diversify both in an intra-industry fund and in an economy-wide index fund. In this extension, a firm deciding whether to marginally increase its employment must consider the effect of that increase on three relative prices: (i) the increase would reduce the relative price of the firm's own products, (ii) it would boost real wages, and (iii) it would increase the relative price of products in other industries—that is, because overall consumption would increase. This third effect, referred to as *inter-sector pecuniary externality*, is internalized only if there is common ownership involving the firm and firms in other industries. In this case, the markdown of real wages relative to the marginal product of labor increases with the H values for the labor market and product markets but decreases with the pecuniary externality (weighted by the extent of competitor profit internalization due to common ownership). We find that common ownership always has an anti-competitive effect when increasing intra-industry diversification but that it can have a pro-competitive effect when increasing economy-wide diversification if the elasticity of labor supply is high in relation to the elasticity of substitution among product varieties. In this case the relative impact of profit internalization in the level of market power in product markets is higher than in the labor market. It is worth to remark that when the elasticity of labor supply is high enough, an increase in economy-wide common ownership has always a pro-competitive effect, no matter how many sectors the economy has.

We then consider the limiting case when the number of sectors tends to infinity.[6] This formulation allows us to check for whether—and, if so, under what circumstances—the monopolistically competitive

[6]See d'Aspremont et al. (1996) for rigorous formulations of those large economies.

limit of Dixit and Stiglitz (1977) is attained, in the presence of common ownership, when firms become small relative to the market; it also enables a determination of how ownership structure affects that competitive limit. We find that with incomplete asymptotic diversification, as the number of sectors N in the economy grows, the monopolistically competitive limit is attained if there is one firm per sector or, alternatively, full intra-industry common ownership. If full diversification is attained at least as fast as $1/\sqrt{N}$, then profit internalization is positive in the limit and the Dixit-Stiglitz limit is not attained. We obtain that the limit degree of profit internalization is increasing in concentration and in how fast diversification is achieved. The limit markdown may increase or decrease with profit internalization.

Competition policy in the one-sector economy can foster employment and increase real wages by reducing market concentration (with non-increasing returns) and/or the level of diversification (common ownership), which serve as complementary tools. When there are multiple sectors, it is optimal for worker-consumers to have full diversification (common ownership) economy wide, but no diversification intra-industry, when the elasticity of substitution in product markets is low in relation to the elasticity of labor supply. In this case, competition policy should seek to alter only intra-industry ownership structure.

The rest of our paper proceeds as follows. Section 2 presents some further connection with the literature. Section 3 develops a one-sector model of general equilibrium oligopoly with labor as the only factor of production; this is where we derive comparative statics results with respect to the effect of market concentration on employment, wages, and the labor share. In Section 4 we extend the model to allow for multiple sectors with differentiated products, and we then derive results that characterize the limit economy as the number of sectors approaches infinity. We point outs also to some illustrative calibrations of the model. Section 5 discusses the implications for competition policies, and we conclude in Section 6 with a summary and suggestions for further research. The proofs of most results are given in Appendix B. Appendix A provides more detail about the case of increasing returns in production.

2 Connections with the literature

2.1 Theory

Our paper is related to four strands of the literature.

The first is the general equilibrium with oligopoly à la Cournot models of Gabszewicz and Vial (1972), Novshek and Sonnenschein (1978), and Mas-Colell (1982) with the proposed Cournot-Walras equilibrium concept assuming firms maximize profit. We assume instead that the manager of a firm maximizes a weighted average of shareholder utilities and consider an ownership structure that allows for common ownership.

The second are the macroeconomic models with Keynesian features that have incorporated market power. A precursor of those models is the work by the contemporary of Keynes, Michal Kalecki on the macroeconomic effects of market power in two class economy (Kalecki, 1938, 1954). The most closely

related papers are perhaps Hart (1982) and d'Aspremont et al. (1990).[7] Hart (1982)'s work differs from ours in assuming that firms are small relative to the overall economy and have separate owners. Unions have the labor market power in his model and so equilibrium real wages are higher than the marginal product of labor; in our model's equilibrium, real wages are lower than that marginal product.

In d'Aspremont et al. (1990) firms are large relative to the economy, but it is still assumed that firms maximize profits in terms of an arbitrary numéraire and that they compete in prices while taking wages as given with an inelastic labor supply. We consider instead the more realistic case of an elastic labor supply, which yields a positive equilibrium real wage even when market power reduces employment to below the competitive level. Our focus differs from theirs also in that we derive measures of market concentration, discuss competition policy in general equilibrium, and consider effects on the labor share.[8] Furthermore, instead of assuming consumer-worker-owners as typical in the literature, we follow Kalecki (1954) and distinguish between two groups: worker-consumers and owner-consumers. Our model has Kaleckian flavor also in relating product market power to the labor share since in Kalecki (1938), the labor share is determined by the economy's average Lerner index.

The third strand are international trade models with oligopolistic firms. Neary (2003a) considers a continuum of industries with Cournot competition in each industry, taking the marginal utility of wealth (instead of the wage) as given. Workers supply labor inelastically and firms maximize profits. He finds a negative relationship between the labor share and market concentration. Our work differs in that firms are large relative to the economy, and therefore have market power in both product and labor markets, and in considering the effects of firms' ownership structure. He also assumes a perfectly inelastic labor supply, so that changes in market power can affect neither employment nor output in equilibrium. In contrast, we allow for an increasing labor supply function and examine more possible effects of competition policy. Atkeson and Burstein (2008) also consider a continuum of sectors with Cournot competition in each industry. The authors assume that goods produced in a country within a sector are better substitutes than across sectors. The aim of the paper is to reproduce stylized facts regarding international relative prices.

The fourth strand relates to ownership structure and oligopoly in partial equilibrium. In our model managers internalize the control of the firm by the different owners as in Rotemberg (1984) and O'Brien and Salop (2000), but ours is not a model of the stakeholder corporation as in Magill et al. (2015) since managers only internalize the welfare of owners. The fact that overlapping ownership may relax competition was observed by Rubinstein and Yaari (1983) and explored by Reynolds and Snapp (1986), and Bresnahan and Salop (1986). Since overlapping ownership may internalize externalities between firms it may have ambiguous welfare effects. Indeed, it may increase market power and raise margins but at the same time internalize technological spillovers and increase productivity (López and Vives, 2019; see He and Huang, 2017 for compatible evidence and Geng et al., 2016 for how vertical common ownership links may improve the internalization of patent complementarities). In the present paper we will see

[7] See Silvestre (1993) for a survey of the market power foundations of macroeconomic policy.

[8] Gabaix (2011) also considers firms that are large in relation to the economy but with no strategic interaction among them; his aim is to show how microeconomic shocks to large firms can create meaningful aggregate fluctuations. Acemoglu et al. (2012) pursue a similar goal but assume that firms are price takers.

how common ownership may have procompetitive effects in a multisector economy.[9]

2.2 Empirics

Our approach may speak about macro trends in the economy in relation to the effects of the evolution of institutional investment and common ownership patterns, product and labor market concentration, markups and the declining labor share, the consequences for competition and investment, and the implications for policy.

The world of dispersed ownership of Berle and Means (1932) in the US no longer exists. The rise in institutional stock ownership in the last 35 years has been formidable. Pension, mutual and exchange traded funds, now have the lion share of publicly traded US firms. The asset management industry is concentrated with around the four largest (BlackRock, Vanguard, State Street and Fidelity) and there has been a shift from active to passive investors, who are more diversified. This evolution of the asset management industry has transformed the ownership structure of firms. Today it is prevalent for large firms in any industry to have common shareholders with significant shares (see, e.g. Azar et al., 2018b).[10]

Before surveying the evidence on the macroeconomic trends let us look at what evidence there is available on how common ownership may affect the incentives of managers. Common owners in an industry may have the ability and incentive to influence management. Indeed, both voice and exit can strengthen with common ownership (Edmans et al., 2019) and not pushing for aggressiveness in management contracts is a mechanism by which common owners can relax competition (Antón et al., 2018). It is worth noticing also that even if a fund follows a passive strategy, and a good part of the increase in common ownership is due to rise of passive funds, it does not mean the fund is a passive owner (Appel et al., 2016).[11]

Recent empirical research has renewed interest in the issue of aggregate market power and its consequences for macroeconomic outcomes. Grullon et al. (2019) claim that concentration has increased in more than 75% of US industries over the last two decades and also that firms in industries with larger increases in product market concentration have enjoyed higher profit margins and positive abnormal stock returns, which suggests that market power is the driver of these outcomes. De Loecker and Eeckhout (forth.) show a large increase in markups and economic profits in the US economy since 1955. The markup increase increases is in excess of the increase in overheads. Barkai (forth.) documents declining labor and capital shares in the US economy in the past 30 years which consistent with an increase in markups.[12] There is also considerable evidence that large firms have market power not just in product

[9]See Vives (forth.) for an exposition of the tension between market power and efficiency as an outcome of common ownership and a parallel with the 1960's and 1970's debates around the Structure-Conduct-Performance paradigm in Industrial Organization.

[10]Minority cross-ownership is also common and it has anti-competitive effects (see Brito et al., 2018, Nain and Wang, 2018, and Dietzenbacher et al., 2000).

[11]Furthermore, portfolio managers have incentives to increase even marginally the value of firms in their portfolio since this increases management fees (Lewellen and Lewellen, 2018). There are, however, countervailing agency problems: Bebchuk and Hirst (forth.) point out that index fund managers may not have incentives to max wealth of beneficial investors; Hansen and Lott (1996) observe that larger agency costs may be associated with more managerial discretion when managers internalize externalities with portfolio value maximization.

[12]Blonigen and Pierce (2016) attribute the US increase in markups to increased merger activity. Autor et al. (2017) argue that

markets but also in labor markets.[13] Furthermore, there are claims also of increasing labor market concentration (Benmelech et al., 2019). In addition to increases in concentration as traditionally measured, recent research has shown that increased overlapping ownership of firms by financial institutions (in particular, funds)—what we refer to as *common ownership*—has led to substantial increases in effective (i.e. augmented by common ownership) concentration indices in the airline and banking industries, and that this greater concentration is associated with higher prices (Azar et al., 2016, 2018b). Gutiérrez and Philippon (2017b) suggest that the increase in index and quasi-index fund ownership has played a role in declining aggregate investment.[14] Summers (2016) and Stiglitz (2017) link then the increase in market power to the potential secular stagnation of developed economies.

Some of the recent empirical papers also develop theoretical frameworks that link changes in market power to the labor share (Barkai, forth.; Eggertsson et al., 2018) and to investment and interest rates (Brun and González, 2017; Gutiérrez and Philippon, 2017a; Eggertsson et al., 2018). The models described by Barkai (forth.), Brun and González (2017), Gutiérrez and Philippon (2017a), and Eggertsson et al. (2018) are based on the monopolistic competition framework with markups determined exogenously by the parameter reflecting the elasticity of substitution among products. In all cases, only product market power is considered and the firms are assumed to have *no* market power in labor or capital markets. Our theoretical framework differs from these because we explicitly model oligopoly and strategic interaction between firms in general equilibrium, which enables the study of how competition policy affects the macroeconomy.

The concern over market power in both product and labor markets is a subject of policy debate. For example, the Council of Economic Advisers produced two reports (CEA, 2016a,b) on the issue of market power. The increase in common ownership has also raised antitrust concerns (Baker, 2016; Elhauge, 2016) and some bold proposals for remedies (Posner et al., 2016; Scott Morton and Hovenkamp, 2018) as well as calls for caution (Rock and Rubinfeld, 2017).

There is an empirical debate about the trends in concentration and markups. Indeed, Rossi-Hansberg et al. (2018) find diverging trends for aggregate (increasing) and (decreasing) concentration. Rinz (2018) and Berger et al. (2019) find also that local labor market concentration has gone down. Traina (2018) and Karabarbounis and Neiman (2018) find flat markups when accounting for indirect costs of production. Increases in concentration are modest overall in both product and labor markets and/or on too broadly

globalization and technological change lead to concentration and the rise of what they call "superstar" firms, which have high profits and a low labor share. As the importance of superstar firms rises (with the increase in concentration), the aggregate labor share falls. Acemoglu and Restrepo (2019a), summarizing a body of work, argue that automation always reduces the labor share in industry value added and that it will tend to reduce the overall labor share in the economy. For example, in Acemoglu and Restrepo (2019b) it is found that the labor share declines relatively in industries that are more amenable to automation, e.g. manufacturing.

[13] A thriving literature in labor economics documents that individual firms face labor supply curves that are imperfectly inelastic, which is indicative of substantial labor market power (Falch, 2010; Ransom and Sims, 2010; Staiger et al., 2010; Matsudaira, 2013; Azar et al., 2017 and 2018a).

[14] There is an empirical debate on the validity and robustness of the results since the Modified HHI is endogenous (see O'Brien and Waehrer (2017), Dennis et al. (2019), Kennedy et al. (2017), Gramlich and Grundl (2017)). Backus et al. (2018) use a structural approach in the cereal industry and find large potential (but not actual) implied effects of common ownership relative to mergers. Boller and Morton (2019) use an event study of inclusion in the SP 500 index to conclude that common ownership increases profits.

defined industries to generate severe product market power problems (e.g, HHIs remain below antitrust thresholds in relevant product and geographic markets, e.g. Shapiro (2018)).

The question then is how to reconcile the evolution of concentration in relevant markets with the evidence of the evolution of margins, rise in corporate profits and decreases in labor share. According to the monopolistic competition model margins increase when products become less differentiated. It is however not plausible that large changes in product differentiation happen in short spans of time. We provide an alternative framework where market concentration and ownership structure have a role to play.

3 One-sector economy with large firms

In this section we first describe the model in detail. We then characterize the equilibrium and comparative static properties before providing a constant elasticity example.

3.1 Model setup

We consider an economy with (a) a finite number of firms, each of them large relative to the economy as a whole, and (b) an infinite number (a continuum) of people, each of them infinitesimal relative to the economy as a whole. There are two types of people: workers and owners. Workers and owners both consume the good produced by firms. The workers obtain income to pay for their consumption by offering their time to a firm in exchange for wages. The owners do not work for the firms. Instead, an owner's income derives from ownership of the firm's shares, which entitles the owner to control the firm as well as a share of its profits. There is a unit mass of workers and a unit mass of owners, and we use I_W and I_O to denote (respectively) the set of workers and the set of owners. There are a total of J firms in the economy.

There are two goods: a consumer good, with price p; and leisure, with price w. Each worker has a time endowment of T hours but owns no other assets. Workers have preferences over consumption and leisure; this is represented by the utility function $U(C_i, L_i)$, where C_i is worker i's level of consumption and L_i is i's labor supply. We assume that the utility function is twice continuously differentiable and satisfies $U_C > 0$, $U_L < 0$, $U_{CC} < 0$, $U_{LL} < 0$, and $U_{CL} \leq 0$.[15] The last of these expressions implies that the marginal utility of consumption is decreasing in labor supply.

The owners hold all of the firms' shares. We assume that the owners are divided uniformly into J groups, one per firm, with owners in group j owning $1 - \phi + \phi/J$ of firm j and ϕ/J of the other firms; here $\phi \in [0, 1]$. Thus ϕ can be interpreted as representing the level of portfolio diversification, or (quasi-)indexation, in the economy.[16] Owners in group j own $1 - \phi + \phi/J$ in firm j, and ϕ/J of the other firms.

[15] In the notation used here, U_x is the partial derivative of U with respect to variable x, and U_{xy} is the cross derivative of U with respect to x and y.

