



## Management Science

Publication details, including instructions for authors and subscription information:  
<http://pubsonline.informs.org>

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To cite this article:

Jordi Mondria, Xavier Vives, Liyan Yang (2021) Costly Interpretation of Asset Prices. *Management Science*

Published online in Articles in Advance 12 Feb 2021

. <https://doi.org/10.1287/mnsc.2020.3871>

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# Costly Interpretation of Asset Prices

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Received: January 21, 2020

Revised: July 25, 2020

Accepted: September 10, 2020

Published Online in Articles in Advance:  
February 12, 2021

<https://doi.org/10.1287/mnsc.2020.3871>

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**Abstract.** We propose a model in which investors cannot costlessly process information from asset prices. At the trading stage, investors are boundedly rational, and their interpretation of prices injects noise into the price, generating a source of endogenous noise trading. Our setup predicts price momentum and yields excessive return volatility and excessive trading volume. In an overall equilibrium, investors optimally choose sophistication levels by balancing the benefit of beating the market against the cost of acquiring sophistication. There can exist strategic complementarity in sophistication acquisition, leading to multiple equilibria.

**History:** Accepted by Gustavo Manso, finance.

**Funding:** J. Mondria and L. Yang gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada [Grants 430-2018-00173, 435-2015-0655, 435-2016-0046]. X. Vives acknowledges financial support of the Ministry of Science, Innovation and Universities with [Project PGC2018-096325-B-I00 (MCIU/AEI/FEDER, UE)]. Support is also acknowledged from the Spanish Ministry of Economy and Competitiveness [Grant ECO2015-63711-P].

**Keywords:** investor sophistication • price momentum • asset prices • complementarity

## 1. Introduction

Data can be viewed as information only after it has been analyzed. Interpreting data is often costly in terms of effort and other resources, which is particularly true for market data given the complexity of modern financial markets. In the existing frameworks—such as the noisy rational expectations equilibrium (REE) model (e.g., Grossman and Stiglitz 1980, Hellwig 1980, Vives 2008) and the recent REE-disagreement hybrid models (e.g., Banerjee 2011)—investors perfectly comprehend the price function and, thus, can costlessly read into the asset price to uncover value-relevant information. Apparently, such an argument requires a high degree of sophistication on the part of market participants.<sup>1</sup> What if interpreting price information is costly and investors commit errors in the inference process? How to determine the sophistication levels of investors in interpreting asset prices? How does investor sophistication affect market prices and trading volume? In this paper, we propose a behavioral model to address these questions.

In our model, a continuum of investors interacts with each other in two periods ( $t = 0$  and 1). At date 1, investors trade on private information in a financial market. As in the standard REE, the asset price aggregates information, and investors make inferences from the price. However, at the trading stage, investors are boundedly rational and do not fully understand how to read information from prices. A fully

sophisticated investor would extract the best signal possible from the price (which is endogenously determined in equilibrium), and a less sophisticated investor introduces noise in interpreting the price. After investors form their beliefs based on the personalized price signals, they behave as Bayesian given their own beliefs and optimize accordingly. Through market clearing, investors' optimal asset demands, in turn, endogenously determine the equilibrium price function and, hence, the best price signal (i.e., the "truth" in investors' personalized price signals).

At date 0, investors optimally choose their sophistication levels to maximize ex ante expected utilities understanding that, at date 1, they are boundedly rational in reading information from prices. On the one hand, increasing sophistication reduces the bounded rationality at the later trading stage, which, therefore, benefits investors ex ante. On the other hand, acquiring sophistication is costly. For instance, if we think of investors as individual investors, then, in order to become more sophisticated, investors may need better education/training (which costs wealth) or simply need to think harder (which is involved with mental costs). The optimal sophistication level is determined by balancing the benefit from reduced bounded rationality against the cost of sophistication acquisition.

We first analyze the equilibrium in the date 1 financial market, which can be viewed as an REE extended with bounded rationality. We find that costly

price interpretation can inject noise into the price system. Specifically, in our setting, the equilibrium price is a linear function of the asset fundamental and a noise term. The fundamental term comes from aggregating investors' private value-relevant information, which is the root reason why investors care to learn from the price. The noise term in the price arises from a common error in investors' personalized price signals, which is meant to capture the idea that, in processing price data, investors may suffer a common cognitive error (such as "sentiment" or "misperception"). Compared with the standard REE, costly interpretation of prices leads to price momentum (future returns depend positively on the current price), excessive return volatility, and excessive trading volume, which is consistent with the existing empirical evidence (e.g., Jegadeesh and Titman 1993, Moskowitz et al. 2012 on momentum, Shiller 1981, LeRoy and Porter 1981 on excess volatility, and Odean 1999, Barber and Odean 2000 on excessive trading).

After analyzing the date 1 financial market equilibrium, we return to date 0 and examine how sophistication levels are determined in an overall equilibrium. The incentive to acquire sophistication comes primarily from beating the average sophistication level across the market, which allows the investor to interpret the price better and trade better (i.e., more likely to buy low and sell high). We analyze two types of sophistication choices by investors: (1) sophistication levels to tame the exposure to the common sentiment and (2) sophistication levels to curb idiosyncratic errors of processing prices. We find that sentiment sophistication choice can exhibit strategic complementarity, leading to the possibility of multiple equilibria.

We then extend our setting to allow exogenous noise trading. In this extension, returns can exhibit either momentum or reversal, depending on the size of the exogenous noise trading. This is due to the interaction between two forces. First, exogenous noise trading tends to cause returns to exhibit reversal: large noise demand pushes prices too high, and thus, a high price predicts a future price decline (see Banerjee et al. 2009, Vives 2008, chapter 4). Second, costly interpretation of prices leads to underreaction of prices to information, giving rise to price momentum. The second effect dominates if and only if the size of noise trading is small. Nonetheless, independent of the size of noise trading, acquiring sophistication about taming sentiment can always lead to strategic complementarity and multiple equilibria. Other results, such as excess trading volume and complementarity in sophistication choice, remain robust in this setting extended with exogenous noise trading.

We finally compare our model to four alternative existing theories that are conceptually related to our story: supply information (Ganguli and Yang 2009),

dismissive traders (Banerjee 2011), cursed traders (Eyster et al. 2019), and operation risk (Basak and Buffa 2019). We show that supply information and operation risk deliver reversal, and dismissiveness and cursedness can deliver price momentum. Relative to dismissiveness and cursedness, our theory predicts different patterns on other variables, such as trading volume and price informativeness (see next section for details).

## 2. Related Literature

A recent literature explores environmental complexity that makes agents fail to account for the informational content of other players' actions in game settings. Eyster and Rabin (2005) develop the concept of "cursed equilibrium," which assumes that each player correctly predicts the distribution of other players' actions but underestimates the degree to which these actions are correlated with other players' information. Esponda and Pouzo (2016) propose the concept of "Berk–Nash equilibrium" to capture that people can have a possibly mis-specified view of their environment. Although these models are cast in a game-theoretical framework, the spirit of our financial market model is similar. In our model, investors' interactions are mediated by an asset price, which can be viewed as a summary statistic for all the other players' actions.

Eyster et al. (2019) have applied the cursed equilibrium concept to a financial market setting. In their setting, an investor is a combination of a fully rational REE investor (who correctly reads information from the price) and a naive Walrasian investor (who totally neglects the information in the asset price). Thus, the notion of "cursedness" in Eyster et al. (2019) is conceptually related to the notion of "sophistication/attention" in our setting because both notions aim to capture the fact that investors sometimes partially ignore information contained in asset prices.<sup>2</sup> Our paper complements Eyster et al. (2019) in four important ways. First, their central results refer to explaining trading volume. For example, they show that, as the number of traders diverges to infinity, the total trading volume goes to infinity in their framework. Instead, at the trading stage, we conduct a comparative static analysis with respect to investors' sophistication level rather than with respect to the number of traders. This exercise allows us to compare our setting to a fully REE benchmark. Moreover, this exercise helps to differentiate our framework from Eyster et al. (2019) in terms of testable volume predictions. Eyster et al. (2019) predict that trading volume always increases with cursedness (i.e., decreases with sophistication). By contrast, volume is either increasing or hump-shaped in sophistication in our setting. Second and more importantly, our setting has an extra stage to determine the equilibrium level of investor sophistication,

which generates novel theory results, such as strategic complementarity in sophistication acquisition.<sup>3</sup> In contrast, Eyster et al. (2019) do not explore these issues. We have analyzed a setting with endogenous cursedness and a common error, and our analysis suggests that such a setting fails to deliver complementarity in the cursedness choice. Third, another testable difference between the models is regarding price informativeness. Our analysis shows that price informativeness is nonmonotonic with sophistication although price informativeness is independent of cursedness. Fourth, our analysis incorporates a common error in interpreting prices, which generates a form of endogenous noise trading. This result leads to the complementarity result in the sophistication-acquisition stage, and again, these features are absent in Eyster et al. (2019).

Banerjee et al. (2009) and Banerjee (2011) combine REE and disagreement frameworks to allow investors to underestimate the precision of other investors' private information (hence, labeled as "Dismissiveness" models). A dismissive investor can be roughly viewed as a combination of a fully sophisticated and a naive agent and, thus, conceptually related to our investors at the trading stage. As we show in Section 7, in a dismissiveness model, returns exhibit momentum only when investors' risk aversion is sufficiently small. In addition, dismissiveness and our model deliver different volume and price informativeness predictions. Dismissive volume decreases with sophistication, and in our setting, volume is either increasing or hump-shaped in sophistication. Price informativeness does not depend on dismissiveness although it is nonmonotonic in our setting. Basak and Buffa (2019) consider models of operation risk, in which investors add idiosyncratic noise into their optimal demand. In Section 7, we explore a setting of operation risk extended with common noise and show that this type of model delivers return reversals.

Banerjee et al. (2019) propose a framework to allow investors to choose their beliefs and study when some empirically relevant behavioral biases, such as overconfidence and dismissiveness, naturally arise in equilibrium. To the extent that behavioral biases are negatively related to sophistication, this model is conceptually related to ours. Nonetheless, the trade-offs and mechanisms in the two models are different and complementary. In our setting, investors would like to be fully sophisticated, but this is infeasible because of the exogenous cost. In contrast, in Banerjee et al. (2019), investors often optimally choose not to be fully sophisticated because wrong beliefs benefit investors by offering anticipated utility.

### 3. A Model of Costly Interpretation of Asset Prices

#### 3.1. Setup

We consider an economy with three dates,  $t = 0, 1$ , and 2. At  $t = 1$ , two assets are traded in a competitive market: a risk-free and a risky asset. The risk-free asset has a constant value of one and is in unlimited supply. The risky asset is traded at an endogenous price  $\tilde{p}$  and is in zero supply. It pays an uncertain cash flow  $\tilde{v}$  at date 2, when  $\tilde{v} \sim N(0, \tau_v^{-1})$  with  $\tau_v > 0$ .

There is a continuum  $[0, 1]$  of investors who derive expected utility from their date 2 wealth. They have constant absolute risk aversion (CARA) utility with a risk-aversion coefficient of  $\gamma$ . Investors have fundamental information and trade on it. Specifically, prior to trading, investor  $i$  is endowed with the following private signal about the risky asset payoff  $\tilde{v}$ :

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N(0, \tau_\varepsilon^{-1}), \quad (1)$$

where  $\tau_\varepsilon > 0$ . We refer to  $\tilde{v}$  as "fundamentals."

Each investor has two selves, self 0 and self 1, who make decisions at dates 0 and 1, respectively. Self 0 does not observe any of the realizations in the later date 1 financial market but, at date 0, builds an understanding of how the market works. Self 0 is fully rational and instructs self 1 to extract fundamental information from the price using the best signal  $\tilde{s}_p = c_0 + c_1 \tilde{p}$ , where  $c_0$  and  $c_1$  are endogenous constants (that are implied by the equilibrium price function). Self 1, however, is boundedly rational and adds receiver noise to the intercept  $c_0$  of the price signal  $\tilde{s}_p$ .<sup>4</sup> Specifically, self 1 adds noise to  $\tilde{s}_p$  and observes

$$\tilde{s}_{p,i} = \left( c_0 + \frac{1}{\sqrt{\tau_{u,i}}} \tilde{u} + \frac{1}{\sqrt{\tau_{e,i}}} \tilde{\varepsilon}_i \right) + c_1 \tilde{p}, \quad (2)$$

where  $\tilde{u} \sim N(0, 1)$  is an error common for all investors,  $\tilde{\varepsilon}_i \sim N(0, 1)$  is an individual processing error, and  $(\tilde{v}, \{\tilde{\varepsilon}_i\}, \tilde{u}, \{\tilde{\varepsilon}_i\})$  are mutually independent.