[16] Each owner in group j is endowed with a fraction $(1 - \phi + \phi/J)/(1/J)$ of firm j and a fraction $(\phi/J)/(1/J) = \phi$ of each of the other firms. Since the mass of the group is $1/J$, it follows that the combined ownership in firm j of all the owners in group j is $1 - \phi + \phi/J$ and that their combined ownership in each of the other firms is ϕ/J. The combined ownership shares of

If we use π_k to denote the profits of firm k, then the financial wealth of owner i in group j is given by

$$W_i = \frac{1-\phi+\phi/J}{1/J}\pi_j + \sum_{k\neq j}\phi\pi_k.$$

Total financial wealth is $\sum_{k=1}^{J}\pi_k$, the sum of the profits of all firms. The owners obtain utility from consumption only, and for simplicity we assume that their utility function is $U^O(C_i) = C_i$. A firm produces using only labor as a resource, and it has a twice continuously differentiable production function $F(L)$ with $F' > 0$ and $F(0) \geq 0$. We use L_j to denote the amount of labor employed by firm j. Firm j's profits are $\pi_j = pF(L_j) - wL_j$.

We assume that the objective function of firm j is to maximize a weighted average of the (indirect) utilities of its owners, where the weights are proportional to the number of shares. That is, we suppose that ownership confers control in proportion to the shares owned.[17] In this simple case, because shareholders do not work and there is only one consumption good, their indirect utility (as a function of prices, wages, and their wealth level) is $V^O(p, w; W_i) = W_i/p$. Hence the objective function of the firm's manager is

$$\underbrace{\left(1-\phi+\frac{\phi}{J}\right)}_{\substack{\text{Control share of}\\ \text{group } j \text{ in firm } j}} \underbrace{\frac{(1-\phi+\frac{\phi}{J})\pi_j + \frac{\phi}{J}\sum_{k\neq j}\pi_k}{p}}_{\text{Indirect utility of shareholder group } j} + \sum_{k\neq j} \underbrace{\frac{\phi}{J}}_{\substack{\text{Control share of}\\ \text{group } k \text{ in firm } j}} \underbrace{\frac{(1-\phi+\frac{\phi}{J})\pi_k + \frac{\phi}{J}\sum_{s\neq k}\pi_s}{p}}_{\text{Indirect utility of shareholder group } k}.$$

After regrouping terms, we can write the objective function as

$$\left[\left(1-\phi+\frac{\phi}{J}\right)^2 + (J-1)\left(\frac{\phi}{J}\right)^2\right]\frac{\pi_j}{p} + \left[2\left(1-\phi+\frac{\phi}{J}\right)\frac{\phi}{J} + (J-2)\left(\frac{\phi}{J}\right)^2\right]\sum_{k\neq j}\frac{\pi_k}{p}.$$

After some algebra we obtain that, for firms' managers, the objective function simplifies to maximizing (in terms of the consumption good) the sum of own profits and the profits of other firms—discounted by a coefficient λ. Formally, we have

$$\frac{\pi_j}{p} + \lambda\sum_{k\neq j}\frac{\pi_k}{p},$$

where

$$\lambda = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi}.$$

all shareholders sum to 1 for every firm:

$$\underbrace{\frac{1-\phi+\phi/J}{1/J}}_{\substack{\text{Ownership of firm } j\\ \text{by an owner in group } j}} \times \underbrace{1/J}_{\text{Mass of group } j} + (J-1)\times \underbrace{\frac{\phi/J}{1/J}}_{\substack{\text{Ownership of firm } j\\ \text{by an owner in group } k\neq j}} \times \underbrace{1/J}_{\text{Mass of group } k} = 1.$$

[17] See O'Brien and Salop (2000) for other possibilities that allow for cash flow and control rights to differ.

We interpret λ as the weight—due to common ownership—that each firm's objective function assigns to the profits of other firms relative to its own profits. This term was called the coefficient of "effective sympathy" between firms by Edgeworth (1881) (also used by Cyert and DeGroot, 1973). It increases with ϕ, the level of portfolio diversification in the economy, and also with market concentration $1/J$. We remark that $\lambda = 0$ if $\phi = 0$ and $\lambda = 1$ if $\phi = 1$, so all firms behave "as one" when portfolios are fully diversified.

Next we define our concept of equilibrium.

3.2 Equilibrium concept

An *imperfectly competitive equilibrium with shareholder representation* consists of (a) a price function that assigns consumption good prices to the production plans of firms, (b) an allocation of consumption goods, and (c) a set of production plans for firms such that the following statements hold.

(1) The prices and allocation of consumption goods are a competitive equilibrium relative to the production plans of firms.

(2) Production plans constitute a Cournot–Nash equilibrium when the objective function of each firm is a weighted average of shareholders' indirect utilities.

It follows then that if a price function, an allocation of consumption goods, and a set of production plans for firms is an imperfectly competitive equilibrium with shareholder representation, then also a scalar multiple of prices will be an equilibrium with the same allocation of goods and productions. The reason is that the indirect utility function is homogeneous of degree zero in prices and income and if a consumption and production allocation satisfies (1) and (2) with the original price function then it will continue to do so when prices are scaled.

We start by defining a competitive equilibrium relative to the firms' production plans—in the particular model of this section, a Walrasian equilibrium conditional on the quantities of output announced by the firms. To simplify notation, we proxy firm j's production plan by the quantity L_j of labor demanded, leaving the planned *production quantity* implicitly equal to $F(L_j)$.

Definition 1 (Competitive equilibrium relative to production plans)**.** *A competitive equilibrium relative to* $(L_1, \ldots, L_J)$ *is a price system and allocation* $[\{w, p\}; \{C_i, L_i\}_{i \in I_W}, \{C_i\}_{i \in I_O}]$ *such that the following statements hold.*

(i) For $i \in I_W$, (C_i, L_i) *maximizes* $U(C_i, L_i)$ *subject to* $pC_i \leq wL_i$*; for* $i \in I_O$, $C_i = W_i/p$.

(ii) Labor supply equals labor demand by the firms: $\int_{i \in I_W} L_i \, di = \sum_{j=1}^{J} L_j$.

(iii) Total consumption equals total production: $\int_{i \in I_W \cup I_O} C_i \, di = \sum_{j=1}^{J} F(L_j)$.

A price function $\mathbb{W}(\mathbf{L})$ and $\mathbb{P}(\mathbf{L})$ assigns prices $\{w, p\}$ to each labor (production) plan vector $\mathbf{L} \equiv (L_1, \ldots, L_J)$, such that for any $\mathbf{L}$, $[\mathbb{W}(\mathbf{L}), \mathbb{P}(\mathbf{L}); \{C_i, L_i\}_{i \in I_W}, \{C_i\}_{i \in I_O}]$ is a competitive equilibrium for

some allocation $\{\{C_i, L_i\}_{i\in I_W}, \{C_i\}_{i\in I_O}\}$. A given firm makes employment and production plans conditional on the price function, which captures how the firm expects prices will react to its plans as well as its expectations regarding the employment and production plans of other firms. The economy is in equilibrium when every firm's employment and production plans coincide with the expectations of all the other firms.

Definition 2 (Cournot–Walras equilibrium with shareholder representation)**.** *A Cournot–Walras equilibrium with shareholder representation is a price function* $(\mathbb{W}(\cdot), \mathbb{P}(\cdot))$*, an allocation* $(\{C_i^*, L_i\}_{i\in I_W}, \{C_i^*\}_{i\in I_O})$*, and a set of production plans* $\mathbf{L}^*$ *such that the next two statements hold.*

(i) $[\mathbb{W}(\mathbf{L}^*), \mathbb{P}(\mathbf{L}^*); \{C_i^*, L_i\}_{i\in I_W}, \{C_i^*\}_{i\in I_O}]$ *is a competitive equilibrium relative to* $\mathbf{L}^*$*.*

(ii) The production plan vector $\mathbf{L}^*$ *is a pure-strategy Nash equilibrium of a game in which players are the* J *firms, the strategy space of firm* j *is* $[0, T]$*, and the firm's payoff function is*

$$\frac{\pi_j}{p} + \lambda \sum_{k\neq j} \frac{\pi_k}{p};$$

here $p = \mathbb{P}(\mathbf{L})$*,* $w = \mathbb{W}(\mathbf{L})$*, and* $\pi_j = pF(L_j) - wL_j$ *for* $j = 1, \ldots, J$*.*

Note that the objective function of firm j depends only on the real wage $\omega = w/p$, which is invariant to normalizations of prices.

3.3 Characterization of equilibrium

Given firms' production plans, we derive the real wage—under a competitive equilibrium—by assuming that workers maximize their utility $U(C_i, L_i)$ subject to the budget constraint $C_i \leq \omega L_i$. This constraint is always binding because utility is increasing in consumption but decreasing in labor. Substituting the budget constraint into the utility function of the representative worker yields the following equivalent maximization problem:

$$\max_{L_i\in[0,T]} U(\omega L_i, L_i).$$

Our assumptions on the utility function guarantee that the second-order condition holds. Thus the first-order condition for an interior solution implicitly defines a labor supply function $h(\omega)$ for worker i such that labor supply is given by $L_i = \min\{h(\omega), T\}$ (which coincides with aggregate (average) labor supply is then $\int_{i\in I} L_i\, di$). Let η denote the elasticity of labor supply. We assume that preferences are such that $h(\cdot)$ is increasing.[18]

Maintained assumption. $h'(\omega) > 0$ for $\omega \in [0, \infty)$.

[18]We can obtain the slope of h by taking the derivative with respect to the real wage in the first-order condition. This procedure yields

$$\text{sgn}\{h'\} = \text{sgn}\{U_C + (U_{CC}\omega + U_{CL}) \int_{i\in I} L_i\, di\}.$$

This assumption is consistent with a wide range of empirical studies that show that the elasticity of labor supply with respect to wages is positive. A meta-analysis of empirical studies based on different methodologies (Chetty et al., 2011) concludes that the long-run elasticity of aggregate hours worked with respect to the real wage is about 0.59. We also assume that the range of the labor supply function is $[0, T]$. This together with the maintained assumption, guarantees the existence of an increasing inverse labor supply function h^{-1} that assigns a real wage to every possible labor supply level on $[0, T]$. In a competitive equilibrium relative to the vector of labor demands by the firms, labor demand has to equal labor supply:

$$\sum_{j=1}^{J} L_j = \int_{i \in I} L_i di.$$

Any competitive equilibrium relative to firms' production plans $\mathbf{L}$ must satisfy $\omega = h^{-1}(L)$ if $L = \sum_{j=1}^{J} L_j < T$ or $\omega \geq h^{-1}(T)$ if $L = T$.[19] In what follows we will use the price function that assigns $\omega = h^{-1}(T)$ if $L = T$. Given that the relative price depends only on L, we can define (with some abuse of notation) the competitive equilibrium real-wage function $\omega(L) = h^{-1}(L)$.

3.4 Cournot–Walras equilibrium: Existence and characterization

Here we identify the conditions under which symmetric equilibria exist. We shall also provide a characterization that relates the markdown of wages relative to the marginal product of labor to the level of market concentration in the economy.

The objective of the manager of firm j is to choose L_j so that the following expression is maximized:

$$F(L_j) - \omega(L)L_j + \lambda \sum_{k \neq j} [F(L_k) - \omega(L)L_k].$$

First of all, note that firm j's best response depends only on the aggregate response of its rivals: $\sum_{k \neq j} L_k$. This claim follows because the marginal return to firm j is $F'(L_j) - \omega(L) - \left(L_j + \lambda \sum_{k \neq j} L_k\right)\omega'(L)$. Let $E_{\omega'} \equiv -\omega'' L/\omega'$ denote the elasticity of the inverse labor supply's slope. Then a sufficient condition for the game (among firms) to be of the "strategic substitutes" variety is that $E_{\omega'} < 1$. In this case, one firm's increase in labor demand is met by reductions in labor demand by the other firms and so there is an equilibrium (Vives, 1999, Thm. 2.7). Furthermore, if $F'' \leq 0$ and $E_{\omega'} < 1$, then the objective of the firm is strictly concave and the slope of its best response to a rival's change in labor demand is greater than -1. In that event, the equilibrium is unique (Vives, 1999, Thm. 2.8).

Proposition 1. *Let $E_{\omega'} < 1$. Then the game among firms is one of strategic substitutes and an equilibrium exists. Moreover, if returns are non-increasing (i.e., if $F'' \leq 0$), then the equilibrium is unique, symmetric, and locally stable under continuous adjustment (unless $F'' = 0$ and $\lambda = 1$). In an interior symmetric equilibrium with $L^* \in (0, T)$, the following statements hold.*

[19] The implication here is that the competitive equilibrium real wage as a function of $(L_1, \ldots, L_J)$ depends on firms' individual labor demands *only* through their effect on aggregate labor demand L.

(a) *The markdown of real wages is given by*

$$\mu \equiv \frac{F'(L^*/J) - \omega(L^*)}{\omega(L^*)} = \frac{H}{\eta(L^*)}, \tag{3.4.1}$$

where $H \equiv (1 + \lambda(J-1))/J$ *is the modified HHI.*

(b) *The total employment level* L^* *and the real wage* ω^* *are each increasing in* J *and decreasing in* ϕ.

(c) *The share of income going to workers,* $(\omega(L^*)L^*)/(JF(L^*/J))$, *decreases with* ϕ.

Remark. To ensure a unique equilibrium it is enough that $-F''(L_j) + (1-\lambda)\omega'(L) > 0$ if the second-order condition holds. In this case we may have a unique (and symmetric) equilibrium with moderate increasing returns. Note that $F'' < 0$ is required if the condition is to hold for all λ. Furthermore, it is possible to show that the conditions $-F'' + (1-\lambda)\omega' > 0$ together with $\omega' > 0$, ensure by themselves that a symmetric equilibrium exists and that there are no asymmetric equilibria. And if in addition $F'' \leq 0$ and $E_{\omega'} < 2$ when evaluated at a candidate symmetric equilibrium, then the symmetric equilibrium is unique for any λ (and stable provided that $\lambda < 1$). This relaxed conditions allow for strategies to be strategic complements.

Remark. If $F'' = 0$ (constant returns) and $\lambda = 1$ ($\phi = 1$, firm cartel), then there is a unique symmetric equilibrium and also multiple asymmetric equilibria, with each firm employing an arbitrary amount between zero and the monopoly level of employment and the total employment by firms equal to that under monopoly. The reason is that the shareholders in this case are indifferent over which firm engages in the actual production.

Remark. The market power friction at a symmetric equilibrium can also be expressed in terms of the markup of product prices over effective marginal cost of labor $\left(\text{mc} \equiv \frac{w}{F'(L/JN)}\right)$,

$$\tilde{\mu} \equiv \frac{p - \text{mc}}{p} = \frac{\mu}{1+\mu},$$

rather than in terms of the markdown

$$\mu = \frac{F' - w/p}{w/p} = \frac{p - \text{mc}}{\text{mc}}.$$

The Lerner-type misalignment of the marginal product of labor and the real wage (i.e., the markdown μ of real wages) is equal to the modified HHI divided by the elasticity η of labor supply. The question then arises: Why does there seem to be no effect of product market power? The reason is that, when there is a single good, this effect (equal to product market modified HHI divided by demand elasticity) is exactly compensated by the effect of owners internalizing their consumption—that is, since they are also consumers of the product that the oligopolistic firms produce. Owners use firms' profit only to purchase the good.[20]

[20]Note that, unlike in the partial equilibrium model of Farrell (1985), in our model the equilibrium markdown is not zero

3.5 Additively separable isoelastic preferences and Cobb–Douglas production

We now consider a special case of the model, one in which consumer-workers have separable isoelastic preferences over consumption and leisure:

$$U(C_i, L_i) = \frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{L_i^{1+\xi}}{1+\xi},$$

where $\sigma \in (0,1)$ and $\chi, \xi > 0$. The elasticity of labor supply is $\eta = (1-\sigma)/(\xi+\sigma) > 0$, and the equilibrium real wage in the competitive equilibrium—given firms' aggregate labor demand—can be written as

$$\omega(L) = \chi^{1/(1-\sigma)} L^{1/\eta}$$

with elasticities

$$\frac{\omega' L}{\omega} = \frac{1}{\eta} \quad \text{and} \quad E_{\omega'} = 1 - \frac{1}{\eta} < 1.$$

Because $E_{\omega'} < 1$, firms' decisions are strategic substitutes. The production function is $F(L_j) = AL_j^{\alpha}$, where $A > 0$ and $0 < \alpha \leq 1$, returns are non-increasing.

The objective function of each firm is strictly concave and Proposition 1 applies. It is easily checked that total employment under the unique symmetric equilibrium is

$$L^* = \left(\chi^{-1/(1-\sigma)} J^{1-\alpha} \frac{A\alpha}{1+H/\eta} \right)^{1/(1-\alpha+1/\eta)}.$$

Figure 1 illustrates that an increase in common ownership (or a decrease in the number of firms) reduces equilibrium employment and real wages. With increasing returns to scale, however, reducing the number of firms involves a trade-off between market power and efficiency. In that case, a decline in the number of firms can increase real wages under some conditions.

[[INSERT Figure 1 about Here]]

The symmetric equilibrium is locally stable if $\alpha - 1 < (1-\lambda)(J\eta)^{-1}(1+H/\eta)^{-1}$, so that a range of increasing returns may be allowed provided that equilibrium exists. If $\alpha > 1$, then neither the inequality $-F'' + (1-\lambda)\omega' > 0$ nor the payoff global concavity condition need hold. In Appendix B we characterize the case where $\alpha \in (1,2)$ and $\eta \leq 1$ and display a necessary and sufficient condition for an interior symmetric equilibrium to exist under increasing returns. Then L^* is decreasing in ϕ, but it may either increase or decrease with J depending on whether the effect on the mark down or the economies of scale prevail.

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even when ownership is proportional to consumption because of the labor market power effect. If the labor market is competitive, i.e., $\eta = \infty$, then the equilibrium markdown is zero. See also Mas-Colell and Silvestre (1991).

3.6 Summary and investment extension

To summarize our results so far, the simple model developed in this section can help make sense of some recent macroeconomic stylized facts, including persistently low output, employment and wages in the presence of high corporate profits and financial wealth, as a response to a permanent increase in effective concentration (due either to common ownership or to a reduced number of competitors). Because we have yet to incorporate investment decisions into the model, there is no real interest rate and so we have nothing to say about how it is affected. However, the model can be extended to include saving, capital, investment, and the real interest rate. In Azar and Vives (2019a) we present a model with workers, owners and savers and show that—for investors who are not fully diversified—either a fall in the number J of firms or a rise in ϕ, the common ownership parameter, will lead to an equilibrium with *lower* levels of capital stock, employment, real interest rate, real wages, output, and labor share of income. Under certain (reasonable) conditions the described changes will lead also to a declining capital share.

When firms are large relative to the economy, an increase in market power implies that firms have an incentive to reduce both their employment and investment below the competitive level; this follows because, even though such firms sacrifice in terms of output, they benefit from lower wages and lower interest rates on every unit of labor and capital that they employ. The effect described here is present only when firms' shareholders perceive that they can affect the economy's equilibrium level of real wages and real interest rates by changing their production plans. Thus, when oligopolistic firms have market power over the economy as a whole, their owners can extract rents from both workers and savers.[21]

4 Multiple sectors

In this section we extend the model to multiple sectors in a Cobb–Douglas isoelastic environment. We characterize the equilibrium, uncover new and richer comparative static results, and proceed to analyze large markets and convergence to the monopolistic competition outcome as the number of sectors grows large. We end the section with a note on calibration of the model.

4.1 Model setup

Consider now the case in which there are N sectors, each offering a different consumer product. We assume that both the mass of workers and the mass of owners are equal to N. So as we scale the economy by increasing the number of sectors, the number of people in the economy scales proportionally. The utility function of worker i is as in the additively separable isoelastic model: $U(C_i, L_i) = C_i^{1-\sigma}/(1-\sigma) - \chi L_i^{1+\xi}/(1+\xi)$ for $\sigma \in (0,1)$ and $\chi, \xi > 0$, where

$$C_i = \left[\left(\frac{1}{N} \right)^{1/\theta} \sum_{n=1}^{N} c_{ni}^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)};$$

[21] Our model does not account for possible technological spillovers among firms due to investment. López and Vives (2019) show that if spillovers are high enough then increasing common ownership may increase R&D investment as well.

where c_{ni} is the consumption of worker i in sector n, and $\theta > 1$ is the elasticity of substitution indicating a preference for variety.[22]

For each product, there are J firms that can produce it using labor as input. The profits of firm j in sector n are given by

$$\pi_{nj} = p_n F(L_{nj}) - wL_{nj};$$

here, as before, the production function is $F(L_{nj}) = AL_{nj}^{\alpha}$ for $A > 0$ and $\alpha > 0$.

The ownership structure is similar to the single-sector case, except now there are $J \times N$ groups of shareholders and that now shareholders can diversify both in an industry fund and in a economy-wide fund. Group nj owns a fraction $1 - \phi - \tilde{\phi} \geq 0$ in firm nj directly; an industry index fund with a fraction $\tilde{\phi}/J$ in every firm in sector n; and an economy-wide index fund with a fraction ϕ/NJ in every firm. The owners' utility is simply their consumption of the composite good C_i. Solving the owners' utility maximization problem yields the indirect utility function of shareholder i, or $V(P, w; W_i) = W_i/P$, when: prices are $\{p_n\}_{n=1}^{N}$, the level of wages is w, the shareholder's wealth is W_i, and $P \equiv \left(\frac{1}{N}\sum_{n=1}^{N} p_n^{1-\theta}\right)^{1/(1-\theta)}$ is the price index.

The objective function of the manager of firm j in sector n is to choose the firm's level of employment, L_{nj}, that maximizes a weighted average of shareholder (indirect) utilities. By rearranging coefficients so that the coefficient for own profits equals one, we obtain the following objective function:

$$\underbrace{\frac{\pi_{nj}}{P}}_{\text{own profits}} + \lambda_{intra} \underbrace{\sum_{k \neq j} \frac{\pi_{nk}}{P}}_{\text{industry } n \text{ profits, other firms}} + \lambda_{inter} \underbrace{\sum_{m \neq n} \sum_{k=1}^{J} \frac{\pi_{mk}}{P}}_{\text{profits, other industries}},$$

where the lambdas are a function of $\left(\phi, \widetilde{\phi}, J, N\right)$.

The Edgeworth sympathy coefficient for other firms in the same sector as the firm is given by:

$$\lambda_{intra} = \frac{(2-\phi)\phi + \left[2(1-\phi) - \widetilde{\phi}\right]\widetilde{\phi}N}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi) - \widetilde{\phi}\right]\widetilde{\phi}N(J-1)},$$

while the Edgeworth sympathy coefficient for firms in other sectors is given by:

$$\lambda_{inter} = \frac{(2-\phi)\phi}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi) - \widetilde{\phi}\right]\widetilde{\phi}N(J-1)}$$

Observe that λ_{intra} is no lees than λ_{inter}. This is so since the former adds the profits weights of both the industry and the economy-wide fund on the firms of an industry. We can show (see Lemma in the Appendix) that λ_{intra} and λ_{inter} are always in $[0, 1]$, increasing in ϕ and $\widetilde{\phi}$, and, for $\phi > 0$ and $\phi + \widetilde{\phi} < 1$, decreasing in N and in J.

[22]The form of C_i is the one used by Allen and Arkolakis (2016). The weight $(1/N)^{1/\theta}$ in C_i implies that, as N grows, the indirect utility derived from C_i does not grow unboundedly, and is consistent with a continuum formulation for the sectors (replacing the summation with an integral) of unit mass. More precisely: if the equilibrium is symmetric then, regardless of N, C_i is equal to the consumer's income divided by the price.

When $\phi + \widetilde{\phi} = 1$, we have $\lambda_{intra} = 1$ and $\lambda_{inter} = \left(1 - \widetilde{\phi}^2\right) / \left[1 + \widetilde{\phi}^2 (N-1)\right]$, so when agents are fully invested in the two index funds, $\lambda_{intra} = 1$ regardless of the share in each fund, while the sympathy for firms in other sectors λ_{inter} decreases as shares are moved from the economy index fund to the own industry index fund $\widetilde{\phi}$.[23] And, indeed, when $\widetilde{\phi} = 1$, $\lambda_{intra} = 1$ and $\lambda_{inter} = 0$. When $\phi = 0$ (only industry index funds), $\lambda_{inter} = 0$ and $\lambda_{intra} = \frac{\left(2-\widetilde{\phi}\right)\widetilde{\phi}}{J-\left(2-\widetilde{\phi}\right)\widetilde{\phi}(J-1)}$. When $\tilde{\phi} = 0$ (only economy-wide index fund), $\lambda_{intra} = \lambda_{inter} = \frac{(2-\phi)\phi}{(1-\phi)^2 JN + (2-\phi)\phi}$,[24] and when $\phi = 1$, $\lambda_{intra} = \lambda_{inter} = 1$.

Thus the firm accounts for the effects of its actions not only on same-sector rivals but also on firms in other sectors. Note that the manager's objective function depends on $N+1$ relative prices: w/P in addition to $\{p_n/P\}_{n=1}^{N}$ for $N > 1$.

4.2 Cournot–Walras equilibrium with N sectors

Given the production plans of the J firms operating in the N sectors, $\mathbf{L} \equiv \{\mathbf{L}_1, ..., \mathbf{L}_J\}$ where $\mathbf{L}_j \equiv \left(L_{1j}, ..., L_{Nj}\right)$, we characterize first the competitive equilibrium in terms of w/P, and $\{p_n/P\}_{n=1}^{N}$. Second, we characterize the equilibrium in the plans of the firms.

4.2.1 Relative prices in a competitive equilibrium given firms' production plans

Because the function that aggregates the consumption of all sectors is homothetic, workers face a two-stage budgeting problem. First, workers choose their consumption across sectors (conditional on their aggregate level of consumption) to minimize expenditures; second, they choose labor supply L_i and consumption level C_i to maximize their utility $U(C_i, L_i)$ subject to $PC_i = wL_i$, where P is the aggregate price level.

We can therefore write the first-stage problem as

$$\min_{\{c_{ni}\}_{n=1}^{N}} \sum_{n=1}^{N} p_n c_{ni}$$

subject to

$$\left[\sum_{n=1}^{N} \left(\frac{1}{N}\right)^{1/\theta} c_{ni}^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} = C_i.$$

The solution to this problem yields the standard demand for each consumer product conditional on aggregate consumption:

$$c_{ni} = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta} C_i. \tag{4.2.1}$$

It follows from homotheticity that, for every consumer, total expenditure equals the price index multi-

[23] Note that when $\phi + \widetilde{\phi} = 1$, two firms in the same industry have the same ownership structure, each with ϕ and $\widetilde{\phi}$ proportions of each fund. Therefore, there is shareholder unanimity in maximizing joint industry profits and $\lambda_{intra} = 1$.

[24] Here λ is the resulting Edgeworth sympathy coefficient, given as in the one-sector economy by replacing J with JN.

plied by their respective level of consumption:

$$\sum_{n=1}^{N} p_n c_{ni} = PC_i.$$

In the second stage, the first-order condition for an interior solution is

$$\frac{w}{P} = -\frac{U_L\left(\frac{w}{P}L_i, L_i\right)}{U_C\left(\frac{w}{P}L_i, L_i\right)}. \tag{4.2.2}$$

Since workers are homogeneous, it follows that total labor supply $\int_{i\in I} L_i\, di$ is simply N times the individual labor supply L_i; moreover, because total labor demand L must equal total labor supply, equation (4.2.2) implicitly defines the equilibrium real wage (now relative to the price of the composite good) as a function $\omega(L)$ of the firms' total employment plans. We retain the assumptions for increasing labor supply that ensure $\omega' > 0$. When elasticity is constant we have $\omega(L) = \chi^{1/(1-\sigma)}(L/N)^{1/\eta}$; once again, $\eta = (1-\sigma)/(\xi+\sigma)$ is the elasticity of labor supply.

Shareholders maximize their aggregate consumption level conditional on their income. Their consumer demands, conditional on their respective levels of consumption, are identical to those of workers. Adding up the demands across both owners and workers, we obtain

$$\underbrace{\int_{i\in I_W\cup I_O} c_{ni}\, di}_{c_n} = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta} \underbrace{\int_{i\in I_W\cup I_O} C_i\, di}_{C}.$$

In a competitive equilibrium, consumption demand must equal the sum of all firms' production of each product:

$$c_n = \sum_{j=1}^{J} F(L_{nj}). \tag{4.2.3}$$

Using equation (4.2.1) and integrating across consumers, we have that $c_n = \frac{1}{N}\left(\frac{p_n}{P}\right)^{-\theta} C$. So given firms' production plans, the following equality holds in a competitive equilibrium:

$$\frac{p_n}{P} = \left(\frac{1}{N}\right)^{1/\theta}\left(\frac{c_n}{C}\right)^{-1/\theta}. \tag{4.2.4}$$

The elasticity of the relative price of sector n, p_n/P, in relation to the aggregate production of the sector c_n for given productions in the other sectors, c_m for $m \neq n$, evaluated at a symmetric equilibrium, is given by $-(1-1/N)/\theta$. Its absolute value is decreasing in the elasticity of substitution of the varieties θ and increasing in the number of sectors N. Increasing c_n has a direct negative impact on p_n/P of $-1/\theta$ for a given C, and an indirect positive impact on p_n/P by increasing aggregate real income C, yielding $1/\theta N$. When there is only one sector ($N = 1$) there is obviously no impact on the relative price. Furthermore, the overall impact increases in the number of sectors N since then the indirect effect diminishes.

We can now use equations (4.2.3) and (4.2.4) to obtain an expression for p_n/P in a competitive equilibrium (ρ_n) conditional on firms' production plans $\mathbf{L}$:

$$\rho_n(\mathbf{L}) = \left(\frac{1}{N}\right)^{1/\theta} \left\{ \frac{\sum_{j=1}^{J} F(L_{nj})}{\left[\sum_{m=1}^{N} \left(\frac{1}{N}\right)^{1/\theta} \left(\sum_{j=1}^{J} F(L_{mj})\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}} \right\}^{-1/\theta}.$$

Observe that—unlike the previous case of a real-wage function, where the dependence was only through total employment plans—relative prices under a competitive equilibrium depend directly on the employment plans of each individual firm.

Proposition 2. *Given the production plans $\mathbf{L} \equiv \{L_{mj}\}$ of firms with aggregate labor demand L, the competitive equilibrium is given by the real wage $\omega(L)$ and on the relative price in sector n: $\rho_n(\mathbf{L})$ for $n = 1, \ldots, N$. If firm j in sector n expands its employment plans, then ω increases; in addition, ρ_n decreases ($\partial\rho_n/\partial L_{nj} < 0$) while ρ_m, $m \neq n$, increases ($\partial\rho_m/\partial L_{nj} > 0$).*

An increase in employment by a firm in sector n increases the relative supply of the consumption good of that sector relative to other sectors, thereby reducing the relative price of that sector's good. Since the increased employment increases overall supply of the aggregate consumption good while leaving supply of the other sectors unchanged, the relative prices of goods in the other sectors increase.