We follow Angeletos and La'O (2013) and refer to the common error  $\tilde{u}$  as "sentiment," which captures aggregate extrinsic movements in agents' expectations. Angeletos and La'O (2013) consider a bilateral trading setting in which each player receives a noisy signal about its trading partner's private information. The noise term in the received signal takes the form of a common component plus an idiosyncratic component, and the common component is defined as a sentiment shock. In our setting, each investor trades with all other investors (and noise traders in Section 6); the trading process summarizes the aggregate information of investor  $i$ 's trading partners as  $\tilde{s}_p = c_0 + c_1 \tilde{p}$ , and investor  $i$  observes this information with noise

$\frac{1}{\sqrt{\tau_{u,i}}}\tilde{u} + \frac{1}{\sqrt{\tau_{e,i}}}\tilde{e}_i$ , where the common component  $\tilde{u}$ , therefore, corresponds to sentiment. Recently, Dávila and Parlatore (2021) adopt a similar notion of sentiment in studying trading costs and market efficiency. In practical terms,  $\tilde{u}$  represents those factors, such as investment mood, weather, and seasonal affective disorder, that affect investors' understanding of asset markets.<sup>5</sup>

At date 0, self 0 of investor  $i$  chooses loadings  $\frac{1}{\sqrt{\tau_{u,i}}}$  and  $\frac{1}{\sqrt{\tau_{e,i}}}$  to  $\tilde{u}$  and  $\tilde{e}_i$ , taking into account a cost function  $C(\tau_{u,i}, \tau_{e,i})$ . When these loadings are small, self 1 adds little noise in the price interpretation, and investors are sophisticated. Therefore, these loadings (inversely) measure investors' sophistication levels in handling sentiment error and handling individual processing error. We assume that investors can separately control the precision of common and idiosyncratic errors. Investors could tame the sentiment/common noise by learning about the history of trade episodes, bubbles, and mistakes done by following the crowd. For instance, to refine their technical analysis on assets, investors could follow the crowd by analyzing Twitter data or the investor expectation surveys (e.g., Shiller 2000, Greenwood and Shleifer 2014). In addition, investors could tame idiosyncratic noise by using self-control techniques that filter personal emotional views (e.g., delegating portfolio choices to robo advisers as a self-control device).

At date 1, self 1 of investor  $i$  chooses the investment  $D_i$  in the risky asset to maximize the subjective expected utility of ex post wealth

$$\tilde{W}_i = (\tilde{v} - \tilde{p})D_i - C(\tau_{u,i}, \tau_{e,i}), \quad (3)$$

where we normalize investor  $i$ 's initial wealth level to zero and take  $\tau_{u,i}$  and  $\tau_{e,i}$  as given. Self 1 is a price taker who infers information from the price  $\tilde{p}$  although adding noise in the inference process.

Our treatment of the two selves is standard and consistent with the bounded-rationality literature (e.g., Berk-Nash equilibrium, Espóneda and Pouzo 2016; optimal expectations, Brunnermeier and Parker 2005; constrained-rational expectations, Molavi et al. 2020).<sup>6</sup> In these existing models, the optimal action at date 1 is made under the subjective belief of self 1, and then, self 0 at date 0 chooses future self 1's belief by evaluating some well-being measure computed under the true probability. Our theory models a situation in which it is costly to learn from prices, and thus, it is costly for an investor to be fully rational at the trading stage, which captures the behavioral constraints on the complexity of investors' statistical models. We then follow the existing bounded-rationality literature and use self 0's behavior to discipline the subjective belief of self 1.

### 3.2. Timeline and Equilibrium Concept

The timeline of our economy is as follows:

$t = 0$ : Self 0 of each investor chooses  $\tau_{u,i}$  and  $\tau_{e,i}$  to maximize ex ante utility.

$t = 1$ : Self 1 is boundedly rational in reading information from the price. Self 1 of each investor receives the private fundamental signal  $\tilde{s}_i$  and the price signal  $\tilde{s}_{p,i}$  and submits demand schedules. This implies that the demand schedule is  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$ . Market clears at price  $\tilde{p}$ .

$t = 2$ : Asset payoff  $\tilde{v}$  is realized, and investors get paid and consume.

The overall equilibrium in our model is composed of a date 1 trading equilibrium in the financial market and a date 0 sophistication determination equilibrium. In the date 1 financial market equilibrium, self 1 of each investor maximizes the conditional subjective expected utility, and the asset market clears for given sophistication levels  $\tau_{u,i}$  and  $\tau_{e,i}$ . This equilibrium determines the price function and, hence, the best price signal  $\tilde{s}_p$ . In the sophistication determination stage, self 0 of each investor optimally chooses the sophistication levels  $\tau_{u,i}$  and  $\tau_{e,i}$  to maximize the investor's ex ante expected utility, taking into account future equilibrium demands. In Section 4, we first consider a financial market equilibrium taking investors' sophistication levels  $\{\tau_{u,i}\}_{i \in [0,1]}$  and  $\{\tau_{e,i}\}_{i \in [0,1]}$  as given. In Section 5, we then deal with the overall equilibrium and the determination of sophistication levels.

## 4. Financial Market Equilibrium

At date 1, taking as given the sophistication levels  $\tau_{u,i}$  and  $\tau_{e,i}$ , self 1 of each investor  $i$  chooses investment  $D_i$ , that is, a demand function  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  to maximize

$$E\left[-e^{-\gamma(\tilde{v}-\tilde{p})D_i}|\tilde{s}_i, \tilde{s}_{p,i}\right]. \quad (4)$$

The CARA-normal setting implies that investor  $i$ 's demand for the risky asset is

$$D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) = \frac{E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}) - \tilde{p}}{\gamma \text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})}, \quad (5)$$

where  $E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  and  $\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  are the conditional expectation and variance of  $\tilde{v}$  given information  $\{\tilde{s}_i, \tilde{s}_{p,i}\}$ . In (5), we have explicitly incorporated  $\tilde{s}_{p,i}$  in the demand function to reflect the informational role of the price (i.e., the price helps to predict  $\tilde{v}$ ) and used  $\tilde{p}$  to capture the substitution role of the price (i.e., a higher price directly leads to a lower demand). Here, the dependence of  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  on the price  $\tilde{p}$  in (5) reflects the fact that the investor knows that purchasing one unit of the asset costs  $\tilde{p}$ , and learning on fundamentals operates through the private signal  $\tilde{s}_{p,i}$  or "price interpretation."

The financial market clears as follows:

$$\int_0^1 D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) di = 0 \text{ almost surely.} \quad (6)$$

This market-clearing condition, together with demand function (5), determines an equilibrium price function

$$\tilde{p} = p(\tilde{v}, \tilde{u}), \quad (7)$$

where  $\tilde{v}$  and  $\tilde{u}$  come from the aggregation of signals  $\tilde{s}_i$  and  $\tilde{s}_{p,i}$ . In equilibrium, price function (7) endogenously determines the informational content in the best signal  $\tilde{s}_p$ .

A financial market equilibrium for given sophistication levels  $\{\tau_{u,i}\}_{i \in [0,1]}$  and  $\{\tau_{e,i}\}_{i \in [0,1]}$  is characterized by a price function  $p(\tilde{v}, \tilde{u})$  and demand function  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  such that

a.  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  is given by (5), which maximizes investors' conditional subjective expected utilities given their date 1 beliefs.

b. The market clears almost surely; that is, Equation (6) holds.

c. Investors' date 1 beliefs are given by (1) and (2), where  $\tilde{s}_p$  in (2) is implied by the equilibrium price function  $p(\tilde{v}, \tilde{u})$ .

#### 4.1. Equilibrium Construction

We consider a linear price function as follows:

$$\tilde{p} = a_v \tilde{v} + a_u \tilde{u}, \quad (8)$$

where the coefficients  $a_v$  and  $a_u$  are endogenous. By Equation (8), provided that  $a_v \neq 0$  (which is true in equilibrium), the best price signal that a fully sophisticated investor can achieve is

$$\tilde{s}_p = \frac{\tilde{p}}{a_v} = \tilde{v} + \alpha \tilde{u} \text{ with } \alpha \equiv \frac{a_u}{a_v}. \quad (9)$$

Note that the first equation in (9) indicates that the best price signal  $\tilde{s}_p$  is linear in  $\tilde{p}$ ; that is,  $\tilde{s}_p = c_0 + c_1 \tilde{p}$  with coefficients  $c_0 = 0$  and  $c_1 = 1/a_v$ . Conceptually, self 0 knows how to compute the equilibrium and knows  $(c_0, c_1, \alpha)$  but cannot impose a demand function on self 1. Self 1 is boundedly rational and adds receiver noise to the best price signal. Thus, investor  $i$  cannot costlessly process the price information and can only read a coarser signal as follows:

$$\begin{aligned} \tilde{s}_{p,i} &= \left( c_0 + \frac{1}{\sqrt{\tau_{u,i}}} \tilde{u} + \frac{1}{\sqrt{\tau_{e,i}}} \tilde{e}_i \right) + c_1 \tilde{p} \\ &= \tilde{v} + \alpha \tilde{u} + \frac{1}{\sqrt{\tau_{u,i}}} \tilde{u} + \frac{1}{\sqrt{\tau_{e,i}}} \tilde{e}_i. \end{aligned} \quad (10)$$

Using Bayes' rule, we can compute

$$\begin{aligned} E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}) &= \left( \tau_{e,i} \tilde{s}_i + \frac{\tau_{e,i} \tau_{u,i}}{\tau_{u,i} + \tau_{e,i} (\alpha \sqrt{\tau_{u,i}} + 1)^2} \tilde{s}_{p,i} \right) \\ &\quad \times \text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}), \end{aligned} \quad (11)$$

$$\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$$

$$= \frac{\tau_{u,i} + \tau_{e,i} (\alpha \sqrt{\tau_{u,i}} + 1)^2}{\tau_{e,i} \tau_{u,i} + [\tau_{u,i} + \tau_{e,i} (\alpha \sqrt{\tau_{u,i}} + 1)^2] (\tau_v + \tau_e)}. \quad (12)$$

Inserting these two expressions into (5), we can compute the expression of  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$ , which is, in turn, inserted into (6) to compute the equilibrium price as a function of  $\tilde{v}$  and  $\tilde{u}$ . Comparing coefficients with the conjectured price function (8), we can form a system of equations to determine the two unknown price coefficients  $a_v$  and  $a_u$ .

**Proposition 1** (Financial Market Equilibrium). *Suppose that investors have the same sophistication level (i.e.,  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$  for  $i \in [0, 1]$ ). There exists a unique linear equilibrium price function*

$$\tilde{p} = a_v \tilde{v} + a_u \tilde{u},$$

where

$$\begin{aligned} a_v &= \frac{\tau_e \tau_u + \tau_e [\tau_u + \tau_e (\alpha \sqrt{\tau_u} + 1)^2]}{\tau_e \tau_u + [\tau_u + \tau_e (\alpha \sqrt{\tau_u} + 1)^2] (\tau_v + \tau_e)}, \\ a_u &= \frac{\tau_e \sqrt{\tau_u} (\alpha \sqrt{\tau_u} + 1)}{\tau_e \tau_u + [\tau_u + \tau_e (\alpha \sqrt{\tau_u} + 1)^2] (\tau_v + \tau_e)}, \end{aligned}$$

where  $\alpha \equiv \frac{a_u}{a_v} \in (0, \frac{\tau_e \sqrt{\tau_u}}{\tau_e \tau_c + \tau_u \tau_e})$  is uniquely determined by the positive real root of the following cubic equation:

$$\tau_e \tau_u \tau_c \alpha^3 + 2\tau_e \sqrt{\tau_u} \tau_c \alpha^2 + (\tau_e \tau_c + \tau_u \tau_c) \alpha - \tau_e \sqrt{\tau_u} = 0. \quad (13)$$

Costly interpretation of asset prices brings an endogenous noise  $\tilde{u}$  into the price system. Even if investors were to get rid of the noise  $\tilde{u}$  in the price system by setting  $\tau_u \rightarrow \infty$ , investors still would not be fully sophisticated, and thus, they would not be able to extract the best signal from the price if  $\tau_e < \infty$ . In the limiting case in which both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$ , we have  $\tilde{s}_{p,i} = \tilde{s}_p$ , and the noise  $\tilde{u}$  vanishes in the price function, making the economy degenerate into the full REE setting.

**Corollary 1.** *Suppose  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$  for  $i \in [0, 1]$ . Given  $(\tau_v, \tau_e) \in \mathbb{R}_{++}^2$ ,*

1. Fix  $\tau_e \in (0, \infty)$  as  $\tau_u \rightarrow \infty$ ; then,  $a_v = \frac{\tau_e + \tau_c}{\tau_c + \tau_p + \tau_e}$  and  $a_u = 0$ .
2. Fix  $\tau_u \in (0, \infty)$  as  $\tau_e \rightarrow \infty$ ; then,  $a_v = \frac{\tau_u + \tau_c (\alpha \sqrt{\tau_u} + 1)^2}{\tau_u + (\alpha \sqrt{\tau_u} + 1)^2 (\tau_v + \tau_e)}$  and  $a_u = \frac{\sqrt{\tau_u} (\alpha \sqrt{\tau_u} + 1)}{\tau_u + (\alpha \sqrt{\tau_u} + 1)^2 (\tau_v + \tau_e)}$ .
3. As both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$ , the price function converges almost surely to  $\tilde{p}^{\text{REE}} = \tilde{v}$  and  $\tilde{s}_{p,i} = \tilde{s}_p$ .

#### 4.2. Price Momentum

We now present one main result of our model—price momentum—which says that the current price  $\tilde{p}$

positively predicts asset returns  $\tilde{v} - \tilde{p}$ . Empirically, one can run a linear regression from  $\tilde{v} - \tilde{p}$  on  $\tilde{p}$ , that is,  $\tilde{v} - \tilde{p} = \text{intercept} + m \times \tilde{p} + \text{error}$ . The regression coefficient is  $m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})}$ . In the traditional noisy REE setting with exogenous noise trading (e.g., Hellwig 1980), returns exhibit reversals; that is,  $m < 0$  (see Banerjee et al. 2009). This is because exogenous noise demand pushes the price too high, and exogenous noisy supply depresses the price too low. In contrast, in our setting with endogenous noise trading resulting from costly interpretation of prices, returns exhibit momentum:  $m > 0$ . This result provides an explanation for the price momentum documented in the data (e.g., Jegadeesh and Titman 1993, Moskowitz et al. 2012).