4.2.2 Cournot-Walras equilibrium

The optimization problem of firm j in sector n is given by

$$\max_{L_{nj}} \left\{ \underbrace{\frac{\pi_{nj}}{P}}_{\text{own profits}} + \lambda_{intra} \underbrace{\sum_{k \neq j} \frac{\pi_{nk}}{P}}_{\text{industry } n \text{ profits, other firms}} + \lambda_{inter} \underbrace{\sum_{m \neq n} \sum_{k=1}^{J} \frac{\pi_{mk}}{P}}_{\text{profits, other industries}} \right\},$$

where $\pi_{nj}/P = \rho_n F(L_{nj}) - \omega(L) L_{nj}$. The first-order condition for the firm is

$$\underbrace{\rho_n(\mathbf{L}) F'(L_{nj})}_{\text{VMPL}} - \underbrace{\omega(L)}_{\text{real wage}} \underbrace{- \underset{(+)}{\frac{\partial \omega}{\partial L_{nj}}} \left[L_{nj} + \lambda_{intra} \sum_{k \neq j} L_{nk} + \lambda_{inter} \sum_{m \neq n} \sum_{k=1}^{J} L_{mk} \right]}_{\text{(i) wage effect}}$$

$$\underbrace{+ \underset{(-)}{\frac{\partial \rho_n}{\partial L_{nj}}} \left[F(L_{nj}) + \lambda_{intra} \sum_{k \neq j} F(L_{nk}) \right]}_{\text{(ii) own-industry relative price effect}} + \underbrace{\lambda_{inter} \sum_{m \neq n} \underset{(+)}{\frac{\partial \rho_m}{\partial L_{nj}}} \left[\sum_{k=1}^{J} F(L_{mk}) \right]}_{\text{(iii) other industries' relative price effect}} = 0$$

When a firm in a given sector considers expanding employment, it faces the following trade-offs. On the one hand, expanding employment increases profits by the value of the marginal product of labor

(VMPL), which the shareholders can consume after paying the new workers the real wage. On the other hand, expanding employment will increase real wages for all workers because the labor supply is upward sloping. So when there is common ownership, the owners will take into account the wage effect not only for firms that expand employment (or just for the firms in the same industry) but for firms in all industries. Furthermore, expanding employment increases output in the firm's sector and thereby reduces relative prices in that sector, which again the owners internalize not just for the firm itself but for all firms in the sector in which they have common ownership. Finally, expanding output in the firm's sector decreases consumption in all the other sectors and thus increases their relative prices; the owners of the firm, if they have common ownership involving other sectors, internalize these increased relative prices as a positive pecuniary externality. However, we will show that the own-sector negative price effect always dominates the effect of increased demand in other sectors.

As we show in the Appendix, a firm's objective function is strictly concave if $\alpha \leq 1$. We can thus establish the following existence and characterization result.[25]

Proposition 3. *Consider a multi-sector economy with additive separable isoelastic preferences and a Cobb–Douglas production function under non-increasing returns to scale* ($\alpha \leq 1$). *There exists a unique symmetric equilibrium, and equilibrium employment is given by*

$$L^* = N\left(J^{1-\alpha}\frac{\chi^{-1/(1-\sigma)}A\alpha}{1+\mu^*}\right)^{1/(1-\alpha+1/\eta)}.$$

The equilibrium markdown of real wages is

$$\mu^* = \frac{1+H_{labor}/\eta}{1-\left(H_{product}-\lambda_{inter}\right)(1-1/N)/\theta} - 1,$$

where $H_{labor} \equiv (1+\lambda_{intra}(J-1)+\lambda_{inter}(N-1)J)/NJ$ *is the labor market modified HHI and* $H_{product} \equiv (1+\lambda_{intra}(J-1))/J$ *is each sector's product market modified HHI.*

The markdown μ^* *decreases with* J *(for* $\phi+\widetilde{\phi}<1$*, with* $\mu^* \to 0$ *as* $J \to \infty$*),* η*, and* θ (for $\phi<1$), *increases in* $\tilde{\phi}$*, and can be nonmonotone in* ϕ*. When* $\tilde{\phi}=0$ *(no industry fund,* $\lambda_{intra}=\lambda_{inter}=\lambda$*),* $H_{product}-\lambda = (1-\lambda)/J$ and

$$\text{sgn}\left\{\frac{\partial\mu^*}{\partial\phi}\right\} = \text{sgn}\left\{\frac{\theta}{1+\eta}-\frac{N-1}{JN-1}\right\}.$$

Remark: Simulations show that μ^* may be *nonmonotone* in ϕ also if $\widetilde{\phi}>0$. In fact, it can be shown that when η is large enough, μ^* is decreasing in ϕ for $\widetilde{\phi}>0$ small, and increasing in ϕ for JN large. Furthermore, μ^* is found to be either increasing or decreasing in N.

As the elasticity of substitution parameter θ tends to infinity, the products of the different sectors become close to perfect substitutes; then the equilibrium is as in the one-industry case but with JN firms

[25] As in the one-sector case, if $\phi=1$ and $\alpha=1$ then there is a unique symmetric equilibrium and there also exist asymmetric equilibria, since shareholders are indifferent to *which* firms employ the workers as long as total employment is at the monopoly level.

instead of J firms. This outcome should not be surprising given that, in the case of perfect substitutes, all firms produce the same good and so—for all intents and purposes—there is but a single industry in the economy.

In the multiple-industry case we find that the equilibrium real wage, employment, and output are analogous—as a function of the markdown—to those in the single-industry case. The only difference is that the markdown is now more complicated owing to the existence of multiple sectors and of product differentiation across firms in different sectors. An important result that contrasts with the single-sector case is that employment, output, and the real wage may all be increasing in the diversification in the economy-wide fund ϕ.

The markdown of wages below the marginal product of labor can be thought of as consisting of two "wedges", one reflecting labor market power, and one reflecting product market power. In particular, the labor market wedge is $1 + H_{labor}/\eta$. The markdown is increasing in H_{labor}/η, which reflects the level of labor market power (and so is decreasing in JN and η). The product market wedge is $1 - \left(H_{product} - \lambda_{inter}\right)(1 - 1/N)/\theta$. This wedge has two components: the first is $H_{product}(1 - 1/N)/\theta$ reflecting the level of market power in the firm's sector, and the second is $\lambda_{inter}(1 - 1/N)/\theta$, reflecting the inter-sectoral externality (note that the latter diminishes as products become more substitutable and *theta* increases). The markdown is increasing in the first component of the product market wedge, and decreasing in the second component. Recall that, when evaluated at a symmetric equilibrium, the (absolute value of the) elasticity of "inverse demand" p_n/P with respect to c_n is $(1 - 1/N)/\theta$; this explains why $H_{product}(1 - 1/N)/\theta$ is the indicator of product market power (note that the indicator decreases with J and θ but increases with N). This explains that μ^* is positively associated with λ_{intra} since both H_{labor} and $H_{product}$ are increasing in λ_{intra}.

However, μ^* may be positively or negatively associated with λ_{inter}. This is so since when $\lambda_{inter} > 0$, the effect of expanding employment by firm j in sector n on the profits of other firms must be taken into account. Expanding employment in one sector benefits firms in other sectors by increasing the relative prices in those sectors (pecuniary externality) via the increase in overall consumption generated by firm nj's expanded employment plans. The result is that $H_{product}$ is diminished then by λ_{inter} (note that $H_{product} \geq \lambda_{inter}$ always). When an increase in λ_{inter} increases the labor market wedge more than it reduces the product market wedge, then μ^* is decreasing in λ_{inter} (and conversely).

When $\tilde{\phi} = 0$ (no industry fund, $\lambda_{intra} = \lambda_{inter} = \lambda$), the net effect is that an increase in λ (due to an increase in ϕ) will *more than* compensate for the product market power's effect on the equilibrium markdown. To see this, note that $(H_{product} - \lambda)(1 - 1/N)/\theta = (1 - \lambda)(1 - 1/N)/\theta J$. In the limit, when $\lambda = 1$ (or $N = 1$), we have a cartel or monopoly and the two product market effects cancel each other out exactly. The $N = 1$ case is the one-sector model developed in Section 3. Here $\lambda = 1$ can be understood in similar terms, except that in this case we have an aggregate good C. When portfolios are perfectly diversified ($\phi = 1$), we can view the economy as consisting of a single large firm that produces the composite good. Since the owner-consumers own shares in each of the components of the composite good in the same proportion, and since they use profits only to purchase that good, these owner-consumers are to the same extent shareholders and consumers of the composite good. So just as

in the single-sector economy, the effects cancel out exactly. It is worth noting that μ^* may either increase or decrease with portfolio diversification ϕ depending on whether labor market effects or rather product market effects prevail. The markdown will be decreasing in ϕ when the increase in the labor market wedge due to the higher ϕ is more than compensated by the lower product market wedge due to the pro-competitive intersector pecuniary externality. That is, when the relative impact of profit internalization in the level of market power in product markets is higher than in the labor market. This happens when the elasticity of substitution θ is small in relation to the elasticity of labor supply η. When $\eta \to \infty$, common ownership has always a pro-competitive effect. When N is large then the anticompetitive effect of common ownership prevails if $\eta < 1$. This is so since then $\frac{\theta}{1+\eta} > \frac{1}{2}$ and

$$\text{sgn}\left\{\frac{\partial \mu^*}{\partial \phi}\right\} = \text{sgn}\left\{\frac{\theta}{1+\eta} - \frac{1}{J}\right\} > 0.$$

Under the parameter configurations for the elasticities considered in Azar and Vives (2019a), $\theta = 3$ and with a conservative $\eta = .6$, we have that $\frac{\theta}{1+\eta} > \frac{1}{2}$. In consequence, the anticompetitive effect will prevail for N large.

4.3 Large economies

Most of the literature on oligopoly in general equilibrium considers the case of an infinite number of sectors such that each sector, and therefore each firm, is small relative to the economy. Monopolistic competition can be considered a special case of a model with infinite sectors in which there is only one firm per industry. Here we consider what happens when the number of sectors, N, tends to infinity. Recall that, according to the Lemma in Appendix B, λ_{intra} and λ_{inter} are increasing in ϕ and $\widetilde{\phi}$, and, for $\phi > 0$ and $\phi + \widetilde{\phi} < 1$, decreasing in N and in J. We consider first the case of asymptotic imperfect diversification on funds and then the case with full asymptotic diversification.

4.3.1 Incomplete asymptotic diversification

Let ϕ, $\widetilde{\phi}$ be constant for simplicity. If there is an economy-wide fund with imperfect diversification $\phi < 1$ and no intra-industry fund, $\tilde{\phi} = 0$, then

$$\lambda_{intra} = \lambda_{inter} \to 0, \text{ and } \mu^* \to 1/\left(\theta J - 1\right) \text{ as } N \to \infty.$$

If $\tilde{\phi} > 0$, then as $N \to \infty$, $\lambda_{inter} \to 0$ (and oligopsony power vanishes since $H_{labor} \to \lambda_{inter(\infty)} = 0$), but

$$\lambda_{intra} \to \lambda_{intra(\infty)} \equiv \frac{2\gamma - 1}{\gamma^2 J - (2\gamma - 1)(J - 1)},$$

where $\gamma \equiv (1-\phi)/\tilde{\phi} > 0$. The parameter γ rages from 1 to infinity; $\gamma = 1$ when $1 - \phi = \tilde{\phi}$, as for example when $\tilde{\phi} = 1$, and γ tends to infinity as $\tilde{\phi}$ tends to 0. When $\gamma = 1$ then $\lambda_{intra(\infty)} = 1$ and when γ tends to infinity then $\lambda_{intra(\infty)} = 0$. Furthermore, $\lambda_{intra(\infty)}$ is decreasing in γ, which in turn is decreasing

in ϕ and $\tilde{\phi}$. We have that

$$\mu^* \to \mu^*_\infty \equiv \frac{1+(J-1)\,\lambda_{intra(\infty)}}{J\theta - \left(1+(J-1)\,\lambda_{intra(\infty)}\right)},$$

which is increasing in $\lambda_{intra(\infty)}$ when $J > 1$. When $\lambda_{intra(\infty)} = 1$ then

$$\mu_\infty = 1/(\theta - 1).$$

For N large only the intraindustry effect is present and increases in either ϕ or $\tilde{\phi}$ will lead to increases in $\lambda_{intra(\infty)}$ and therefore μ^*_∞.

Recall that the market power friction at a symmetric equilibrium can also be expressed in terms of the markup of product prices over effective marginal cost of labor $\left(\text{mc} \equiv \frac{w}{F'(L/JN)}\right)$,

$$\tilde{\mu} \equiv \frac{p - \text{mc}}{p} = \frac{\mu}{1+\mu},$$

rather than in terms of the markdown. When the sequence of economies is such that $\lambda_{intra(\infty)} = 0$, e.g. $\tilde{\phi}$ tends to 0, we have that $\tilde{\mu}^* \to 1/\theta J$ (Neary's oligopoly markup). We have that $\tilde{\mu}^* \to 1/\theta$ (Dixit-Stiglitz's monopolistic competition markup), when you have essentially one firm per sector (either $J = 1$ or $\lambda_{intra(\infty)} = 1$, e.g. $\tilde{\phi}$ tends to 1).

In summary, with imperfect asymptotic diversification, the Dixit and Stiglitz (1977) monopolistically competitive limit is attained if there is one firm per sector (alternatively, full intra-industry common ownership). The Neary oligopoly solution when there are several firms in each sector is attained only with no intraindustry internalization in the limit.

4.3.2 Full asymptotic diversification

Let us illustrate the case when there is no industry fund ($\tilde{\phi} = 0$, $\lambda_{intra} = \lambda_{inter} = \lambda$). Consider a sequence of economies (ϕ_N, N) with potentially varying degrees ϕ_N of (economy-wide) common ownership. We use λ_N, μ_N to denote the lambda, markdown of economy N in the sequence. If $\phi_N \to \phi < 1$ then $\lambda_N \to 0$—as, for example, when $\phi_N = \phi < 1$ for all N. To have $\lambda_N \to \lambda \in (0,1]$, we need for ϕ_N to approach unity (full diversification) at least as rapidly as $1/\sqrt{N}$ (i.e., $\sqrt{N}(1-\phi_N) \to k$ for $k \in [0,\infty)$). If the convergence rate is faster than $1/\sqrt{N}$ with $k = 0$, then the limiting λ is always equal to 1. For sequences $1-\phi_N$ with convergence rates equal to $1/\sqrt{N}$, the value of λ in the limit is determined by k, the constant of convergence. Therefore, if $\sqrt{N}(1-\phi_N) \to k$ then $\lim_{N\to\infty} \lambda_N = \frac{1}{1+Jk^2}$.[26] If $\lambda_N \to \lambda$, then the limit markdown is:

$$\mu^*_\infty \equiv \lim_{N\to\infty} \mu^*_N = \frac{1+\lambda/\eta}{1-(1-\lambda)/\theta J} - 1.$$

[26]This result follows from the expression for λ_N by noting that $(1-\phi_N)^2 N$ is of order k^2 and that $\phi_N \to 1$ as $N \to \infty$. Note that the limit sympathy coefficient λ is increasing in concentration $1/J$ and also in the speed of convergence of $\phi_N \to 1$, as measured by the constant $1/k$. When diversification increases faster (k smaller), profit internalization is larger. Hence, in order for λ to be positive, the limiting portfolio must be fully diversified: $\phi_N \to 1$. Indeed, if $\phi_N = 1$ for all N then also $\lambda_N = 1$ for all N. When $\phi_N = 1 - \sqrt{(1-\lambda)/(\lambda JN + (1-\lambda))}$, λ is constant for all N.

The impact of λ on the markdown depends, as before, on whether its effect on the labor market wedge effect dominates its effect on the product market wedge. The labor market wedge effect dominates the product market wedge effect if and only if the elasticity η of labor supply is *lower* than $\theta J - 1$. These results are summarized in our next proposition.

Proposition 4. *Consider a sequence of economies $(\tilde{\phi}_N, \phi_N, N)$ where $\tilde{\phi}_N = 0$ for all N but attaining full diversification $\phi_N \to 1$. If $\sqrt{N}(1-\phi_N) \to k$ for $k \in [0, \infty)$, then, as $N \to \infty$, $\lambda_N \to 1/\left(1 + Jk^2\right)$ which is increasing in concentration $1/J$ and in the speed of convergence of $\phi_N \to 1$ as measured by the constant $1/k$. The limit markdown is $\mu^*_\infty = \frac{1+\lambda_\infty/\eta}{1-(1-\lambda_\infty)/\theta J} - 1$, which is increasing in λ_∞ if and only if $\frac{\theta}{1+\eta} > \frac{1}{J}$ or $\theta J - 1 > \eta$.*

In words, if full diversification is attained at least as fast as $1/\sqrt{N}$ as the economy grows large, then profit internalization is positive in the limit, and it is increasing in concentration and in how fast diversification is achieved. The limit markdown increases in profit internalization if and only if $\frac{\theta}{1+\eta} > \frac{1}{J}$.

Only if $\lambda_\infty = 0$ we obtain the markdown associated to Dixit-Stiglitz or Neary $\mu^*_\infty = 1/(J\theta - 1)$. When $\lambda_\infty > 0$, however, we obtain a different limit. In this case, if $J \to \infty$ then there is no product market power and so the markdown λ_∞/η is due only to labor market power and the markdown increases in λ_∞. When $\eta \to \infty$ then the labor market is competitive and the markdown decreases in λ_∞. When $\lambda_\infty = 1$, we obtain the monopoly solution $\mu^*_\infty = 1/\eta$.

4.4 Calibration

The model is parsimonious enough so that it can be calibrated with a few parameters. In the US economy, under the maintained assumption of proportional control, the weights that managers put on rivals' profits (i.e. the lambdas) have increased in an important way in the last decades. For example, for the US 1500 largest firms according to market capitalization the calibrated average intraindustry lambdas double from about .37 in 1985 to about .70 in 2015 (see Figure 2 and Azar and Vives, 2019b). This increase can explain the evolution of markups observed in the empirical studies in our Cournot model as well as in the calibration in Backus et al. (2019) using a Bertrand competition model.

[[INSERT Figure 2 about Here]]

The model has been extended in Azar and Vives (2019a) to include savings and capital and shown able to reproduce macroeconomic trends such as the secular decline in the labor share in the US economy from 1985 to 2005 as well as approximate also the decline in the capital share. The key to the approximation is to use the evolution of effective (i.e including the influence of common ownership) concentration in product and labor markets combining market power in product and labor markets with the evolution of common ownership.[27] With this we do not claim that common ownership is the cause of the evolution of mark ups and mark downs and of the decline of labor and capital shares but only that it has the potential to explain it.

[27]Note that firms need not be symmetric as in our model since we can input the modified HHI for an asymmetric market structure. Indeed, an industry with a very uneven distribution of firms' market shares may have a high HHI even with a large number of firms.

A question arises as to what explains the increase in the lambdas. Banal-Estañol et al. (2018) find that passive investors increased their holdings relative to active shareholders post-crisis (with data for 2004-2012 of all publicly listed firms in the U.S.). This need not lead necessarily to a higher degree of internalization of rivals' profits since passive investors could, in principle, exert less control than active ones. However, passive shareholders are more diversified and the shift toward passive investors explains the increase in profit internalization. The authors also find that an increase in the intra-industry lambda is positively associated with an increase markups in a cross-section of industries.

5 Competition policy

In this section, we show how equilibrium outcomes in oligopolistic economies are suboptimal from a social welfare perspective. We then consider the effects of competition policies that could have a positive effect on aggregate equilibrium outcomes. Our model is static and should therefore be interpreted as capturing only long-run phenomena. In this model, then, the low levels of output and employment are of a long-run nature and so could be affected by fiscal but not by monetary policy.[28]

Competition policy can influence aggregate outcomes by directly affecting product and labor market concentration—that is, by affecting the number of firms and also the extent of their ownership overlap. We illustrate the analysis with the one-sector model Cobb-Douglas isoelastic specification. We look in turn at the social planner allocation (first best), then competition policy (second best), and conclude with some remarks on the multiple-sector model.

5.1 Social planner's solution in the one-sector model

Here we characterize, in the one-sector Cobb–Douglas additively separable isoelastic model, the allocation that would be chosen by a benevolent social planner who maximizes a weighted sum of the utilities of all worker-consumers with weight $1-\kappa$ and the utilities of all owner-consumers with weight $\kappa \in [0,1]$.[29] We assume that the social planner can choose the allocation of labor and consumption as well as the number of firms (with access to a large number $J^{\max}$). Let (C, L) be the consumption and labor supply of a representative worker, and let C_O be the consumption of a representative owner; then the social planner's problem is constrained by $C + C_O \leq JA(L/J)^{\alpha} = AL^{\alpha}(1/J)^{\alpha-1}$. This constraint will always hold with equality, since otherwise it would be possible to increase welfare by increasing workers' consumption until the constraint binds. Therefore, the problem can be rewritten as

$$\max_{C,L,J}(1-\kappa)\left(\frac{C^{1-\sigma}}{1-\sigma} - \chi\frac{L^{1+\xi}}{1+\xi}\right) + \kappa[AL^{\alpha}J^{1-\alpha} - C].$$

We solve this problem in two steps. First we choose the welfare-maximizing C and L conditional

[28] We look at the effects of government employment policies in Azar and Vives (2019b) .

[29] One can interpret κ as determining the welfare standard used by society. Thus $\kappa = 0$ represents the case of a "worker-consumer welfare standard" in which owners' utilities are assigned zero weight; this case is analogous—in our general equilibrium oligopoly model—to that of the usual *partial* equilibrium *consumer* welfare standard. The case $\kappa = 1/2$ corresponds to a "total welfare standard" in which all agents' utilities are equally weighted.

on the number J of firms that are used (symmetrically) in production. Second, we maximize over J to obtain the optimal number of firms from the social planner's perspective.

The first-order conditions (which are sufficient under non-increasing returns to scale) for the first maximization problem ensure that, in an interior solution, $C^{-\sigma}$, the marginal utility of workers' consumption, is equal to $\kappa/(1-\kappa)$ multiplied by the owners' marginal utility of consumption (which is constant and equals 1) and that it is equal also to the marginal disutility from working divided by the marginal product of labor: $\chi L^{\xi}/(A\alpha(L/J)^{\alpha-1})$.[30] This condition cannot hold in an oligopolistic equilibrium because the markdown of wages relative to the marginal product of labor is positive; that outcome follows, in turn, because worker-consumers equalize the marginal utility of labor to the ratio of the marginal disutility of work and the real wage. Thus a positive markdown introduces a wedge between the marginal product of labor and the real wage:

$$\underbrace{C^{-\sigma} = \frac{\chi L^{\xi}}{w/p} = \frac{\chi L^{\xi}}{A\alpha(L/J)^{\alpha-1}}(1+\mu)}_{\text{Oligopoly equilibrium condition}} > \frac{\chi L^{\xi}}{A\alpha(L/J)^{\alpha-1}}.$$

How many firms will the social planner choose to use in the production process? If there are *decreasing* returns to scale, then social benefits are increasing in J and so the optimal choice is $J^{\max}$. With *constant* returns to scale, the number of firms in operation is irrelevant. Under *increasing* returns to scale, the social planner would choose to produce using only one firm; however, the planner would still set—contra the monopolistic outcome—the marginal product of labor equal to the marginal rate of substitution between consumption and labor.[31] Thus, from the viewpoint of a social planner, there is no Williamson trade-off because the planner can set the "shadow" markdown to zero and still benefit fully from the economies of scale due to producing with only one firm. Next we address the second-best allocation, where the planner can affect the oligopoly equilibrium only by controlling the variables J and ϕ.

5.2 Competition policy

The models developed so far illustrate how the level of competition in the economy has macroeconomic consequences, from which it seems reasonable to conclude that competition policy may stimulate the economy by boosting output and inducing a more egalitarian distribution of income. We showed that if returns to scale are non-increasing then employment, output, real wages, and the labor share all decrease under higher market concentration and more common ownership.

In the one-sector case, the equilibrium modified HHI (H) was the same for the product and labor markets and also was proportional to the markdown of wages relative to the marginal product of labor in the economy. In the multi-sector case, the markdown was a function of both the within-industry

[30] It is possible, however, for low enough values of κ, to have a corner solution, such that all the output is assigned to the workers, and the consumption of the owners is zero, i.e., $C = AL^{\alpha}$ and $C_O = 0$.

[31] With increasing returns to scale, and $\alpha < 1+\xi$, the objective of the social planner is convex in L below a threshold, and concave in L above that threshold. This guarantees that the optimal L is strictly positive (however, just like in the non-increasing returns case, there can be a corner solution for the consumption of the workers and the owners, that is $C = AL^{\alpha}$ and $C_O = 0$). If $\alpha > 1+\xi$, in some cases there could be a corner solution with $L = 0$.

and the economy-wide modified HHIs, of which the latter is most relevant for the labor market. (In practice, labor markets are segmented and so the labor market modified HHI would differ from the economy-wide one; however, the insight would be similar.)

5.2.1 Worker-consumer welfare

We can think of the competition policy in our model as setting a policy environment that affects—in a symmetric equilibrium—the number of firms per industry and/or the extent of common ownership. We start by showing that $1-\phi$ and J are complements as policy tools. Then common ownership mitigates the effect of "traditional" competition policy on employment because increasing the number of firms has a diminished effect on concentration when firms have more similar shareholders.

Proposition 5. *Let $\alpha < 1 + 1/\eta$ and let L^* be a symmetric equilibrium. Then reducing common ownership (increasing $1-\phi$) and reducing concentration (increasing J) are complements as policy tools to increase equilibrium employment.*

The proposition follows because $\text{sgn}\{\frac{\partial^2 \log L^*}{\partial(1-\phi)\partial J}\} = \text{sgn}\{-(J-1)(1-\lambda)\frac{\partial\lambda}{\partial(1-\phi)}\} > 0$ for $J > 1$, $\eta < \infty$ and $\frac{\partial\lambda}{\partial(1-\phi)} < 0$. We remark that this proposition holds under decreasing returns and also in our increasing returns example (see Appendix A) with $\eta \leq 1$ and $\alpha \in (1,2)$.

We claim that, under either constant or decreasing returns to scale, it is always welfare-increasing for worker-consumers if the planner's policy decreases diversification (common ownership) and increases the number of firms—although the latter claim need not apply under increasing returns. Under non-increasing returns, the result follows because L^* increases with both $1-\phi$ and J, equilibrium real wages increase with employment, and worker-consumer utility increases with real wages. Under increasing returns, however, there is a trade-off between market power and efficiency; in this scenario, the optimal number of firms (from the perspective of worker-consumer welfare) is limited.[32] In short: if returns to scale are increasing, then a decrease in the equilibrium markdown does not always translate into an increase in worker-consumer welfare. The following proposition presents these results formally.

Proposition 6. *Employment, real wages, and the welfare of worker-consumers are maximized by setting $\phi = 0$ and*:

(a) *$J = J^{\max}$ with non-increasing returns ($\alpha \leq 1$); and*

(b) *J equal to the greatest integer less than $\frac{2-\alpha}{\alpha-1}\eta^{-1}$ when returns are increasing, $\alpha \in (1,2)$ and $\eta \leq 1$.*[33]

In the case of non-increasing returns, competition policy can lead to equilibria arbitrarily close to the social planner's as $J^{\max}$ becomes large. This is because the markdown then becomes arbitrarily close to zero.

[32] One can easily check that, under our assumptions, L^* is increasing in $1-\phi$ and that it peaks for J (when considered as a continuous variable) at $\eta^{-1}(2-\alpha)/(\alpha-1)$.

[33] If $J > \eta^{-1}(2-\alpha)/(\alpha-1)$, then $\alpha - 1 > (\eta J)^{-1}(1+(\eta J)^{-1})^{-1}$ and the equilibrium would be unstable (see Section 2.5).

5.2.2 Positive weight on owner-consumer welfare

The polar case of $\kappa = 1$, when the social planner maximizes the utility of the owner-consumers only, it is easily seen to imply, under the assumption that $\eta \leq 1$, setting $\phi = 1$ to have a completely concentrated economy in terms of the modified HHI, while choosing the number of firms to produce as efficiently as possible, which implies setting $J = J^{\max}$ in the case of decreasing returns, $J = 1$ in the case of increasing returns, and any $J \in \{1, \ldots, J^{\max}\}$ in the case of constant returns.

For intermediate values of κ, there is no simple analytic solution to the problem of choosing a competition policy that maximizes social welfare. Yet we do know that, as κ increases, owner-consumer welfare increases while worker-consumer welfare declines; this implies that equilibrium employment and wages are both lower, in equilibrium, when κ is higher. In Figure 3, we present results of simulations from which we derive the optimal policy—and the resulting employment and welfare of each type of agent—as a function of κ when $\alpha = 0.8$. The optimal policy always sets $J = J^{\max}$ ($= 100$ in this simulation). The parameter ϕ starts at 0 and remains there for an interval corresponding to κ values between 0 and about 0.4; thereafter, ϕ increases rapidly and reaches $\phi = 1$ when $\kappa = 1$. Employment and worker welfare are highest in the range of κ for which $\phi = 0$, after which they both decrease monotonically and achieve their lowest value at $\kappa = 1$. Owner-consumer welfare is lowest for low values of κ; then it increases because larger κ values result in larger values of ϕ, the common ownership parameter.[34]

[[INSERT Figure 3 about Here]]

5.2.3 Competition policy with multiple sectors

In the one-sector case, with the worker-consumer welfare standard ($\kappa = 0$) it is always efficient to force completely separate ownership of firms—that is, regardless of how many firms there are—because there are no efficiencies associated with common ownership. In the multi-sector case, however, common ownership is associated with internalization of demand effects in other sectors; this means that—depending on the elasticity of substitution, the elasticity of labor supply, and the number of firms per industry—worker-consumers could be better-off under complete indexation of the economy. In any case, it is better to eliminate intra-industry common ownership (i.e, letting $\tilde{\phi} = 0$) and reach the maximum number of firms $J^{\max}$ if the goal is to maximize employment. Along these lines, our next result is a corollary of our previous results.

Proposition 7. *Suppose the economy has N sectors and non-increasing returns to scale. Then employment, real wages, and the welfare of worker-consumers are maximized when $J = J^{\max}$, $\tilde{\phi} = 0$, and when $\phi = 0$ (resp., $\phi = 1$) if $\theta\left(J^{\max} - 1/N\right) > (1+\eta)\left(1 - 1/N\right)$ (resp., if inequality is reversed).*[35]

[34] With increasing returns to scale it easy to generate examples where it is optimal—even from the worker-consumers' standpoint, $\kappa = 0$—if some market power is allowed so as to exploit economies of scale. Typically, the number of firms declines as κ increases.

[35] When the inequality becomes an equality, the employment-population ratio, real wages, and worker-consumer welfare are maximized (in a large economy) by $J = J^{\max}$ for any $\phi \in [0, 1]$.

So if the product market wedge effect dominates the labor market wedge effect (i.e., with low θ and high η), then allowing full economy-wide common ownership increases equilibrium employment. Conversely, if the labor market wedge effect dominates the product market wedge effect then the optimal policy is no common ownership, as in the one-sector case.