The price momentum in our model is an under-reaction story. When investors are fully sophisticated ( $\tilde{s}_{p,i} = \tilde{s}_p$ ), the price fully aggregates their private information, and there is no return predictability. Formally, by part 3 of Corollary 1, the price is a martingale ( $\tilde{p}^{REE} = E(\tilde{v}|\tilde{p}^{REE})$ ), and hence, the price change is not predictable ( $\text{Cov}(\tilde{v} - \tilde{p}^{REE}, \tilde{p}^{REE}) = 0$ ). When investors have limited sophistication, their forecasts do not fully use the information in the price, which, in turn, causes their trading not to fully incorporate information, thereby making the price underreact to information. Our mechanism shares similarity with Hong and Stein (1999) and Eyster et al. (2019), who generate momentum via traders that make forecasts based on private information but do not fully infer the information from prices. Because these investors fail to extract information from prices, the slow diffusion generates momentum.

**Proposition 2** (Return Predictiveness). *Returns exhibit price momentum:*

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} > 0.$$

### 4.3. Implications of Investor Sophistication

In this section, we examine how investor sophistication affects asset prices and trading volume. We assume that all investors have a common sophistication level  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$  and conduct comparative static analysis with respect to  $\tau_u$  and  $\tau_e$ . In a full equilibrium setting,  $\tau_{u,i}$  and  $\tau_{e,i}$  are determined endogenously, which is explored in Section 5.

**4.3.1. Sophistication Level  $\tau_u$  of Taming Sentiment.** The parameter values in Figure 1 follow from the calibration exercise conducted by Kovalenkov and Vives (2014). Specifically, we interpret one period as one year and let the total asset payoff volatility match its historical value 20% of the aggregate stock market (i.e.,  $\sqrt{\text{Var}(\tilde{v})} = 20\%$ ). Regarding private information quality, Kovalenkov and Vives (2014) consider a range

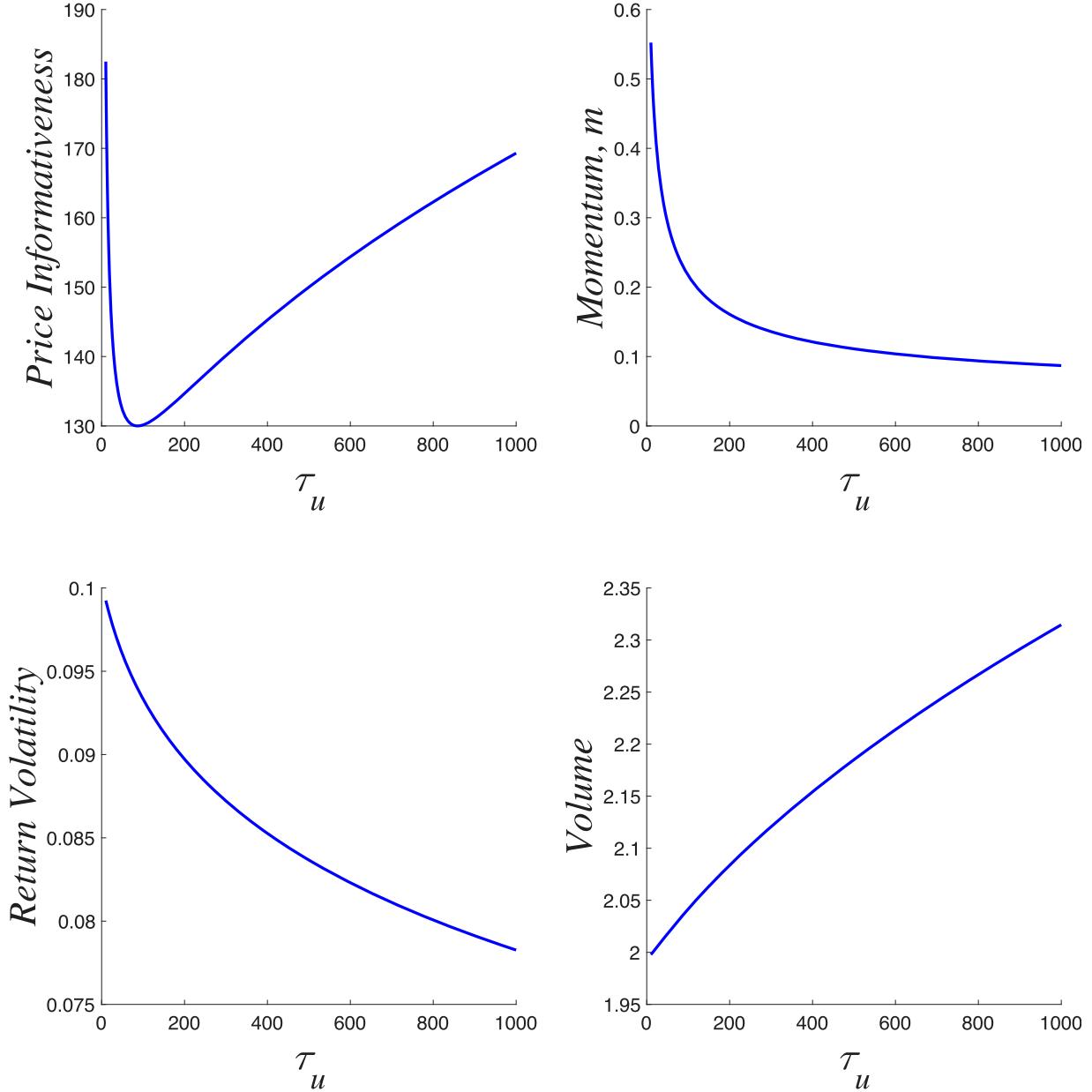
of signal-to-noise ratios  $\frac{\tau_e}{\tau_v}$  from as low as one basis point to as high as 16. We assume  $\frac{\tau_e}{\tau_v} = 1$ . We also follow Kovalenkov and Vives (2014) and set the risk-aversion coefficient  $\gamma$  at two. Finally, we assume that the sophistication level of curbing the individual processing error is constant at  $\tau_e = 500$ , which implies that investors make very small errors in forming their date 1 beliefs (i.e.,  $\text{Var}(\frac{1}{\sqrt{\tau_e}} \tilde{e}_t) = \frac{1}{\tau_e} = 0.2\%$ ). We do so deliberately to illustrate that even small errors in interpreting price information can aggregate into a significant effect on equilibrium outcomes.

**4.3.1.1. Price Informativeness.** As is standard in the literature (e.g., Vives 2008, Peress 2010), we use the precision  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$  of stock payoff conditional on its price to measure “price informativeness” (or “market efficiency,” “informational efficiency,” and “price efficiency”). By Equation (8), applying Bayes’ rule delivers  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \tau_v + \frac{1}{\alpha^2}$ . Because  $\tau_v$  is an exogenous constant, price informativeness is negatively related to  $\alpha$ .

In the top left panel of Figure 1, we plot price informativeness  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$  against the investor sophistication  $\tau_u$  of taming the sentiment error. When  $\tau_u = 0$  or  $\tau_e = 0$ , we have  $\alpha = 0$  and  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \infty$ . When either of these two cases occurs, traders do not observe a price signal. Hence, common noise  $u$  does not get incorporated into the price so that  $a_u = 0$  and  $a_v = \frac{\tau_e}{\tau_v + \tau_e}$ , which implies that  $\alpha = 0$  and  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \infty$ . Also, as a comparison, the  $\alpha$  value in the standard REE economy with both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  is zero. We observe (a) that costly interpretation of prices injects noise into the price as long as investors are not fully sophisticated (i.e.,  $\alpha > 0$  and  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} < \infty$  for  $\tau_u \in (0, \infty)$ ) and (b) that price informativeness  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$  is non-monotonic in sophistication  $\tau_u$ . Intuitively, if  $\tau_u$  increases from zero to some positive value, trading injects noise  $u$  into the price, lowering informativeness. However, if  $\tau_u$  continues to increase further to infinity, then the price becomes a more precise signal. This price informativeness result has important implications for determining the sophistication level in Section 5.

**4.3.1.2. Return Volatility.** Return volatility is measured by the standard deviation of asset returns,  $\sigma(\tilde{v} - \tilde{p})$ . In the bottom left panel of Figure 1, we plot return volatility  $\sigma(\tilde{v} - \tilde{p})$  against the investor sophistication  $\tau_u$  of taming the sentiment error. Return volatility in the standard REE economy with both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  is zero. Costly interpretation of prices generates higher return volatility than the full REE benchmark. This may help to address the volatility puzzle (LeRoy and Porter 1981, Shiller 1981), which states that it is difficult to explain the historical volatility of stock returns with any model in which investors are rational and discount rates are constant.

**Figure 1.** (Color online) Implications of  $\tau_u$  for Asset Prices and Trading Volume



*Notes.* This figure plots price informativeness ( $\frac{1}{\sqrt{\text{Var}(\tilde{v}|\tilde{p})}}$ ), return volatility ( $\sigma(\tilde{v} - \tilde{p})$ ), price momentum ( $m$ ), and trading volume against the investor sophistication  $\tau_u$  of taming the sentiment error. The parameters are set as follows:  $\tau_v = \tau_e = 25$ ,  $\tau_e = 500$ , and  $\gamma = 2$ .

Also, note that the excess return volatility is nonnegligible even though investors only make very small mistakes.

**4.3.1.3. Return Predictability.** In Proposition 2, we show that our model generates price momentum  $m > 0$ . In the top right panel of Figure 1, we plot  $m$  against the investor sophistication  $\tau_u$  of taming the sentiment error. Return predictability in the standard REE economy with both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  is zero. We observe that, consistent with Proposition 2,  $m$  is indeed positive, indicating that there exists price momentum in our economy. In addition,  $m$  is

decreasing in  $\tau_u$  as the information obtained from the price signal becomes more precise.

**4.3.1.4. Trading Volume.** In the bottom right panel of Figure 1, we plot trading volume  $\int_0^1 |D(p; s_i, s_{p,i})| di$  against  $\tau_u$ . Trading volume in the standard REE economy with both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  is zero. We see that costly price interpretation generates excess trading volume. Trading volume in the model is positive because traders have private information and do not fully learn all the information about fundamentals from prices. In our numerical exercise,

trading volume is increasing in  $\tau_u$ . Intuitively, a high  $\tau_u$  decreases the weight of the common sentiment component  $\tilde{u}$  in the price signal, which indirectly makes the individual processing error  $\tilde{e}_i$  relatively more important in the price signal. Hence, there is an increase in disagreement and an increase in volume as a result of an increase in  $\tau_u$ .

#### **Proposition 3** (Implications of $\tau_u$ ).

##### a. Price informativeness

1. For sufficiently small  $\tau_u$ , as investors become more sophisticated by increasing  $\tau_u$ , price  $\tilde{p}$  conveys less precise information about fundamental  $\tilde{v}$ .

##### b. Return volatility

1. As  $\tau_u \rightarrow \infty$ , return volatility approaches  $\sqrt{\frac{\tau_v}{(\tau_e + \tau_v + \tau_e)^2}}$ .

2. As  $\tau_u \rightarrow 0$ , return volatility approaches  $\sqrt{\frac{\tau_e^2 \tau_v}{(\tau_e \tau_v + \tau_e \tau_e)^2}}$ .

##### c. Trading Volume

1. As  $\tau_u \rightarrow \infty$ , the total trading volume does not vanish (i.e.,  $\lim_{\tau_u \rightarrow \infty} \text{Volume} > 0$ ).

2. As  $\tau_u \rightarrow 0$ , the total trading volume does not vanish (i.e.,  $\lim_{\tau_u \rightarrow 0} \text{Volume} > 0$ ).

#### **4.3.2. Sophistication Level $\tau_e$ of Curbing Individual Processing Error.**

In Figure 2, we analyze the effects of changes in  $\tau_e$  on asset prices and trading volume. The parameter values in Figure 2 are the same as Figure 1 with the exception that the sophistication level of taming sentiment is now fixed at  $\tau_u = 500$ , which implies small sentiment-driven errors in self 1 beliefs (i.e.,  $\text{Var}(\frac{1}{\sqrt{\tau_u}} \tilde{u}_i) = \frac{1}{\tau_u} = 0.2\%$ ).

**4.3.2.1. Price Informativeness.** In the top left panel of Figure 2, we plot price informativeness  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$  against the investor sophistication  $\tau_e$  of curbing individual processing errors. As a comparison, the  $\alpha$  value in the standard REE economy with both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  is zero, and hence,  $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \tau_v + \frac{1}{\alpha^2} = \infty$ . Price informativeness is monotonically decreasing in  $\tau_e$ . Intuitively, at  $\tau_e = 0$ , the price is extremely noisy, and investors completely disregard the price signal. Hence, the price is fully revealing. As  $\tau_e$  increases, investors start using the price signal, and the sentiment noise  $\tilde{u}$  is added to the price. At the same time, as  $\tau_e$  increases, the weight of sentiment  $\tilde{u}$  in the individual price signal increases relative to the weight of the individual processing error, making the signal more dependent on sentiment and, hence, the price more noisy.

**4.3.2.2. Return Volatility.** In the bottom left panel of Figure 2, we plot return volatility  $\sigma(\tilde{v} - \tilde{p})$  against the investor sophistication  $\tau_e$  of curbing individual processing errors. Return volatility in the standard REE economy is zero. Return volatility is nonmonotonic in  $\tau_e$ . As  $\tau_e$  increases, (a) traders have a more precise individual price signal, bringing the price closer to fundamentals

and reducing volatility, and (b) the weight of sentiment  $\tilde{u}$  in the individual price signal increases relative to the weight of the individual processing error, making the signal more dependent on sentiment and, hence, the price more noisy and further away from fundamentals.