For large economies, the following analogous proposition holds. There is an $\hat{N}$ such that, to maximize employment, for economies with $N > \hat{N}$: (i) set $\tilde{\phi} = 0$, $J = J^{\max}$; and (ii) set $\phi = 0$ if $\theta J - 1 > \eta$, but $\phi = 1$ if $\theta J - 1 < \eta$. (Note that the inequality $\theta J - 1 > \eta$ is the limit of $\theta(J^{\max} - 1/N) > (1 + \eta)(1 - 1/N)$ as $N \to \infty$.) It is noteworthy that, even under Neary's (2003b) assumption of no common ownership, competition policy has an effect when firms across all sectors employ the same constant-returns technology. This result follows because we have an elastic supply of labor (and so changes in the real wage affect both employment and output) and because we have two types of agents. If our model included only worker-owner-consumers, then the representative agent would always choose the optimal level of employment.

6 Conclusion

We have provided a tractable model of oligopoly in general equilibrium which accommodates the influence of ownership structure. By assuming that managers maximize a weighted sum of utilities of shareholders in a firm we find a numéraire-free Cournot-Walras equilibrium and we characterize it. In our model, firms' employment decisions affect prices in both product and factor markets. We find that a higher *effective* market concentration (which accounts for portfolio diversification/common ownership) increases markups and reduces both real wages and employment. When there are multiple industries, common ownership can have a positive or negative effect on the equilibrium markup: the sign of the effect depends on the relative magnitudes of the elasticities of product substitution and of labor supply. We find also that the monopolistically competitive limit (e.g. as in Dixit and Stiglitz, 1977) may or may not be attained as the number of sectors in the economy grows large depending on the parallel evolution of diversification.

Competition policy can increase employment and improve welfare. In the one-sector economy we find that controlling common ownership and reducing concentration are complements with respect to fostering employment. With multiple sectors, to foster employment traditional competition policy on market concentration is adequate. However, common ownership can have a positive or negative effect on employment. It will be negative for intra-industry common ownership but could be positive for economy-wide common ownership.

The consideration of vertical relations and possible different patterns of consumption between owners and workers provide caveats to the results. For example, vertical relations imply that products in one sector may be inputs for another sector. Then common ownership may lead to partial internalization of double marginalization and decrease markups.[36]

In general, our results indicate a need to go beyond the traditional partial equilibrium analyses of

[36] Azar (2012) finds that common ownership links across industries are associated with lower markups.

competition policy, where consumer surplus is king. However, traditional competition policy (e.g., lowering market concentration) is fully valid, as well as limiting intra-industry ownership, while policy towards economy-wide common ownership requires a more nuanced approach.

The models presented here are extremely stylized. To start with we have not considered asymmetries in technology and ownership structure across firms. The ownership structure is exogenous with a separation between owners and workers, we consider neither the benefits of diversification in an uncertain world nor the effects of unions' market power on the labor market for example. The models considered are static, dynamic versions incorporating uncertainty and adjustment costs may shed light on the influence of oligopoly on issues such as monetary policy transmission. In other words, there is ample room in future research for extensions and generalizations of our approach.

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Figures

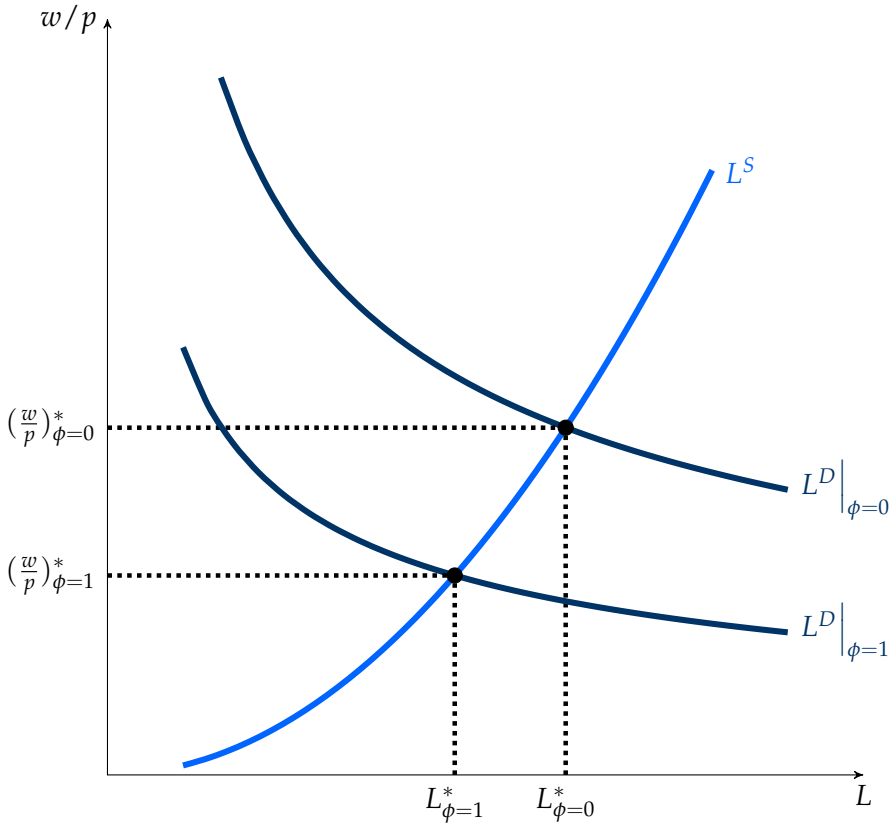

Figure 1. Effect of an increase in market concentration on equilibrium real wages and employment in the one-sector model. The model parameters for the plot are: $A = 6$, $J = 4$, $\alpha = 0.5$, $\xi = 0.5$, $\sigma = 0.5$, $\chi = 0.5$. In the case of $\phi = 0$, the MHHI is $H = 0.25$. In the case of $\phi = 1$, the MHHI is $H = 1$. L^S refers to the labor supply curve. L^D refers to the curve defined by the first-order condition of a firm and imposing symmetry.

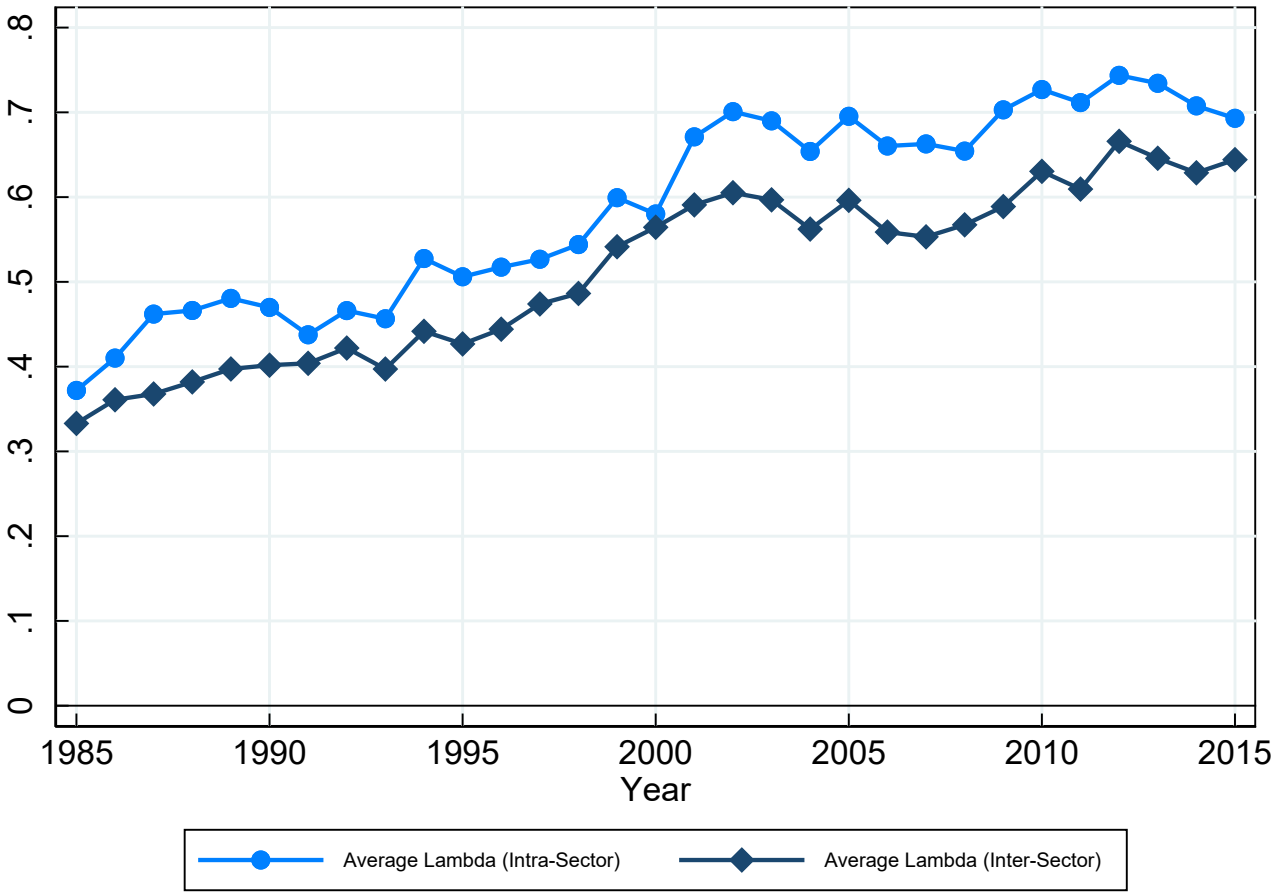

Figure 2. Average Intra-Sector and Inter-Sector Edgeworth Sympathy Coefficients. Source: Authors' calculation using Thomston 13F Institutional ownership data.

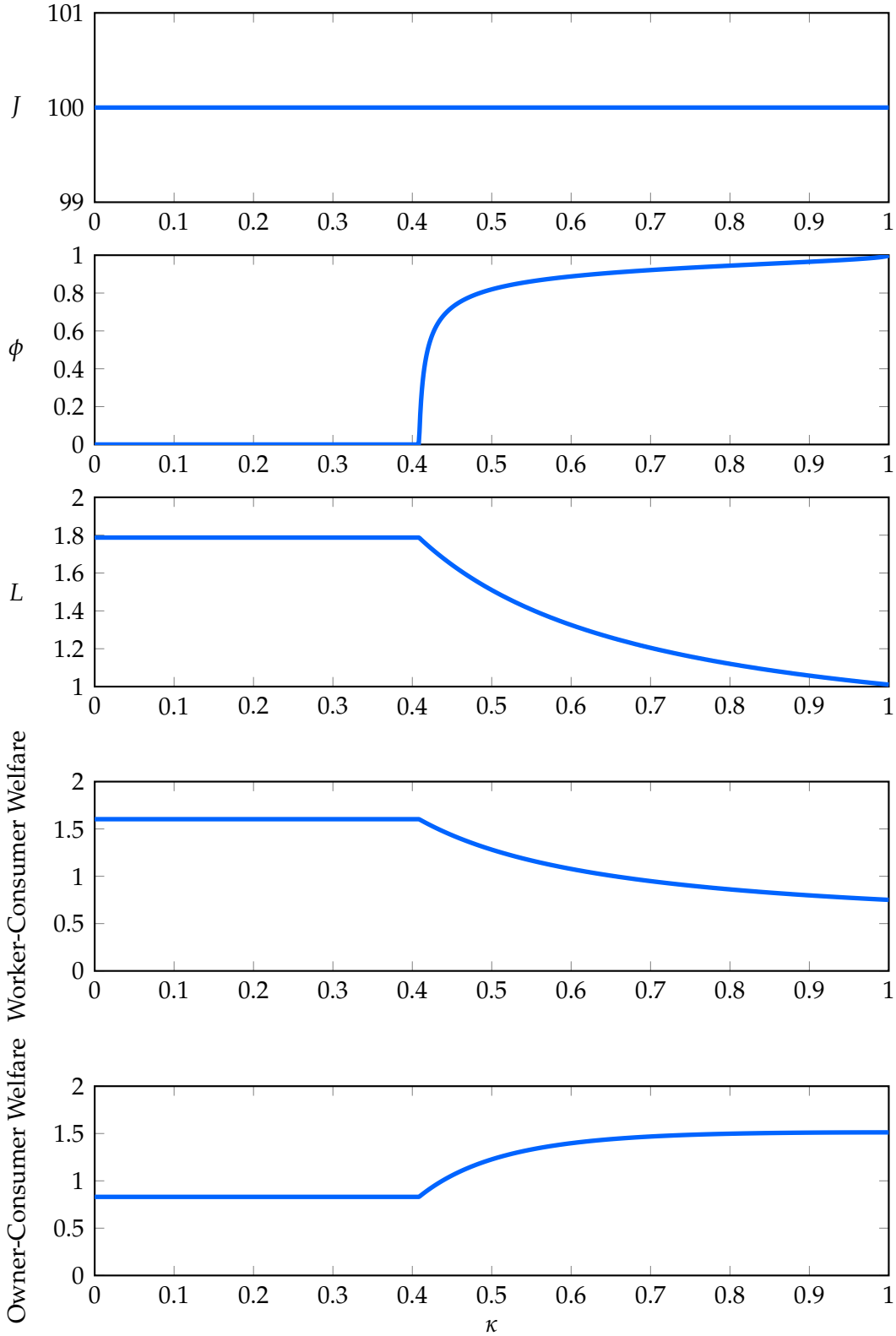

Figure 3. Optimal Competition Policy with Decreasing Returns to Scale ($\alpha = 0.8$). The model parameters for the plot are: $A = 1$, $\chi = 1$, $\sigma = 1/3$, $\xi = 1/3$, $J^{\max} = 100$.

Appendix

A Increasing returns to scale

If $\alpha > 1$, then neither the inequality $-F'' + (1-\lambda)\omega' > 0$ nor the payoff global concavity condition need hold. We characterize the situation where $\alpha \in (1,2)$ and $\eta \leq 1$. Then, with respect to L_j, firm j's objective function has a convex region below a certain threshold and a concave region above that threshold. Hence we conclude that there are no more than two candidate maxima for L_j, when given the other firms' decisions, at a *symmetric* equilibrium: $L_j = 0$; and the critical point in the concave region (if there is any). We identify (after some work) the following necessary and sufficient condition for the candidate interior solution to be a symmetric equilibrium: $\alpha \leq (1 + H/\eta)\{1 + \lambda(J-1)[1-(1-1/J)^{1/\eta}]\}^{-1}$.[37] For small λ we have that when an equilibrium exists it is stable. Here L^* is decreasing in ϕ, but it may either increase or decrease with J:

$$\frac{\partial \log L^*}{\partial J} = \frac{1}{1-\alpha+1/\eta}\frac{1}{J}\left(\underbrace{(1-\lambda)\frac{H/\eta}{1+H/\eta}}_{\text{Markdown effect}} - \underbrace{(\alpha-1)}_{\text{Economies of scale effect}}\right).$$

Increasing the number of firms has two effects on a symmetric equilibrium with increasing returns to scale: a positive effect from fewer markdowns, and a negative effect from diminished economies of scale. That is, a merger between two firms (decreasing J) would involve a so-called Williamson trade-off between higher market power and efficiencies from a larger scale of production. In our example, a merger would increase equilibrium employment if α were high enough to dominate the markdown effect.

A higher MHHI (the H in our formulation) makes it more difficult for the scale effect to dominate. Yet for a given H, a higher internalization λ makes it easier for that effect to dominate because if λ is high enough then firms will act jointly irrespective of the total number J of firms. In fact, if they act fully as one firm i.e., in the case of $\lambda = 1$, the condition is always fulfilled. Indeed, reducing J then improves scale yet does not affect the markdown because it is already at the monopoly level. It is easy to generate examples where, under increasing returns, there are multiple equilibria and some firms do not produce.

B Proofs

PROOF OF PROPOSITION 1: The objective of the manager of firm j is to maximize

$$\zeta(\mathbf{L}) = F(L_j) - \omega(L)L_j + \lambda \sum_{k \neq j} [F(L_k) - \omega(L)L_k].$$

[37] The symmetric equilibrium is locally stable under continuous adjustment provided that $\alpha - 1 \leq (1-\lambda)(J\eta)^{-1}(1 + H/\eta)^{-1}$.

The first derivative $\partial\zeta/\partial L_j$ is given by $F' - \omega - \omega'\left(L_j + \lambda\sum_{k\neq j} L_k\right)$ and therefore the best response of firm j depends only on $\sum_{k\neq j} L_k$. The cross derivative $(\partial^2\zeta//\partial L_j\partial L_m)$ equals

$$-\omega'(1+\lambda) - \left(L_j + \lambda\sum_{k\neq j} L_k\right)\omega'' = -\omega'(1+\lambda) - (s_j + \lambda s_{-j})\omega'' L,$$

where $s_j \equiv L_j/L$ and $s_{-j} \equiv \sum_{k\neq j} L_k/L$. If $E_{\omega'} \equiv -\omega'' L/\omega' < 1$, it follows that the cross derivative is negative since $s_j + \lambda s_{-j} \leq 1$ and

$$-(1+\lambda) - (s_j + \lambda s_{-j})\omega'' L/\omega' < -(1+\lambda) + (s_j + \lambda s_{-j}) < -\lambda.$$

In this case Thm. 2.7 in Vives (1999) guarantees the existence of equilibrium. The second derivative $(\partial^2\zeta/\left(\partial L_j\right)^2)$ equals $F'' - 2\omega' - \left(L_j + \lambda\sum_{k\neq j} L_k\right)\omega''$, and is negative provided that $F'' \leq 0$ also. Let $L_{-j} \equiv \sum_{k\neq j} L_k$ and $R\left(L_{-j}\right)$ denote the best response of firm j. Under the assumptions,

$$R' = -\frac{-\left((1+\lambda)\omega' + \left(L_j + \lambda\sum_{k\neq j} L_k\right)\omega''\right)}{F'' - \left(2\omega' + \left(L_j + \lambda\sum_{k\neq j} L_k\right)\omega''\right)}.$$

If the SOC holds, then $R' > -1$ whenever $-F'' + (1-\lambda)\omega' > 0$ and indeed when $F'' \leq 0$ (except when $F'' = 0$ and $\lambda = 1$). When $R' > -1$, Thm. 2.8 in Vives (1999) guarantees that the equilibrium is unique.

Given that $E_{\omega'} < 1$ and $F'' \leq 0$ we have that $\partial^2\zeta/(\partial L_j)^2 < 0$ and $\partial^2\zeta/\partial L_j\partial L_k < 0$ for $k \neq j$. Then the equilibrium is locally stable under continuous adjustment dynamics if $\partial^2\zeta/(\partial L_j)^2 < \partial^2\zeta/\partial L_j\partial L_k$ (see, e.g., Dixit (1986)). This holds if $F'' < (1-\lambda)\omega'$, which is true if $F'' < 0$ or if $F'' \leq 0$ and $\lambda < 1$.

(a) Dividing the FOC by $\omega(L)$ we have that:

$$\frac{F'\left(L_j\right) - \omega(L)}{\omega(L)} = \frac{\omega'(L)L}{\omega(L)}\left(s_j + \lambda\sum_{k\neq j} s_k\right).$$

In a symmetric equilibrium $s_j = 1/J$ for every j. Thus,

$$\frac{F'\left(\frac{L}{J}\right) - \omega(L)}{\omega(L)} = \frac{\omega'(L)L}{\omega(L)}\left(\frac{1}{J} + \lambda\frac{J-1}{J}\right).$$

(b) The symmetric equilibrium is given by the fixed point of $L_{-j}/(J-1) = R\left(L_{-j}\right)$. Total employment is $L = L_{-j} + R\left(L_{-j}\right)$, which is increasing in L_{-j} since $R' > -1$. Furthermore, R is decreasing in λ since the first derivative of the objective function is decreasing in λ. This implies that L_{-j} and therefore L and $\omega(L)$ are also decreasing in λ (and in ϕ). We have also that L_{-j} is increasing in J since $R' < 0$ and R itself is increasing in J (since R is decreasing in λ and λ is decreasing in J). It follows then that, in equilibrium, L and $\omega(L)$ are increasing in J.

(c) The labor share is $\frac{\omega(L)L}{JF(L/J)}$. The derivative with respect to total employment L is

$$\frac{\omega'(L)L + \omega(L)\left[F\left(\frac{L}{J}\right) - \frac{L}{J}F'\left(\frac{L}{J}\right)\right]}{J(F(\frac{L}{J}))^2} > 0$$

given that returns to scale are non-increasing, $F\left(\frac{L}{J}\right) - \frac{L}{J}F'\left(\frac{L}{J}\right) \geq 0$.[38] Since employment is decreasing in ϕ, that implies the labor share is decreasing in ϕ as well. □

Lemma. *λ_{intra} and λ_{inter} are: (1) increasing in ϕ and $\widetilde{\phi}$, (2) for $\phi > 0$ and $\phi + \widetilde{\phi} < 1$, and for $\phi \in (0,1)$, respectively, decreasing in N; otherwise constant as functions of N, (3) for $\phi + \widetilde{\phi} < 1$ decreasing in J; if $\phi + \widetilde{\phi} = 1$ constant as functions of J, and (4) always in $[0,1]$.*

PROOF:

1. Consider the first point. The sign of the derivative of λ_{intra} with respect to ϕ is given by:

$$\text{sgn}\left\{\frac{\partial \lambda_{intra}}{\partial \phi}\right\} = \text{sgn}\left\{(1-\phi)\left(1-\phi-\widetilde{\phi}\right)^2 + \left(1-\phi-\widetilde{\phi}\right)\left[(2-\phi)\phi + (1-\phi)\widetilde{\phi}N\right]\right\}$$

where the first term is always non-negative and positive if $1-\phi-\widetilde{\phi} > 0$ and the second one is always non-negative and positive if $1-\phi-\widetilde{\phi} > 0$ and ($\phi > 0$ or $\widetilde{\phi} > 0$). Thus, the derivative is positive in the interior of ϕ's domain, so λ_{intra} is increasing in ϕ.

The sign of the derivative of λ_{inter} with respect to ϕ is given by:

$$\text{sgn}\left\{\frac{\partial \lambda_{inter}}{\partial \phi}\right\} = \text{sgn}\left\{\begin{array}{c}(2-\phi)\phi\left[(1-\phi-\widetilde{\phi})J + \widetilde{\phi}\right] + 2(1-\phi)J\left[(1-\phi)^2 - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\right] \\ + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\end{array}\right\}$$

where the first term is always non-negative and positive if $0 < \phi < 1$, the middle one is always non-negative and positive if $1-\phi-\widetilde{\phi} > 0$ and last term is always non-negative and positive if $\widetilde{\phi} > 0$. Thus, the derivative is positive in the interior of ϕ's domain, so λ_{inter} is increasing in ϕ.

The sign of the derivative of λ_{intra} with respect to $\widetilde{\phi}$ is given by:

$$\begin{aligned}\text{sgn}\left\{\frac{\partial \lambda_{intra}}{\partial \widetilde{\phi}}\right\} &= \text{sgn}\left\{\begin{array}{c}\left(1-\phi-\widetilde{\phi}\right)2N\left[(1-\phi)^2JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)\right. \\ \left. + (J-1)\left[(2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N\right]\right]\end{array}\right\} \\ &= \text{sgn}\left\{\left(1-\phi-\widetilde{\phi}\right)2JN\left[(1-\phi)^2N + (2-\phi)\phi\right]\right\}\end{aligned}$$

so the derivative is positive in the interior of $\widetilde{\phi}$'s domain and, given this λ_{intra} is increasing in $\widetilde{\phi}$.

The derivative of λ_{inter} with respect to $\widetilde{\phi}$ is given by:

$$\frac{\partial \lambda_{inter}}{\partial \widetilde{\phi}} = \frac{\left(1-\phi-\widetilde{\phi}\right)2N(J-1)\left[(2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N\right]}{[\cdot]^2}$$

[38] If $F(x)$ is increasing and concave for $x \geq 0$, with $F(0) \geq 0$, then $F(x)/x \geq F'(x)$.

$(2-\phi)\phi \geq 0$ (with inequality if $\phi > 0$); also $\left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi} \geq 0$ (with inequality if $\widetilde{\phi} > 0$). Thus, the derivative is positive in the interior of $\widetilde{\phi}$'s domain, and so λ_{inter} is increasing in $\widetilde{\phi}$.

2. Now consider the second point. The sign of the derivative of λ_{intra} with respect to N is given by:

$$\begin{aligned}
\text{sgn}\left\{\frac{\partial \lambda_{intra}}{\partial N}\right\} &= \text{sgn}\left\{\left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}(2-\phi)\phi - (2-\phi)\phi\left[(1-\phi)^2 J - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}(J-1)\right]\right\} \\
&= -\text{sgn}\left\{(2-\phi)\phi\left[(1-\phi)^2 J - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi} J\right]\right\} \\
&= -\text{sgn}\left\{(2-\phi)\phi\left[(1-\phi)^2 - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\right]\right\} \\
&= -\text{sgn}\left\{(2-\phi)\phi\left(1-\phi-\widetilde{\phi}\right)^2\right\}
\end{aligned}$$

Thus, if $\phi > 0$ and $1-\phi-\widetilde{\phi} > 0$, λ_{intra} is decreasing in N.
Also, for the denominator of λ_{inter} we have $(1-\phi)^2 J - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}(J-1) = J(1-\phi-\widetilde{\phi})^2 + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}$, which is positive for $\phi < 1$. The numerator is positive for $\phi > 0$, so λ_{inter} is decreasing in N for $\phi \in (0,1)$.

3. Now consider the third point. $\left((1-\phi)^2 - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\right) = \left(1-\phi-\widetilde{\phi}\right)^2 \geq 0$ with equality for $\widetilde{\phi} = 1-\phi$, so the denominators of λ_{intra} and λ_{inter} are both increasing in J as long as $1-\phi-\widetilde{\phi} > 0$ (we have shown already that if $1-\phi-\widetilde{\phi} = 0$, they do not depend on J), and, given this condition λ_{intra} and λ_{inter} are decreasing in J.

4. Last, consider the fourth point. Since $\left[2(1-\phi)-\widetilde{\phi}\right] \geq 0$ with equality for $\phi = 1$ it is immediate that the minimum value λ_{intra} and λ_{inter} can assume is 0. We have shown that λ_{intra} and λ_{inter} are either decreasing or constant in N. Thus, they attain their maxima for $N = 1$, for which value we have:

$$\lambda_{intra} = \frac{(2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}}{(1-\phi)^2 J + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}(J-1)}$$

$$\lambda_{inter} = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}(J-1)}$$

Notice that $\lambda_{intra} \geq \lambda_{inter}$. Also, $\left((1-\phi)^2 - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\right) \geq 0$ with equality for $\widetilde{\phi} = 1-\phi$, so for $J = 1$, they both attain their maxima with the one for λ_{intra} given by:

$$\lambda_{intra} = \frac{(2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}}{(1-\phi)^2 + (2-\phi)\phi} = (2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi} = \left(2-\phi-\widetilde{\phi}\right)\left(\phi+\widetilde{\phi}\right)$$

which is maximized for $\widetilde{\phi} + \phi = 1$, which gives a value of 1.

We conclude that $\lambda_{intra} \in [0,1]$ and $\lambda_{inter} \in [0,1]$. □

PROOF OF PROPOSITION 2:

The change in the relative price of the firm's own sector is:

$$\begin{aligned}\frac{\partial \rho_n}{\partial L_{nj}} &= -\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}-1}\frac{F'(L_{nj})C - c_n\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_n^{\frac{\theta-1}{\theta}-1}F'(L_{nj})}{C^2}\\ &= -\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\left[1-\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{\frac{\theta-1}{\theta}}\right]\frac{F'(L_{nj})}{c_n}\\ &= -\frac{1}{\theta}\rho_n\left[1-\left(\frac{p_n c_n}{PC}\right)\right]\frac{F'(L_{nj})}{c_n} < 0.\end{aligned}$$

The change in the relative price of the other sectors ($m \neq n$) is:

$$\begin{aligned}\frac{\partial \rho_m}{\partial L_{nj}} &= -\frac{1}{\theta}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\left(\frac{c_m}{C}\right)^{-\frac{1}{\theta}-1}\frac{c_m}{C}\frac{-1}{C}\frac{\theta}{\theta-1}\frac{C}{C^{\frac{\theta-1}{\theta}}}\left(\frac{1}{N}\right)^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_n^{\frac{\theta-1}{\theta}-1}F'(L_{nj})\\ &= \frac{1}{\theta}\left(\frac{1}{N^2}\right)^{\frac{1}{\theta}}\left(\frac{c_m}{C}\right)^{1-\frac{1}{\theta}}\left(\frac{c_n}{C}\right)^{-\frac{1}{\theta}}\frac{F'(L_{nj})}{c_m}\\ &= \frac{1}{\theta}\left(\frac{p_m c_m}{PC}\right)\rho_n\frac{F'(L_{nj})}{c_m} > 0.\square\end{aligned}$$

PROOF OF PROPOSITION 3:

The expressions in the proof of 2 imply the following relationship between the change in the relative price of sector n and the changes in the relative prices of the other sectors:

$$\frac{\partial \rho_n}{\partial L_{nj}}c_n = -\sum_{m\neq n}\frac{\partial \rho_m}{\partial L_{nj}}c_m. \tag{B.1}$$

Multiplying and dividing by L in the wage effect term, by c_n in the own-industry relative price effect term, and by c_m in the other industry relative price terms, and using equation (B.1), the first-order condition simplifies to:

$$\begin{aligned}&\rho_n F'\left(L_{nj}\right) - \omega(L) - \omega'(L)L\left[s_{nj}^L + \lambda_{intra}s_{n,-j}^L + \lambda_{inter}\left(1 - s_{nj}^L - s_{n,-j}^L\right)\right]\\ &+ \frac{\partial \rho_n}{\partial L_{nj}}c_n\left[s_{nj} + \lambda_{intra}(1-s_{nj}) - \lambda_{inter}\right] = 0,\end{aligned}$$

where $s_{nj} \equiv \frac{F(L_{nj})}{c_n}$ is the share of firm j in the total production of sector n, $s_{nj}^L \equiv \frac{L_{nj}}{L}$ and $s_{n,-j}^L \equiv \left(\sum_{k\neq j} L_{nk}\right)/L$.