**4.3.2.3. Return Predictability.** In Proposition 2, we show that our model generates momentum, that is,  $m > 0$ . In the top right panel of Figure 2, we plot  $m$  against  $\tau_e$ . Return predictability in the standard REE economy is zero. We observe that  $m$  is indeed positive, indicating that there exists price momentum in the economy. In addition,  $m$  is decreasing in  $\tau_e$  as the information obtained from the price signal becomes more precise.

**4.3.2.4. Trading Volume.** In the bottom right panel of Figure 2, we plot trading volume against  $\tau_e$ . Trading volume in the standard REE economy is zero. We see that costly price interpretation generates trading volume because prices are not fully revealing. In our numerical exercise, trading volume is nonmonotonic in  $\tau_e$ . Intuitively, an increase in  $\tau_e$  from  $\tau_e = 0$  provides traders with a price signal, which is an additional source of disagreement between investors, leading to an increase in trading volume. However, as  $\tau_e$  increases even further, the disagreement diminishes, and trading volume decreases as well.

#### **Proposition 4** (Implications of $\tau_e$ ).

##### a. Price informativeness

1. As investors become more sophisticated by increasing  $\tau_e$ , price  $\tilde{p}$  conveys less precise information about fundamental  $\tilde{v}$ .

##### b. Return volatility

1. As  $\tau_e \rightarrow \infty$ , return volatility approaches

$$\sigma(\tilde{v} - \tilde{p}) = \sqrt{\frac{(\alpha\sqrt{\tau_u} + 1)^2 (\tau_v \alpha^2 \tau_u + 2\tau_v \tau_v) \times \sqrt{\tau_u + \tau_u + \tau_v}}{(\tau_v + \tau_e + \tau_u + \alpha^2 \tau_u \tau_v + \alpha^2 \tau_u \tau_e)^2 + 2\alpha\sqrt{\tau_u} \tau_v + 2\alpha\sqrt{\tau_u} \tau_e}^2}$$

2. As  $\tau_e \rightarrow 0$ , return volatility approaches

$$\sqrt{\frac{\tau_u^2 \tau_v}{(\tau_u \tau_v + \tau_u \tau_e)^2}}$$

##### c. Trading volume

1. As  $\tau_e \rightarrow \infty$ , the total trading volume does not vanish (i.e.,  $\lim_{\tau_e \rightarrow \infty} \text{Volume} > 0$ ).

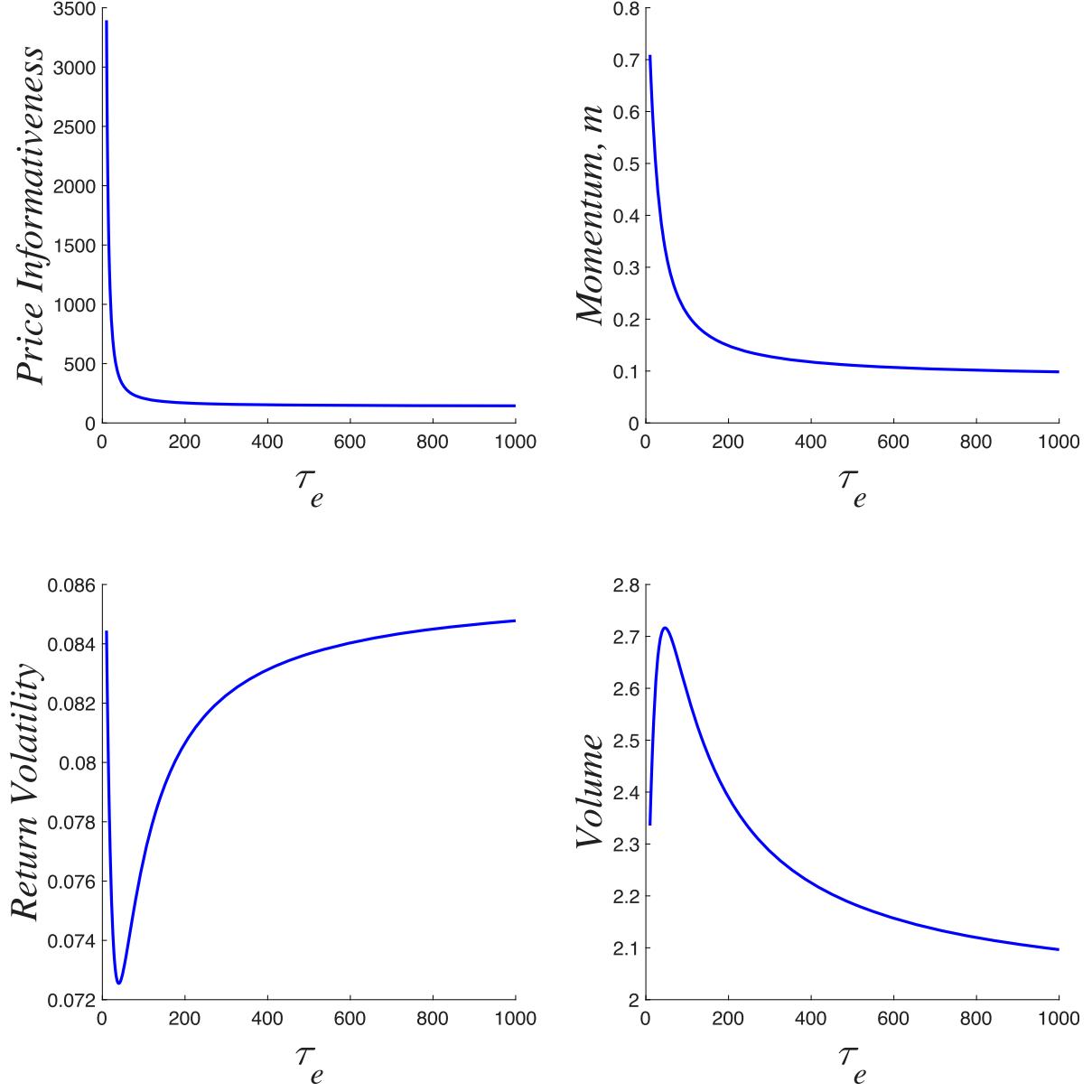
2. As  $\tau_e \rightarrow 0$ , the total trading volume does not vanish (i.e.,  $\lim_{\tau_e \rightarrow 0} \text{Volume} > 0$ ).

## **5. Sophistication Level Equilibrium**

### **5.1. Sophistication Determination**

As discussed in Section 3.1, the sophistication levels are determined by the rational self 0 of each investor

**Figure 2.** (Color online) Implications of  $\tau_e$  for Asset Prices and Trading Volume



*Notes.* This figure plots price informativeness ( $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$ ), return volatility ( $\sigma(\tilde{v} - \tilde{p})$ ), price momentum ( $m$ ), and trading volume against the investor sophistication  $\tau_e$  of curbing individual processing error. The parameters are set as follows:  $\tau_v = \tau_e = 25$ ,  $\tau_u = 500$ , and  $\gamma = 2$ .

at date 0. Inserting the date 1 demand function  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  in (5) into the CARA utility function and taking expectations yields investor  $i$ 's date 0 payoff,  $E[-e^{-\gamma}[(\tilde{v} - \tilde{p})D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) - C(\tau_{u,i}, \tau_{e,i})]]$ . Note that this expectation is computed under the correct distribution because self 0 is fully rational in contemplating the sophistication levels  $\tau_{u,i}$  and  $\tau_{e,i}$  of self 0's future self, which, in turn, determines how much information the boundedly rational self 1 reads from the asset price  $\tilde{p}$ . Formally, trader  $i$  chooses  $\tau_{u,i}$  and  $\tau_{e,i}$  to maximize

$$\max_{(\tau_{u,i}, \tau_{e,i})} E\left[-e^{-\gamma}[(\tilde{v} - \tilde{p})D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) - C(\tau_{u,i}, \tau_{e,i})]\right]. \quad (14)$$

**Definition 1.** An overall equilibrium of the two-stage game is defined as follows:

a. Financial market equilibrium at date 1 is characterized by a price function  $p(\tilde{v}, \tilde{u})$  and demand functions  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  such that

1.  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$  is given by (5), which maximizes investors' conditional subjective expected utilities given their date 1 beliefs.

2. The market clears almost surely; that is, Equation (6) holds.

3. Investors' date 1 beliefs are given by (11) and (12), in which  $\tilde{s}_p$  is implied by the equilibrium price function  $p(\tilde{v}, \tilde{u})$  and in which the sophistication levels  $(\tau_{u,i})_{i \in [0,1]}$  and  $(\tau_{e,i})_{i \in [0,1]}$  are determined at date 0.

b. Sophistication level equilibrium at date 0, which is characterized by sophistication levels  $(\tau_{u,i})_{i \in [0,1]}$  and  $(\tau_{e,i})_{i \in [0,1]}$  such that  $(\tau_{u,i}, \tau_{e,i})$  solves (14), in which investors' date 1 beliefs are given by (11) and (12).

## 5.2. Equilibrium Characterization

Given that investors are ex ante identical, we consider a symmetric equilibrium in which all investors choose the same sophistication level. Let  $W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)$  denote the certainty equivalent of investor  $i$ 's date 0 payoff when investor  $i$  decides to acquire sophistication levels  $\tau_{u,i}$  and  $\tau_{e,i}$  and all the other investors acquire sophistication levels  $\tau_u$  and  $\tau_e$ . That is,

$$\begin{aligned} W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e) \\ \equiv -\frac{1}{\gamma} \ln \left( -E \left[ -e^{-\gamma[(\tilde{v}-\tilde{p})D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) - C(\tau_{u,i}, \tau_{e,i})]} \right] \right) \\ = \frac{1}{2\gamma} \ln \left[ \underbrace{\left[ 1 + \gamma \frac{\text{Cov}(\tilde{v} - \tilde{p}, D_i)}{\text{more informed trading}} \right]^2 - \gamma^2 \text{Var}(\tilde{v} - \tilde{p})}_{\times \underbrace{\text{Var}(D_i)}_{\text{excessive trading}}} \right] \\ - \underbrace{C(\tau_{u,i}, \tau_{e,i})}_{\text{cost}}, \end{aligned} \quad (15)$$

where the second equality follows from the properties of normal distributions.

In Equation (15), improving the sophistication of future self 1 affects the current self 0's payoff in three ways. The first effect is a direct effect: acquiring sophistication incurs a cost,  $C(\tau_{u,i}, \tau_{e,i})$ , which directly harms the investor from self 0's perspective. The other two effects are indirect, which work through affecting the trading in the future financial market. These two indirect effects work in opposite directions.

First, being more sophisticated allows the future self 1 to better read information from the asset price, which, therefore, makes self 1's trading more aligned with price changes—that is, buying low and selling high—and, therefore, benefits the investor at date 0. This positive indirect effect is captured by the term  $\text{Cov}(\tilde{v} - \tilde{p}, D_i)$ . Intuitively, when  $\tau_{u,i} > \tau_u$  and  $\tau_{e,i} > \tau_e$ , investor  $i$ 's forecast beats the market, and thus, investor  $i$ 's trading improves investor  $i$ 's ex ante welfare.

Second, investors engage in speculative trading in the date 1 financial market because there are no risk-sharing benefits in our setting. In this sense, trading is excessive, and the more an investor's future self trades, the more harmful it is from self 0's perspective. Improving the sophistication levels  $\tau_{u,i} > \tau_u$  and  $\tau_{e,i} > \tau_e$  allows self 1 to lower self 1's perceived risk, and thus,

self 1 trades more aggressively, which, in turn, harms self 0 via the excessive trading channel. This negative effect is captured by the term  $\text{Var}(D_i)$  in Equation (15), which measures the size of self 1's trading in the financial market.

The optimal sophistication levels  $\tau_{u,i}^*$  and  $\tau_{e,i}^*$  are determined by the first-order conditions (FOCs) of maximizing  $W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)$ . Formally,  $(\tau_{u,i}^*, \tau_{e,i}^*) = \arg \max_{\tau_{u,i}, \tau_{e,i}} W(\tau_{u,i}, \tau_{e,i}; \tau_u^*, \tau_e^*)$ . The FOCs of the sophistication determination problem are given by

$$\begin{cases} \frac{\partial W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)}{\partial \tau_{u,i}} \Big|_{\tau_{u,i}=0} \leq 0, & \text{if } \tau_{u,i} = 0, \\ \frac{\partial W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)}{\partial \tau_{u,i}} = 0, & \text{if } \tau_{u,i} > 0, \end{cases} \quad (16)$$

and

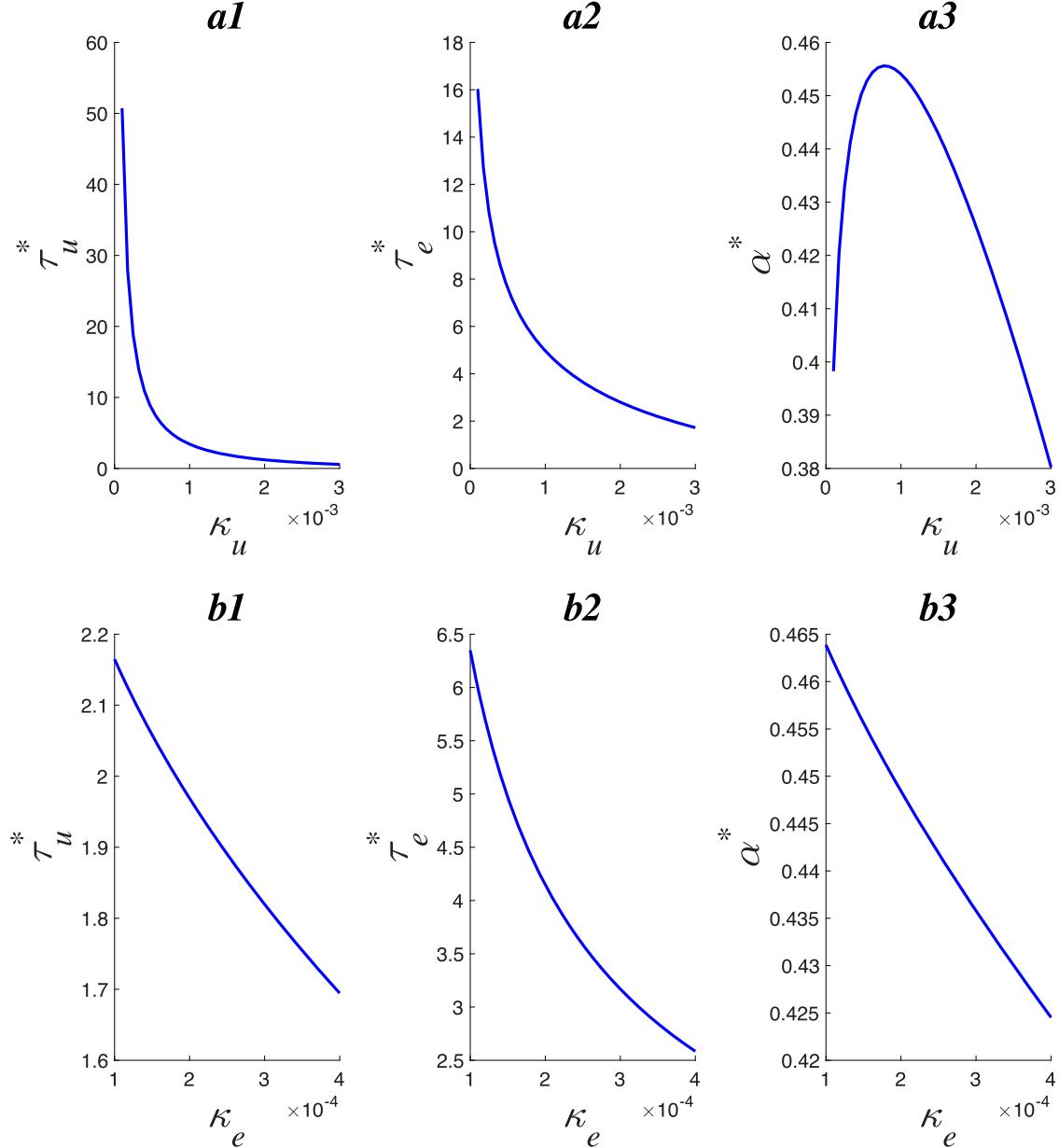
$$\begin{cases} \frac{\partial W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)}{\partial \tau_{e,i}} \Big|_{\tau_{e,i}=0} \leq 0, & \text{if } \tau_{e,i} = 0, \\ \frac{\partial W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)}{\partial \tau_{e,i}} = 0, & \text{if } \tau_{e,i} > 0. \end{cases} \quad (17)$$

In a symmetric equilibrium, we impose  $\tau_{u,i}^* = \tau_u^*$  and  $\tau_{e,i}^* = \tau_e^*$  in the FOCs. The overall equilibrium is characterized by three variables:  $(\tau_u^*, \tau_e^*, \alpha^*)$ . These three variables are jointly determined by a system of three conditions: (a) the cubic Equation (13) characterizing the financial market equilibrium, (b) the FOC (16) characterizing the optimal sophistication level  $\tau_u^*$  of taming the sentiment error, and (c) the FOC (17) characterizing the optimal sophistication level  $\tau_e^*$  of curbing individual processing errors.

The complexity of the problem precludes a full analytical characterization of the equilibrium. We, therefore, rely on numerical analysis. We consider a linear cost function,  $C(\tau_{u,i}, \tau_{e,i}) = \kappa_u \tau_{u,i} + \kappa_e \tau_{e,i}$ , where  $\kappa_u$  and  $\kappa_e$  are positive constants. This linear and separable cost function serves as a simple benchmark to understand the numerical results. Our model admits multiple equilibria. We use Figure 3 to plot the first equilibrium and Figure 4 to plot the second equilibrium. In Panels (a1)–(a3) of both figures, we plot the equilibrium values of  $(\tau_u^*, \tau_e^*, \alpha^*)$  as functions of  $\kappa_u$ , when  $\kappa_e$  is fixed at 0.0002. In Panels (b1)–(b3) of both figures, we vary  $\kappa_e$  and fix the value of  $\kappa_u$  at 0.0015. The other parameters take the same values as in previous figures, that is,  $\tau_v = 25$  and  $\gamma = 2$  except for  $\tau_e = 1$ . We set a smaller  $\tau_e$  because, under coarse private information, there are multiple equilibria at the sophistication determination stage. Numerically, we have only been able to find two equilibria.

The qualitative patterns of  $(\tau_u^*, \tau_e^*, \alpha^*)$  are similar across both figures, and thus, we focus on Figure 3. Intuitively, an increase in the cost  $\kappa_u$  of taming the sentiment error leads to a decrease in both sophistication levels  $\tau_u^*$  and  $\tau_e^*$  in Panels (a1) and (a2), and an increase in the cost  $\kappa_e$  of curbing individual

**Figure 3.** (Color online) The First Overall Equilibrium



*Notes.* This figure plots values of  $(\tau_u^*, \tau_e^*, \alpha^*)$  for the first equilibrium as functions of  $\kappa_u$  in the top panels and  $\kappa_e$  in the bottom panels. The cost function is  $C(\tau_{u,i}, \tau_{e,i}) = \kappa_u \tau_{u,i} + \kappa_e \tau_{e,i}$ , where  $\kappa_u$  and  $\kappa_e$  are positive constants. The parameters are set as follows:  $\tau_v = 25$ ,  $\tau_e = 1$ , and  $\gamma = 2$ .

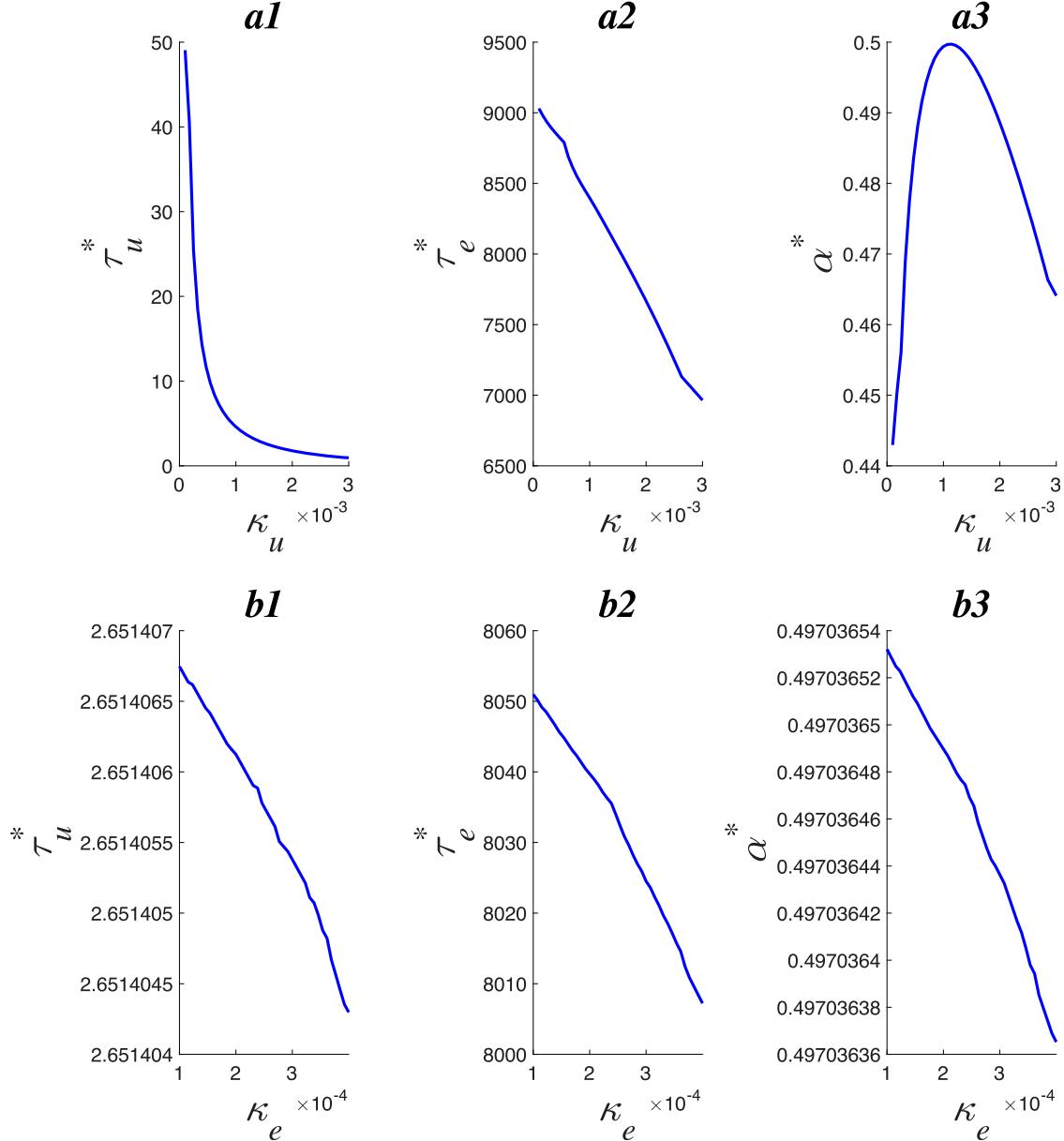
processing errors leads to a decrease in both  $\tau_u^*$  and  $\tau_e^*$  in Panels (b1) and (b2). However, Panels (a3) and (b3) exhibit different patterns of  $\alpha^*$ . Specifically, in Panel (a3),  $\alpha^*$  is nonmonotonic in  $\kappa_u$ , and in Panel (b3),  $\alpha^*$  is monotonically decreasing in  $\kappa_e$ . This difference is caused by the different behavior of  $\tau_u^*$ : in Panel (a1), changes in  $\kappa_u$  have a big impact on the choice of  $\tau_u^*$ , and as we have seen in Figure 1, price informativeness is nonmonotonic; by contrast, changes in  $\kappa_e$  only cause modest variations in  $\tau_u^*$  in Panel (b1), and hence, price informativeness is monotonic.

### 5.3. What Drives Equilibrium Multiplicity?

In this section, we dig deeper into the driving forces of multiple equilibria by distinguishing the choice of sophistication  $\tau_{u,i}$  of taming the sentiment error from the choice of sophistication  $\tau_{e,i}$  of curbing individual processing errors.

**5.3.1. The Choice of  $\tau_u$ .** We now take  $\tau_{e,i} = \tau_e$  as given and focus on the choice of  $\tau_{u,i}$ . Because the cost function  $C(\tau_{u,i}, \tau_{e,i})$  is exogenous, we focus on analyzing the marginal benefit and simply set  $C(\tau_{u,i}, \tau_{e,i}) = 0$ .

**Figure 4.** (Color online) The Second Overall Equilibrium



*Notes.* This figure plots values of  $(\tau_u^*, \tau_e^*, \alpha^*)$  for the second equilibrium as functions of  $\kappa_u$  in the top panels and  $\kappa_e$  in the bottom panels. The cost function is  $C(\tau_{u,i}, \tau_{e,i}) = \kappa_u \tau_{u,i} + \kappa_e \tau_{e,i}$ , where  $\kappa_u$  and  $\kappa_e$  are positive constants. The parameters are set as follows:  $\tau_v = 25$ ,  $\tau_e = 1$ , and  $\gamma = 2$ .

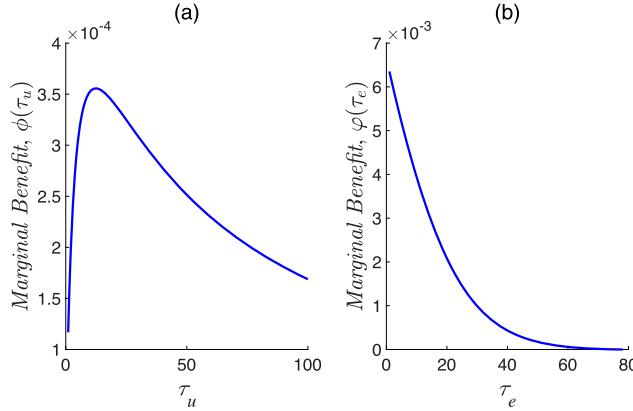
Let us denote the marginal benefit of choosing  $\tau_{u,i} = \tau_u$  as  $\phi(\tau_u)$ . The shape of  $\phi(\tau_u) = \frac{\partial W(\tau_{u,i}, \tau_u)}{\partial \tau_{u,i}}|_{\tau_{u,i}=\tau_u}$  determines whether there is a unique equilibrium or there are multiple equilibria. Formally, if  $\phi(\tau_u)$  is downward sloping, then the equilibrium is unique. In contrast, when  $\phi(\tau_u)$  has an upward sloping segment, multiplicity can arise. Hence, the key to have multiple equilibria is to check the sign of  $\frac{d\phi(\tau_u)}{d\tau_u}$ .