The second derivative of the objective function of firm j in sector n is:

$$\begin{aligned}&\frac{\partial \rho_n}{\partial L_{nj}}F'(L_{nj}) + \rho_n F''(L_{nj}) - 2\omega'(L) - \omega''(L)\left[L_{nj} + \lambda_{intra}\sum_{k\neq j}L_{nk} + \lambda_{inter}\sum_{m\neq n}\sum_{k=1}^{J}L_{mk}\right]\\ &+ \frac{\partial \rho_n}{\partial L_{nj}}F'(L_{nj})(1-\lambda_{inter}) + \frac{\partial^2 \rho_n}{(\partial L_{nj})^2}\left[F(L_{nj})(1-\lambda_{inter}) + (\lambda_{intra}-\lambda_{inter})(c_n - F(L_{nj}))\right].\end{aligned}$$

The second derivative of the relative price of firm j in sector n with respect to its own employment is

$$\frac{\partial^2 \rho_n}{(\partial L_{nj})^2} = \frac{\partial \rho_n}{\partial L_{nj}} \left[\frac{\partial \rho_n}{\partial L_{nj}} \frac{1}{\rho_n} \left(1 + (\theta - 1) \frac{\frac{p_n c_n}{PC}}{1 - \frac{p_n c_n}{PC}} \right) + \frac{F''(L_{nj})}{F'(L_{nj})} - \frac{F'(L_{nj})}{c_n} \right].$$

Replacing this in the second derivative and grouping terms yields

$$\begin{aligned}
&\frac{\partial \rho_n}{\partial L_{nj}} F'(L_{nj}) \left\{ 1 - \lambda_{inter} - \left[(1 - \lambda_{inter}) \frac{F(L_{nj})}{c_n} + (\lambda_{intra} - \lambda_{inter}) \frac{c_n - F(L_{nj})}{c_n} \right] \left[\frac{1}{\theta} \left(1 - \frac{p_n c_n}{PC} \right) + \left(1 - \frac{1}{\theta} \right) \frac{p_n c_n}{PC} \right] \right\} \\
&+ \frac{\partial \rho_n}{\partial L_{nj}} F'(L_{nj}) \left\{ 1 - \left[(1 - \lambda_{inter}) \frac{F(L_{nj})}{c_n} + (\lambda_{intra} - \lambda_{inter}) \frac{c_n - F(L_{nj})}{c_n} \right] \right\} \\
&+ \rho_n F''(L_{nj}) + \frac{\partial \rho_n}{\partial L_{nj}} c_n \frac{F''(L_{nj})}{F'(L_{nj})} \left[(1 - \lambda_{inter}) \frac{F(L_{nj})}{c_n} + (\lambda_{intra} - \lambda_{inter}) \frac{c_n - F(L_{nj})}{c_n} \right] \\
&- 2\omega'(L) - \omega''(L) \left[L_{nj} + \lambda_{intra} \sum_{k \neq j} L_{nk} + \lambda_{inter} \sum_{m \neq n} \sum_{k=1}^{J} L_{mk} \right].
\end{aligned}$$

The first row of this expression is negative because $\frac{\partial \rho_n}{\partial L_{nj}}$ is negative, F' is positive, and the expression in curly brackets is positive because $\left[\frac{1}{\theta} \left(1 - \frac{p_n c_n}{PC} \right) + \left(1 - \frac{1}{\theta} \right) \frac{p_n c_n}{PC} \right] < 1$. The term of the second row is clearly negative. The first term of the third row is non-positive, but the second term is non-negative. The two combined, however, can be rewritten as

$$\frac{\partial \rho_n}{\partial L_{nj}} c_n \frac{F''(L_{nj})}{F'(L_{nj})} \left\{ -\frac{\theta}{1 - \frac{p_n c_n}{PC}} + \left[(1 - \lambda_{inter}) \frac{F(L_{nj})}{c_n} + (\lambda_{intra} - \lambda_{inter}) \frac{c_n - F(L_{nj})}{c_n} \right] \right\},$$

which is the product of three nonpositive factors and therefore the whole expression is non-positive. The fourth row is strictly negative because, with the constant-elasticity utility functional form, it is equal to $-\frac{\omega}{L} \frac{1}{\eta} \left\{ 2 + \left(\frac{1}{\eta} - 1 \right) \left[s^L_{nj} + \lambda_{intra} s^L_{n,-j} + \lambda_{inter} \left(1 - s^L_{nj} - s^L_{n,-j} \right) \right] \right\}$. The expression

$$\left\{ 2 + \left(\frac{1}{\eta} - 1 \right) \left[s^L_{nj} + \lambda_{intra} s^L_{n,-j} + \lambda_{inter} \left(1 - s^L_{nj} - s^L_{n,-j} \right) \right] \right\}$$

is greater than one, and it is multiplying a factor $-\frac{\omega}{L} \frac{1}{\eta}$ that is negative, and therefore the fourth row of the second-order condition is negative.

The objective function of each firm is thus globally strictly concave, and therefore any solution to the system of equation implied by the first-order conditions is an equilibrium. To find the symmetric equilibria, we thus start by simplifying the first-order condition of firm nj when it is evaluated at a symmetric equilibrium, with $c_n = c$ for all n, and $p_n = p$ for all n. Note first that in the symmetric case $c_n/C = c/C = 1/N$.

In a symmetric equilibrium, the marginal product of labor is equal to $F'(\frac{L}{JN})$. Using that fact and replacing $\frac{c_n}{C} = \frac{c_m}{C} = \frac{1}{N}$ in the expression for the change in the relative price of the firm's industry when

the firm expands employment plans it simplifies to

$$\frac{\partial \rho_n}{\partial L_{nj}} = -\frac{1}{\theta}\left(1-\frac{1}{N}\right)\frac{F'(\frac{L}{JN})}{c}.$$

Dividing the first-order condition by the real wage and substituting the derivatives of the relative price that we just derived yields:

$$\begin{aligned}\frac{F'\left(\frac{L}{JN}\right)-\omega(L)}{\omega(L)} &= \frac{\omega'(L)L}{\omega(L)}\left[s^L_{nj}+\lambda_{intra}s^L_{n,-j}+\lambda_{inter}\left(1-s^L_{nj}-s^L_{n,-j}\right)\right] \\ &\quad + \frac{1}{\theta}\left(1-\frac{1}{N}\right)\frac{F'\left(\frac{L}{JN}\right)}{\omega(L)}\left[s_{nj}+\lambda_{intra}(1-s_{nj})-\lambda_{inter}\right].\end{aligned}$$

In a symmetric equilibrium, the employment share of firm j in sector n is equal to $\frac{L_{nj}}{L} = \frac{1}{JN}$ for all sectors n and all firms j within the sector, since the employment shares of all firms are the same. Similarly, the product market share of firm j in sector n is $\frac{F(L_{nj})}{c} = \frac{1}{J}$. Replacing these in the previous equation implies

$$\mu = \frac{1}{\eta}\left[1/NJ+\lambda_{intra}(J-1)/NJ+\lambda_{inter}(N-1)/N\right]+\frac{1+\mu}{\theta}\left(1-\frac{1}{N}\right)\left[1/J+\lambda_{intra}(J-1)/J-\lambda_{inter}\right].$$

We can then express this in terms of MHHIs for the labor market and product markets as follows:

$$\mu = \frac{1}{\eta}\underbrace{\left[1/NJ+\lambda_{intra}(J-1)/NJ+\lambda_{inter}(N-1)/N\right]}_{H_{labor}}+\frac{1+\mu}{\theta}\left\{\underbrace{\left[1/J+\lambda_{intra}(J-1)/J\right]}_{H_{product}}-\lambda_{inter}\right\}\left(1-\frac{1}{N}\right).$$

In this expression, H_{labor} is the MHHI for the labor market, which equals $(1+\lambda_{intra}(J-1)+\lambda_{inter}(N-1)J)/NJ$, and $H_{product}$ is the MHHI for the product market of one industry, which equals $\frac{1}{J}+\lambda_{intra}\left(1-\frac{1}{J}\right)$.

The expression for the markup provides an equation in L:

$$\omega(L) = \frac{F'\left(\frac{L}{JN}\right)}{\frac{1+\frac{H_{labor}}{\eta}}{1-\frac{1}{\theta}\left(H_{product}-\lambda_{inter}\right)\left(1-\frac{1}{N}\right)}}.$$

Combining this equation in L and w/P with the inverse labor supply and imposing labor market clearing yields an equation for the equilibrium level of employment L:

$$-\frac{U_L\left(\frac{w}{P}\frac{L}{N},\frac{L}{N}\right)}{U_C\left(\frac{w}{P}\frac{L}{N},\frac{L}{N}\right)} = \frac{F'\left(\frac{L}{JN}\right)}{\frac{1+\frac{H_{labor}}{\eta}}{1-\frac{1}{\theta}\left(H_{product}-\lambda_{inter}\right)\left(1-\frac{1}{N}\right)}}.$$

We can obtain a closed-form solution for the constant-elasticity labor supply and Cobb-Douglas production function case. In this case, the equation for equilibrium total employment level is:

$$\chi^{\frac{1}{1-\sigma}}\left(\frac{L}{N}\right)^{\frac{\xi+\sigma}{1-\sigma}}=\frac{A\alpha\left(\frac{L}{JN}\right)^{\alpha-1}}{\frac{1+\frac{H_{labor}}{\eta}}{1-\frac{1}{\theta}\left(H_{product}-\lambda_{inter}\right)\left(1-\frac{1}{N}\right)}}.$$

This equation has a unique solution for L:

$$L^{*}=N\left(\frac{\chi^{-\frac{1}{1-\sigma}}A\alpha}{1+\mu^{*}}\right)^{\frac{1}{\frac{1}{\eta}-(\alpha-1)}}J^{-\frac{\alpha-1}{\frac{1}{\eta}-(\alpha-1)}},$$

where $1+\mu^{*}$ is

$$1+\mu^{*}=\frac{1+\frac{H_{labor}}{\eta}}{1-\frac{1}{\theta}\left(H_{product}-\lambda_{inter}\right)\left(1-\frac{1}{N}\right)}.$$

We show the following: the equilibrium markdown of real wages μ^{*} is (1) increasing in $\widetilde{\phi}$, (2) for $\phi+\widetilde{\phi}<1$ decreasing in J; if $\phi+\widetilde{\phi}=1$ constant as function of J, (3) decreasing in the elasticity of labor supply η, and (4) for $\phi<1$ decreasing in the elasticity of substitution among goods by consumers θ; otherwise constant as function of θ.

Consider the first point. From the Lemma we know that λ_{intra} and λ_{inter} are increasing in $\widetilde{\phi}$ and, thus, so is H_{labor}. We also have:

$$\frac{\partial(H_{product}-\lambda_{inter})}{\partial\widetilde{\phi}}=\frac{J-1}{J}\frac{\partial\lambda_{intra}}{\partial\widetilde{\phi}}-\frac{\partial\lambda_{inter}}{\partial\widetilde{\phi}}$$

We can check that its sign is given by:

$$\begin{aligned}\operatorname{sgn}\left\{\frac{\partial(H_{product}-\lambda_{inter})}{\partial\widetilde{\phi}}\right\}&=\operatorname{sgn}\left\{\begin{array}{c}\frac{J-1}{J}\left(1-\phi-\widetilde{\phi}\right)2JN\left[(1-\phi)^{2}N+(2-\phi)\phi\right]\\-\left(1-\phi-\widetilde{\phi}\right)2N(J-1)\left[(2-\phi)\phi+\left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N\right]\end{array}\right\}\\&=\operatorname{sgn}\left\{\left(1-\phi-\widetilde{\phi}\right)\left[(1-\phi)^{2}N-\left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N\right]\right\}\\&=\operatorname{sgn}\left\{\left(1-\phi-\widetilde{\phi}\right)^{3}\right\}\end{aligned}$$

which is positive for $\left(1-\phi-\widetilde{\phi}\right)>0$, so $(H_{product}-\lambda_{inter})$ is increasing in $\widetilde{\phi}$. Also, $(H_{product}-\lambda_{inter})\leq 1$ (it is equal to 1 for $\widetilde{\phi}=1$), so in the fraction in the expression of $\mu^{*}>0$ the numerator is increasing and the denominator is decreasing in $\widetilde{\phi}$, so μ^{*} is increasing in $\widetilde{\phi}$.

Now consider the second point. Examine $H_{product} - \lambda_{inter}$:

$$
\begin{aligned}
H_{product} - \lambda_{inter} &= \frac{1+\lambda_{intra}(J-1)}{J} - \lambda_{inter} = \frac{1 + \dfrac{(J-1)\left[(2-\phi)\phi + \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N\right]}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)}}{J} - \lambda_{inter} \\
&= \frac{(1-\phi)^2 JN + J(2-\phi)\phi}{J\left[(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\right]\widetilde{\phi}N(J-1)} - \lambda_{inter} \\
&= \frac{(1-\phi)^2 N}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)}
\end{aligned}
$$

which is decreasing in J as long as $1-\phi-\widetilde{\phi} > 0$, since $\left((1-\phi)^2 - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}\right) = \left(1-\phi-\widetilde{\phi}\right)$; if $\phi + \widetilde{\phi} = 1$, then $H_{product} - \lambda_{inter}$ is constant in J.

Consider now H_{labor}:

$$
\begin{aligned}
H_{labor} &= \frac{1+\lambda_{intra}(J-1)+\lambda_{inter}(N-1)J}{NJ} = \frac{1}{N}\left[\frac{1+\lambda_{intra}(J-1)}{J} + (N-1)\lambda_{inter}\right] \\
&= \frac{1}{N}\left[\frac{(1-\phi)^2 N + (2-\phi)\phi}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)} + (N-1)\lambda_{inter}\right] \\
&= \frac{1}{N}\frac{(1-\phi)^2 N + (2-\phi)\phi + (N-1)(2-\phi)\phi}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)} \\
&= \frac{1}{(1-\phi)^2 JN + (2-\phi)\phi - \left[2(1-\phi)-\widetilde{\phi}\right]\widetilde{\phi}N(J-1)}
\end{aligned}
$$

which is decreasing in J as long as $1-\phi-\widetilde{\phi} > 0$; otherwise constant in J. We conclude that if $1-\phi-\widetilde{\phi} > 0$ the numerator and the denominator in the fraction in the expression of μ^* are decreasing and increasing in J, respectively, and if $\phi + \widetilde{\phi} = 1$, they are both constant as functions of J. Thus, if $1-\phi-\widetilde{\phi} > 0$, the equilibrium markdown is decreasing in J; otherwise it does not change with J.

Points (3) and (4) are straightforward given that $H_{product} - \lambda_{inter} \leq 1$ always, $H_{product} - \lambda_{inter} > 0$ for $\phi < 1$, and $H_{labor} > 0$ always.

We check now that when $\tilde{\phi} = 0$, $\mu^*$is nonmonotone in ϕ and N. We have that

$$
\frac{\partial \log\left(1+\mu^*\right)}{\partial \phi} = \left\{\frac{\frac{1}{\eta}\left(1-\frac{1}{JN}\right)}{1+\frac{1}{\eta}\left[\frac{1}{JN}+\lambda\left(1-\frac{1}{JN}\right)\right]} - \frac{\frac{1}{\theta J}\left(1-\frac{1}{N}\right)}{1-\frac{1-\lambda}{\theta J}\left(1-\frac{1}{N}\right)}\right\}\frac{\partial \lambda}{\partial \phi}.
$$

This is negative whenever $\frac{\theta J}{1-\frac{1}{N}} - 1 < \frac{\eta}{1-\frac{1}{JN}} + \frac{1}{JN-1}$ or $\theta\left(JN-1\right) < \left(1+\eta\right)\left(N-1\right)$.□

PROOF OF PROPOSITION 5: We have that

$$
\frac{\partial^2 \log L^*}{\partial(1-\phi)\partial J} = \frac{1}{\frac{1}{\eta}-(\alpha-1)}\frac{\frac{1}{\eta}\frac{\partial^2 H}{\partial(1-\phi)\partial J}\left(1+\frac{H}{\eta}\right) - \frac{1}{\eta^2}\frac{\partial H}{\partial(1-\phi)}\frac{\partial H}{\partial J}}{\left(1+\frac{H}{\eta}\right)^2} > 0
$$

since $\operatorname{sgn}\left\{\frac{\partial^2 H}{\partial(1-\phi)\partial J}\left(1+\frac{H}{\eta}\right)-\frac{1}{\eta}\frac{\partial H}{\partial(1-\phi)}\frac{\partial H}{\partial J}\right\} = \operatorname{sgn}\left\{-\left(1-\frac{1}{J}\right)(1-\lambda)\frac{\partial\lambda}{\partial(1-\phi)}\right\}$, which is positive for $J > 1$ since $\frac{\partial\lambda}{\partial(1-\phi)} < 0$. $\square$