We can see this nonmonotonicity in Panel (a) of Figure 5, where we use the same parameters as in Figure 1. Intuitively, when  $\tau_u = 0$  (low  $\tau_u$ ), there is no noise  $\tilde{u}$  in the price. As  $\tau_u$  increases, the noise  $\tilde{u}$  gets

incorporated into the price, and the incentives to tame sentiment increase ( $\phi(\tau_u)$  is increasing). In contrast, for high values of  $\tau_u$ , the price signal is already very informative, and a higher  $\tau_u$  will not provide much additional information ( $\phi(\tau_u)$  is decreasing). Consequently, the marginal benefit  $\phi(\tau_u)$  has a hump shape, and there are multiple equilibria.

Proposition 5 characterizes some sufficient conditions for multiple equilibria. The first condition implies that  $\phi(\tau_u)$  is upward sloping for small values of  $\tau_u$ . The second condition says that the sophistication-acquisition cost must take intermediate values.

**Figure 5.** (Color online) Marginal Benefit of Sophistication



Notes. Panel (a) plots the marginal benefit  $\phi(\tau_u)$  of choosing  $\tau_{u,i} = \tau_u$ . Panel (b) plots the marginal benefit  $\varphi(\tau_e)$  of choosing  $\tau_{e,i} = \tau_e$ . The parameters are set as follows:  $\tau_v = \tau_e = 25$  and  $\gamma = 2$ . In Panel (a),  $\tau_e = 500$ , and in Panel (b),  $\tau_u = 500$ .

Intuitively, if the sophistication-acquisition cost is too high, then no investor acquires additional sophistication to tame the sentiment error (i.e.,  $\tau_u^* = 0$ ). If the sophistication-acquisition cost is too low, then all investors coordinate on a unique high sophistication level of taming the sentiment error.

**Proposition 5.** Suppose  $C(\tau_{u,i}) = \kappa_u \tau_{u,i}$ . There exist multiple equilibrium levels of sophistication if (i)  $\frac{\tau_e^2(4\tau_e^2+8\tau_e^2+4\tau_e\tau_v+7\tau_e\tau_e+4\tau_v\tau_e)}{\tau_v^2(3\tau_e\tau_v+2\tau_e\tau_e+4\tau_v\tau_e)} > 1$  and (ii)  $\kappa_u \in (\underline{\kappa}_u, \bar{\kappa}_u)$ , where  $\underline{\kappa}_u$  and  $\bar{\kappa}_u$  are two constants given in the appendix.

We next take  $\tau_{u,i} = \tau_u$  as given and focus on the choice of  $\tau_{e,i}$ . As before, we focus on analyzing the marginal benefit, and so for simplicity, we assume  $C(\tau_{u,i}, \tau_{e,i}) = 0$ . Let us denote the marginal benefit of choosing  $\tau_{e,i} = \tau_e$  as  $\varphi(\tau_e) = \frac{\partial W(\tau_{e,i}, \tau_e)}{\partial \tau_{e,i}}|_{\tau_{e,i}=\tau_e}$ . In Panel (b) of Figure 5, we numerically check the shape of  $\varphi(\tau_e)$  to study the complementarity result using the same parameter values as Figure 2. We see no evidence of complementarity in the figure. The marginal benefit of reducing individual processing errors is large when  $\tau_e$  is low, and it is small when  $\tau_e$  is large. This result is formalized in the following section.

#### 5.4. Uncorrelated Receiver Noise

This section solves the model with no common sentiment errors  $\tilde{u}$ . To this end, we assume that  $\tau_{u,i} = \tau_u \rightarrow \infty$ . Corollary 1 shows that, in a symmetric equilibrium with  $\tau_{e,i} = \tau_e$ , price coefficients are given by  $a_v = \frac{\tau_e + \tau_v}{\tau_e + \tau_v + \tau_e}$  and  $a_u = 0$ , implying that  $\alpha = 0$ . In this case, returns still exhibit momentum:

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\tau_v}{\tau_e + \tau_e} > 0.$$

Thus, the results on return predictiveness of the model are robust to turning off the common sentiment error term  $\tilde{u}$ . Trading volume is given by

$$\text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi} (\tau_e + \tau_e)}.$$

It is increasing in sophistication  $\tau_e$ . Proposition 6 formally analyzes the existence of a unique equilibrium in sophistication levels when the cost function is  $C(\tau_{e,i}) = \kappa_e \tau_{e,i}$ . This proposition suggests that complementarity and multiplicity are driven by the choice of  $\tau_u$ . When we get rid of the common error term  $\tilde{u}$  in the receiver noise, the complementarity result disappears.

**Proposition 6.** Suppose that  $\tau_u \rightarrow \infty$  and  $C(\tau_{e,i}) = \kappa_e \tau_{e,i}$ . If  $\kappa_e$  is sufficiently small, then there is a unique symmetric equilibrium with  $\tau_e^* > 0$ . Otherwise, there is a unique symmetric equilibrium with  $\tau_e^* = 0$ .

## 6. Extension with Noisy Supply

In this section, we extend the setup in Section 3.1 with exogenous noisy trading. Specifically, the risky asset has a noisy supply  $\tilde{q} \sim N(0, \tau_q^{-1})$ , which is independent of all the other random variables. All the other features of the setup in Section 3.1 remain unchanged. The market-clearing condition in the financial market now becomes

$$\int_0^1 D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) di = \tilde{q} \text{ almost surely.} \quad (18)$$

With noisy supply, the price of the risky asset is given by

$$\tilde{p} = a_v \tilde{v} + a_u \tilde{u} + a_q \tilde{q}, \quad (19)$$

where  $a$ 's are endogenous coefficients. Hence, the price signal can be written as

$$\tilde{s}_{p,i} = \tilde{v} + \alpha \tilde{u} + \theta \tilde{q} + \frac{1}{\sqrt{\tau_{u,i}}} \tilde{u} + \frac{1}{\sqrt{\tau_{e,i}}} \tilde{e}_i,$$

where  $\alpha \equiv \frac{a_u}{a_v}$  and  $\theta \equiv \frac{a_q}{a_v}$ . The investor's beliefs in this case are given by

$$\begin{aligned} E(\tilde{v} | \tilde{s}_i, \tilde{s}_{p,i}) &= \left( \tau_e \tilde{s}_i + \frac{\tau_{e,i} \tau_{u,i} \tau_q}{\tau_{u,i} \tau_q + \tau_{e,i} \tau_q (\alpha \sqrt{\tau_{u,i}} + 1)^2 + \tau_{e,i} \tau_{u,i} \theta^2} s_{p,i} \right) \\ &\times V(\tilde{v} | \tilde{s}_i, \tilde{s}_{p,i}), \end{aligned} \quad (20)$$

$$\begin{aligned} V(\tilde{v} | \tilde{s}_i, \tilde{s}_{p,i}) &= \frac{\tau_{u,i} \tau_q + \tau_{e,i} \tau_q (\alpha \sqrt{\tau_{u,i}} + 1)^2 + \tau_{e,i} \tau_{u,i} \theta^2}{\tau_{e,i} \tau_{u,i} \tau_q + (\tau_{u,i} \tau_q + \tau_{e,i} \tau_q (\alpha \sqrt{\tau_{u,i}} + 1)^2 + \tau_{e,i} \tau_{u,i} \theta^2)} \\ &\times (\tau_v + \tau_e). \end{aligned} \quad (21)$$

The following proposition shows the financial market equilibrium and return predictiveness in this setup with noisy supply.

**Proposition 7.** Suppose that investors have the same sophistication level (i.e.,  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$ ,  $i \in [0, 1]$ ) in the economy with noisy supply. Then,

a. There exists a unique linear equilibrium price function with coefficients

$$a_v = \frac{\tau_e \tau_u \tau_q + \tau_\varepsilon (\tau_u \tau_q + \tau_e \tau_q (\alpha \sqrt{\tau_u} + 1)^2 + \tau_e \tau_u \theta^2)}{\tau_e \tau_u \tau_q + (\tau_u \tau_q + \tau_e \tau_q (\alpha \sqrt{\tau_u} + 1)^2 + \tau_e \tau_u \theta^2)(\tau_v + \tau_\varepsilon)},$$

$$a_u = \frac{\tau_e \sqrt{\tau_u} \tau_q (\alpha \sqrt{\tau_u} + 1)}{\tau_e \tau_u \tau_q + (\tau_u \tau_q + \tau_e \tau_q (\alpha \sqrt{\tau_u} + 1)^2 + \tau_e \tau_u \theta^2)(\tau_v + \tau_\varepsilon)},$$

$$a_q = -\frac{\gamma (\tau_u \tau_q + \tau_e \tau_q (\alpha \sqrt{\tau_u} + 1)^2 + \tau_e \tau_u \theta^2) - \tau_e \tau_u \tau_q \theta}{\tau_e \tau_u \tau_q + (\tau_u \tau_q + \tau_e \tau_q (\alpha \sqrt{\tau_u} + 1)^2 + \tau_e \tau_u \theta^2)(\tau_v + \tau_\varepsilon)},$$

where  $\theta = -\frac{\gamma}{\tau_e}$  and  $\alpha$  is determined by the unique real root of the following cubic equation:

$$\alpha^3 + \frac{2}{\sqrt{\tau_u}} \alpha^2 + \left( \frac{\gamma^2}{\tau_q \tau_\varepsilon^2} + \frac{1}{\tau_e} + \frac{1}{\tau_u} \right) \alpha - \frac{1}{\sqrt{\tau_u} \tau_\varepsilon} = 0. \quad (22)$$

b. There exists a threshold  $\hat{\tau}_q$  such that returns exhibit momentum  $m > 0$  for any  $\tau_q > \hat{\tau}_q$ .

Figure 6 reports the main results of our model with exogenous noise trading and compares them to the results in a standard noisy-REE benchmark (in which we set both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$  but still keep  $\tau_q \in (0, \infty)$ ). We focus on the implications of  $\tau_u$  and report the results for two different values of  $\tau_q \in \{0.1, 5\}$  to examine how our results depend on the size of noise trading. Other parameter values are the same as those in Figure 1.

In the bottom panels of Figure 6 with high  $\tau_q$  (low exogenous noise trading), the results are qualitatively similar to those in our baseline model without exogenous noise trading (Figure 1 and Panel (a) of Figure 5). Relative to the standard noisy-REE benchmark, our model predicts lower price informativeness but higher return volatility and trading volume. In addition, our model predicts price momentum, and the standard noisy-REE benchmark exhibits return reversal. In Panel (b5) of Figure 6, the complementarity result in the sophistication choice prevails.

In the top panels of Figure 6 with low  $\tau_q$  (high exogenous noise trading), the results on price informativeness, trading volume, and sophistication acquisition remain robust. However, returns exhibit reversal in Panel (a2) of Figure 6 although the degree of return reversal is lower than that in the standard noisy-REE benchmark. Intuitively, the sign of return

predictiveness is determined by two counteracting forces in our setting. First, as in the traditional noisy REE setting, exogenous noise trading delivers return reversal (see Vives 2008, Banerjee et al. 2009): exogenous noise demand pushes the current price too high, and exogenous noisy supply depresses the current price too low. Second, in our setting, investors interpret price costly, which causes the price to underreact to information, leading to return momentum. When the size of noise trading is high (low  $\tau_q$ ), the first effect dominates. In contrast, when the size of noise trading is low (high  $\tau_q$ ), the second effect dominates.

## 7. Comparison with Other Related Models

In this section, we investigate other models that are conceptually related to our theory. We consider four alternative theories: (a) information acquisition about noisy supply, (b) dismissive traders, (c) cursed traders, and (d) operation risk. We show that supply information and operation risk deliver return reversal and that dismissiveness and cursedness can deliver price momentum. Relative to dismissiveness and cursedness, our model has different predictions on other variables, such as price informativeness and trading volume.

### 7.1. Information Acquisition About Noisy Supply

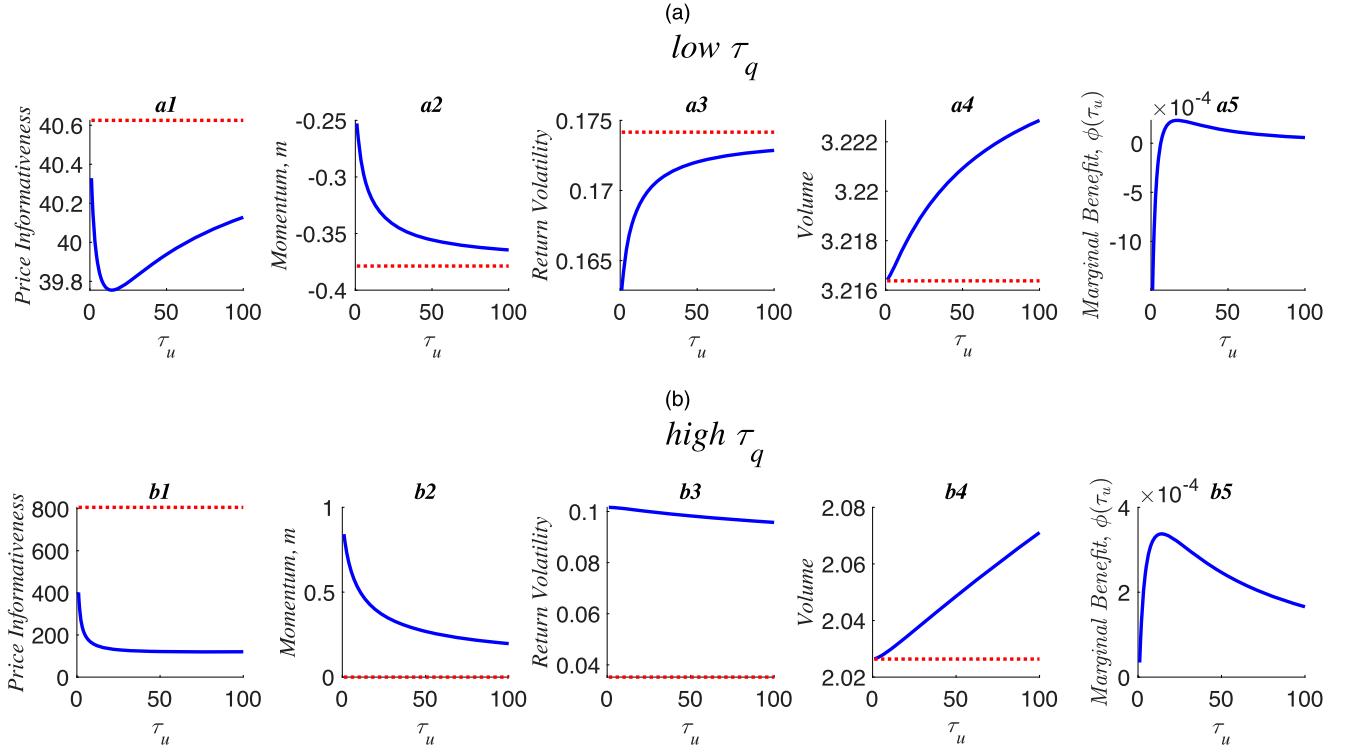
Information acquisition about noisy supply has been analyzed by Ganguli and Yang (2009), Manzano and Vives (2011), and Marmor and Rytchkov (2018), among others. We base our analysis on Ganguli and Yang (2009) and modify our setup in Section 3.1 in two aspects. First, we shut down the sophistication level choice in our model by setting both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$ , which implies that investors are able to observe the best price signal  $\tilde{s}_p$ . Second, we assume that at date 1, investor  $i$  is endowed with  $q_i$  units of the risky asset, where  $\tilde{q}_i = \tilde{q} + \tilde{\eta}_i$  with  $\tilde{q} \sim N(0, \tau_q^{-1})$  and  $\tilde{\eta}_i \sim N(0, \tau_\eta^{-1})$ , which implies that the aggregate supply is  $\int_0^1 \tilde{q}_i di = \tilde{q}$ . With these two modifications, the price of the risky asset is given by  $\tilde{p} = a_v \tilde{v} - a_q \tilde{q}$ . In this supply information model, there can exist multiple linear equilibria at the trading stage. In each of these equilibria, returns exhibit reversal (i.e.,  $m < 0$ ), which contrasts with our setting of costly interpretation of prices.

**Proposition 8.** In the economy with supply information, if  $\gamma^2 > 4\tau_e \tau_\eta$ , then there exist two equilibria with price coefficients given by

$$a_v = (\tau_v + \tau_\varepsilon + \tau_p)^{-1} (\tau_\varepsilon + \tau_p) \text{ and}$$

$$a_q = (\tau_v + \tau_\varepsilon + \tau_p)^{-1} (\beta \tau_q + \gamma),$$

**Figure 6.** (Color online) Extension Setting with Noisy Supply



Notes. This figure plots price informativeness ( $\frac{1}{\text{Var}(\tilde{v}|p)}$ ), return volatility ( $\sigma(\tilde{v} - \tilde{p})$ ), price momentum ( $m$ ), trading volume, and marginal benefit  $\phi(\tau_u)$  against the investor sophistication  $\tau_u$  of taming the sentiment error for low  $\tau_q$  in the top panels and high  $\tau_q$  in the bottom panels. The parameters are set as follows:  $\tau_v = \tau_\epsilon = 25$ ,  $\tau_e = 500$ , and  $\gamma = 2$ . We set  $\tau_q = 0.1$  in the top panels and  $\tau_q = 5$  in the bottom panels. The dotted lines plot the values in a standard REE economy.

with  $\tau_p = \beta^2(\tau_q + \tau_\eta)$  and  $\beta = a_v/a_q$  taking two values

$$\beta_{SUB} = (2\tau_\eta)^{-1} \left( \gamma - \sqrt{\gamma^2 - 4\tau_\epsilon\tau_\eta} \right) \text{ and}$$

$$\beta_{COM} = (2\tau_\eta)^{-1} \left( \gamma + \sqrt{\gamma^2 - 4\tau_\epsilon\tau_\eta} \right).$$

Under SUB-equilibrium, information acquisition is a strategic substitute. Under COM-equilibrium, information acquisition is a complement. Returns exhibit reversal under both equilibria:

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} < 0.$$

## 7.2. Dismissiveness

We incorporate dismissiveness into our model based on Banerjee (2011). We make two modifications to our setup with noise trading in Section 6. First, we shut down the sophistication level choice in our model, and thus, investors are able to observe the best price signal  $\tilde{s}_p$ . Second, investor  $i$ 's beliefs about the private signal of investor  $j$  are given by

$$\tilde{s}_{i,j} = \rho\tilde{v} + \sqrt{(1-\rho^2)}\tilde{\phi} + \tilde{\varepsilon}_j, \text{ where } \tilde{\varepsilon}_j \sim N(0, \tau_\epsilon^{-1}), \\ \tilde{\phi} \sim N(0, \tau_v^{-1}),$$

where  $\rho \in [0, 1]$  and  $\tilde{\phi}$  is independent of  $\tilde{v}$ . The parameter  $\rho$  measures the degree of dismissiveness by investors. When  $\rho < 1$ , each investor believes that the private signal of all other investors includes an additional random variable  $\tilde{\phi}$ . When  $\rho = 1$ , we have a standard REE model. With these modifications, investors perceive that the price of the risky asset is given by

$$\tilde{p} = a_v\rho\tilde{v} + a_v\sqrt{(1-\rho^2)}\tilde{\phi} + a_q\tilde{q}. \quad (23)$$

Dismissiveness can predict either momentum or reversal, depending on the value of investors' risk aversion: when the risk aversion is sufficiently small, returns exhibit momentum, and when the risk aversion is sufficiently large, returns exhibit reversal. This is intuitive because the risk aversion scales how much noise there is in prices and so how much noise-driven reversal there is. Nonetheless, dismissiveness delivers different implications for price informativeness and trading volume from our model. Specifically, price informativeness is independent of the level of dismissiveness, and in our model, price informativeness is nonmonotonic with respect to  $\tau_u$  and decreasing with respect to  $\tau_e$ . In addition, trading volume increases with dismissiveness and, hence, decreases with sophistication (to the extent that dismissiveness is inversely related to sophistication). By contrast, in

our setting, trading volume either increases with sophistication or is hump shaped in sophistication.

**Proposition 9.** *In a setting with dismissive investors, there is a unique linear equilibrium price function with coefficients*

$$a_v = \frac{(\tau_v \tau_\varepsilon \alpha^2 - \tau_q \tau_\varepsilon \rho^2 + \tau_q \tau_v \rho + \tau_q \tau_\varepsilon)}{(\alpha^2 \tau_v^2 + \tau_\varepsilon \alpha^2 \tau_v - \tau_q \tau_\varepsilon \rho^2 + \tau_q \tau_v + \tau_q \tau_\varepsilon)},$$

$$a_q = -\frac{(\gamma \tau_v \alpha^2 - \tau_q \tau_v \alpha \rho - \gamma \tau_q \rho^2 + \gamma \tau_q)}{(\alpha^2 \tau_v^2 + \tau_\varepsilon \alpha^2 \tau_v - \tau_q \tau_\varepsilon \rho^2 + \tau_q \tau_v + \tau_q \tau_\varepsilon)},$$

where  $\alpha \equiv \frac{a_q}{a_v} = -\gamma/\tau_\varepsilon$ . For sufficiently small  $\gamma$ , returns exhibit momentum, and for sufficiently large  $\gamma$ , returns exhibit reversal. Price informativeness is independent of dismissiveness  $\rho$ , and trading volume is increasing in dismissiveness.<sup>7</sup>

### 7.3. Cursed Traders

We analyze cursed traders based on Eyster et al. (2019). We modify our setup in Section 3.1 in four aspects. First, we shut down the sophistication level choice in our model so that investors are able to observe the best price signal  $\tilde{s}_p$ . Second, we assume that there is a finite number of traders  $N$ . Third, the risky asset pays date 2 value  $\tilde{v} + \tilde{\zeta}$ , where  $\tilde{\zeta} \sim N(0, \tau_\zeta^{-1})$  is an unlearnable risk, so the private signal is only about the payoff component  $\tilde{v}$ . Fourth, investors' behavior lies between rationality and full cursedness. Traders maximize the following utility:

$$EU = E[-e^{-\gamma D_i(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i, \tilde{p}]^{1-\chi} E[-e^{-\gamma D_i(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i]^\chi,$$

where parameter  $\chi$  captures the extent of cursedness. With these modifications, the price of the risky asset is given by  $\tilde{p} = a_v \sum_{i=1}^N \tilde{s}_i$ .

This model is able to generate positive momentum as in our model. Intuitively, cursed traders ( $\chi > 0$ ) disregard information from the price, which leads to price underreaction. The mechanism is similar to our model. As explained in Section 2, our framework complements Eyster et al. (2019) in four important ways. First, Eyster et al. (2019) predict that trading volume always increases with cursedness and, hence, decreases with sophistication. By contrast, in our setting, volume is either increasing or hump shaped in sophistication. Intuitively, because cursed volume arises from investors neglecting price information, the more cursed and less sophisticated the investors are, the higher the volume. In our setting, if investors are less exposed to the sentiment error, their posterior beliefs put relative higher weights on individual processing errors, generating more disagreement and higher volume. This difference in sophistication-volume patterns can be taken to the data to differentiate our model from cursedness (for instance, one can use the fraction of institutional trading to proxy for the sophistication degree of a market).

Second and more importantly, our setting has an extra stage to determine the equilibrium level of investor sophistication, which generates novel theory results, such as strategic complementarity in sophistication acquisition. We have also analyzed a setting with endogenous cursedness, a common error, and a linear cost function in a similar way as Proposition 5. Our analysis suggests that such a setting fails to deliver complementarity in the cursedness choice. Third, in the original setting of Eyster et al. (2019), there is no common error term for cursed traders and, thus, price informativeness is independent of the level of cursedness. Instead, in our model, price informativeness is nonmonotonic with respect to sophistication. Fourth, our analysis incorporates a common error in interpreting prices, which generates a form of endogenous noise trading.<sup>7</sup>

**Proposition 10.** *In the economy with cursed traders, the price coefficient is given by*

$$a_v = \frac{\tau_\varepsilon([N - \chi(N - 1)]\tau_v + N\tau_\varepsilon)}{N(\tau_v + \tau_\varepsilon)(\tau_v + N\tau_\varepsilon)}.$$

*This model with cursed traders generates momentum*

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = a_v \frac{\chi(N - 1)(\tau_v + N\tau_\varepsilon)}{(\tau_v + \tau_\varepsilon)(\tau_v + N\tau_\varepsilon)} > 0.$$

*Price informativeness is independent of cursedness  $\chi$ , and trading volume is increasing in cursedness.*

### 7.4. Operation Risk

We incorporate operation risk into our model based on Basak and Buffa (2019). We modify our setup with noise trading in Section 6 in two aspects. First, we shut down the sophistication level choice in our model, and so investors are able to observe the best price signal  $\tilde{s}_p$ . Second, the asset demand contains operational errors, and hence, the asset demand is given by

$$D_{or,i} = D_i + \tilde{x} + \tilde{\eta}_i, \text{ where } \tilde{x} \sim N(0, \tau_x^{-1}) \text{ and} \\ \tilde{\eta}_i \sim N(0, \tau_\eta^{-1}). \quad (24)$$

Variable  $D_i$  is the optimal investment strategy, and  $\tilde{x} + \tilde{\eta}_i$  is the operational error. The operational error  $\tilde{x}$  is common to all investors, and operational errors  $\tilde{\eta}_i$  are identically and independently distributed.

With these modifications, the price of the risky asset is given by

$$\tilde{p} = a_v \tilde{v} + a_x \tilde{x} + a_q \tilde{q}. \quad (25)$$

Hence, the price signal can be written as  $\tilde{s}_{p,i} = \tilde{v} + \alpha \tilde{x} + \theta \tilde{q}$ , where  $\alpha \equiv \frac{a_x}{a_v}$  and  $\theta \equiv \frac{a_q}{a_v}$ . Solving for the optimal asset demand is not straightforward because final wealth is no longer normally distributed, and we cannot use mean-variance results. Final wealth is given by  $W_i = D_i(\tilde{v} - \tilde{p}) + \tilde{x}(\tilde{v} - \tilde{p}) + \tilde{\eta}_i(\tilde{v} - \tilde{p})$ , and we need to take

expectations:  $EU = E[-\exp(-\gamma W_i)|\tilde{s}_i, \tilde{s}_{p,i}]$ . When there is no common operational risk  $\tilde{x}$ , the optimal asset demand  $D_i$  turns out to be the exact same as the standard CARA-normal framework:  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) = \frac{E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}) - \tilde{p}}{\gamma \text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})}$ . With common operational risk  $\tilde{x}$ , the optimal demand is given by  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}) = \frac{E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}) - \tilde{p}}{\gamma \text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})} - E(\tilde{x}|\tilde{s}_i, \tilde{s}_{p,i})$ . Hence, the total demand is

$$D_{or,i} = \frac{E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i}) - \tilde{p}}{\gamma \text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})} + [\tilde{x} - E(\tilde{x}|\tilde{s}_i, \tilde{s}_{p,i})] + \tilde{\eta}_i.$$

Intuitively, because investors know that they will get a common shock  $\tilde{x}$  to their ideal demand, they correct demand by their best forecast of  $\tilde{x}$ . They are able to estimate  $\tilde{x}$  with both signals  $\tilde{s}_i$  and  $\tilde{s}_{p,i}$ .

**Proposition 11.** *The coefficients in the linear equilibrium price function are given by*

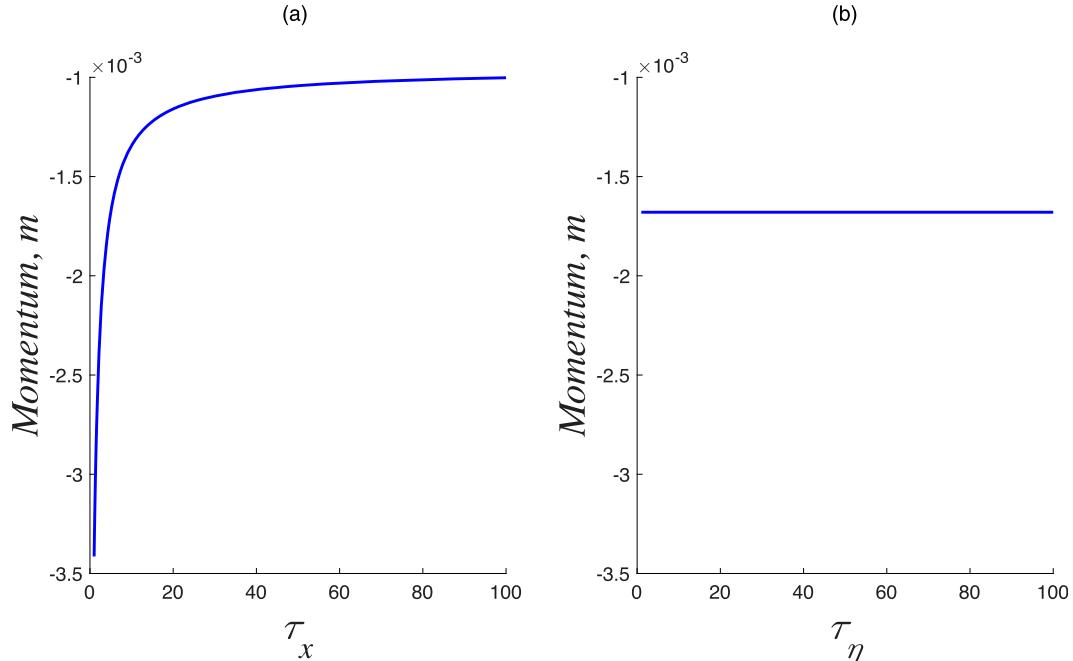
$$a_v = \frac{[\tau_q \tau_x + \alpha^2 \tau_\varepsilon (\tau_q + \tau_x)][\tau_q \tau_x + \alpha^2 (\tau_v + \tau_\varepsilon) (\tau_q + \tau_x)] - \alpha^3 \gamma \tau_q \tau_v (\tau_q + \tau_x)}{[\tau_q \tau_x + \alpha^2 (\tau_v + \tau_\varepsilon) (\tau_q + \tau_x)]^2},$$

$$a_x = \alpha a_v \text{ and } a_q = -\alpha a_v,$$

where  $\alpha$  is a real root to the following cubic equation:

$$(\tau_q \tau_\varepsilon^2 + \tau_x \tau_\varepsilon^2 + \tau_q \tau_v \tau_\varepsilon + \tau_x \tau_v \tau_\varepsilon) \alpha^3 + (\gamma \tau_q \tau_v + \gamma \tau_x \tau_v + \gamma \tau_x \tau_\varepsilon) \alpha^2 + \tau_q \tau_x \tau_\varepsilon \alpha + \gamma \tau_q \tau_x = 0.$$

**Figure 7.** (Color online) Return Predictiveness in Settings of Operation Risk



*Notes.* This figure shows price momentum ( $m$ ) against the precision of the common operational risk  $\tilde{x}$  and the precision of the idiosyncratic operational risk  $\tilde{\eta}_i$ . The parameters are set as follows:  $\tau_v = \tau_\varepsilon = 25$ ,  $\tau_q = 5$ , and  $\gamma = 2$ . Panel (a) sets  $\tau_\eta = 5$ , and Panel (b) sets  $\tau_x = 5$ .

The common error  $\tilde{x}$  behaves in a very similar way as noisy supply with the only difference being that agents can learn about  $\tilde{x}$ , and as a consequence, investors try to correct for the demand shock  $\tilde{x}$  in their ideal demand  $D_i$ . Figure 7 uses the same parameter values as Figure 6 with high  $\tau_q$ , where investors are not able to observe the ideal price signal and there is noisy supply. We assume that the precision  $\tau_x$  of the common error  $\tilde{x}$  is the same as the one assumed for the sentiment term  $\tilde{v}$ . Figure 7 shows that the model with operation risk fails to generate momentum for those values. As mentioned, common operation risks behave in a similar way as a noisy supply and, thus, are not able to generate momentum. We have tried a variety of parameter values and find that the results are robust even for very large values of  $\tau_q$ . Panel (b) shows that momentum under operation risk does not depend on the precision  $\tau_\eta$  of idiosyncratic operational errors. Intuitively, these idiosyncratic errors disappear when aggregating demand, and they have no effect on asset prices.

**Corollary 2.** *In the setting with operation risk, if  $\tau_x \rightarrow \infty$ , then there is a unique linear equilibrium price function with coefficients*

$$a_v = \frac{\tau_\varepsilon \theta^2 + \tau_q}{\tau_q + \theta^2 \tau_v + \theta^2 \tau_\varepsilon} \text{ and } a_q = \frac{-\gamma \theta^2 + \tau_q \theta}{\tau_q + \theta^2 \tau_v + \theta^2 \tau_\varepsilon},$$

where  $\theta = \frac{a_q}{a_v} = -\gamma/\tau_\varepsilon$ . The asset price is isomorphic to a standard REE equilibrium, and returns exhibit reversals (i.e.,  $m < 0$ ).

In the limiting case without common error, because all operational errors are identically and independently distributed, we end up with a standard REE equilibrium because all operational errors cancel with each other when aggregated. Return volatility and momentum are the same as in a standard REE equilibrium, but volume is higher because of the operational errors. In particular, this model of operation risk is not able to generate price momentum.

## 8. Conclusion

We develop a model to capture the notion that investors cannot costlessly process price data in financial markets. Although investors actively infer information from the price, their information processing is noisy. The more sophisticated the investors are, the smaller the processing noise. After reading price data and forming their beliefs, investors hold optimal trading positions according to their own beliefs (and so they are only boundedly rational in extracting information from the price). We find that imperfect price interpretation can inject noise into the price system, which serves as a form of endogenous noise trading in our setting. Our model predicts price momentum because investors fail to make full use of the information in asset prices. Compared with the standard REE and noisy REE (with low levels of noise trading), our model generates excessive return volatility and excessive trading volume. There can exist strategic complementarity in acquiring sophistication of taming sentiment, leading to the possibility of multiple equilibria. Our theory qualitatively differs from many other conceptually related models, such as supply information, dismissiveness, cursedness, and operation risk.

## Acknowledgments

The authors thank Department Editor Gustavo Manso, the associate editor, and three anonymous referees for many insightful suggestions. They also thank George-Marios Angeletos, Efstrathios Avdis, Jess Benhabib, Snehal Banerjee, Larry Blume, Patrick Bolton, Bradyn Breon-Drish, Matthijs Breugem, Georgy Chabakauri, Pierre Chaigneau, Amil Dasgupta, Shaun William Davies, Evan Dudley, Bernard Dumas, David Easley, Thierry Foucault, Simone Galperti, Diego García, Itay Goldstein, Naveen Gondhi, David Hirshleifer, Zhiguo He, Yu Hou, Zhen Huo, Philippe Jehiel, Alexandre Kohlhas, Jennifer La'o, John Leahy, Mina Lee, Samuel Lee, Xuewen Liu, Igor Makarov, Thomas Mertens, Jianjun Miao, Christian Opp, Emre Ozdenoren, Alessandro Pavan, Ryan Riordan, Oleg Rytchkov, Francesco Sangiorgi, Jan Schneemeier, Karl Shell, Vasiliki Skreta, Günter Strobl, Edward D. Van Wesep, Laura Veldkamp, Venky Venkateswaran, Pengfei Wang, Yajun Wang, Pierre-Olivier Weill, Yizhou Xiao, Wei Xiong, Ming Yang, Kathy Yuan, Jing Zeng, Yao Zeng, and participants at various seminars and conferences for helpful comments.

## Appendix A. Proof of Proposition 1

Using Bayes' rule, we can compute investors' beliefs given by Equations (11) and (12). In a symmetric equilibrium with  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$ , all investors have the same conditional variance  $\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$ . Thus, inserting these two expressions into the demand (5), we can compute the expression of  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$ , which is, in turn, inserted into (6) to compute the equilibrium price as a function of  $\tilde{v}$  and  $\tilde{u}$ . Comparing coefficients with the conjectured price function (8), we can form a system of equations to determine the two unknown price coefficients  $a_v$  and  $a_u$  in Proposition 1. Inserting the expressions of  $a$ 's into  $\alpha \equiv \frac{a_u}{a_v}$  and simplifying yields the cubic (13) that determines the value of  $\alpha$ . Denote the left-hand side of (13) by  $f(\alpha)$ . That is,

$$f(\alpha) \equiv \tau_e \tau_u \tau_e \alpha^3 + 2\tau_e \sqrt{\tau_u} \tau_e \alpha^2 + (\tau_e \tau_e + \tau_u \tau_e) \alpha - \tau_e \sqrt{\tau_u}.$$

We can compute  $f(0) = -\tau_e \sqrt{\tau_u} < 0$  and  $f(\frac{\tau_e \sqrt{\tau_u}}{\tau_e \tau_e + \tau_u \tau_e}) > 0$ , and thus, by the intermediate value theorem, there exists a solution  $\alpha \in (0, \frac{\tau_e \sqrt{\tau_u}}{\tau_e \tau_e + \tau_u \tau_e})$  such that  $f(\alpha) = 0$ . This result establishes the existence of a financial market equilibrium. The discriminant of the cubic (13) is negative. Thus, there exists a unique real root, which establishes the uniqueness of a financial market equilibrium. QED.

## Appendix B. Proof of Corollary 1

Given  $(\tau_v, \tau_e) \in \mathbb{R}_{++}^2$ , if we take the following limits of the price coefficients  $a_v$  and  $a_u$  in Proposition 1, we get

1. Fix  $\tau_e \in (0, \infty)$ ; as  $\tau_u \rightarrow \infty$ , then  $a_v = \frac{\tau_e + \tau_e}{\tau_e + \tau_v + \tau_e}$  and  $a_u = 0$ . Hence,  $\alpha = 0$ .

2. Fix  $\tau_u \in (0, \infty)$ ; as  $\tau_e \rightarrow \infty$ , then  $a_v = \frac{\tau_u + \tau_e(\alpha \sqrt{\tau_u} + 1)^2}{\tau_u + (\alpha \sqrt{\tau_u} + 1)^2(\tau_v + \tau_e)}$  and  $a_u = \frac{\sqrt{\tau_u}(\alpha \sqrt{\tau_u} + 1)}{\tau_u + (\alpha \sqrt{\tau_u} + 1)^2(\tau_v + \tau_e)}$ . Inserting the expressions of  $a$ 's into  $\alpha \equiv \frac{a_u}{a_v}$  and simplifying yields the following cubic equation that determines the value of  $\alpha$ :  $\tau_u \tau_e \alpha^3 + 2\sqrt{\tau_u} \tau_e \alpha^2 + \tau_e \alpha - \sqrt{\tau_u} = 0$ .

3. As both  $\tau_u \rightarrow \infty$  and  $\tau_e \rightarrow \infty$ , then  $a_v = 1$  and  $a_u = 0$ . Hence, the price function converges almost surely to  $\tilde{p}^{\text{REE}} = \tilde{v}$  and  $\tilde{s}_{p,i} = \tilde{s}_p$ . QED.

## Appendix C. Proof of Proposition 2

Return predictiveness  $m$  is given by

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\frac{a_v(1-a_v)}{\tau_v} - a_u^2}{\frac{a_v^2}{\tau_v} + a_u^2} = \frac{\tau_v \left( \tau_e \tau_e^2 (\alpha \sqrt{\tau_u} + 1)^4 + 2\tau_e \tau_u \tau_e (\alpha \sqrt{\tau_u} + 1)^2 + \tau_u^2 (\tau_e + \tau_e) \right)}{\left( \tau_e \tau_u + \tau_e (\tau_u + \tau_e (\alpha \sqrt{\tau_u} + 1)^2) \right)^2 + \tau_e^2 \tau_u \tau_v (\alpha \sqrt{\tau_u} + 1)^2} > 0,$$

where  $a_v$  and  $a_u$  are the price coefficients in Proposition 1. QED.

## Appendix D. Proof of Proposition 3

### a. Price informativeness

1. Using Equation (13), we can derive

$$\frac{d\alpha}{d\tau_u} = \frac{-\left(\tau_e \tau_e \alpha^3 + \tau_e (\sqrt{\tau_u})^{-1} \tau_e \alpha^2 + \tau_e \alpha - \tau_e (2\sqrt{\tau_u})^{-1}\right)}{3\tau_e \tau_u \tau_e \alpha^2 + 4\tau_e \sqrt{\tau_u} \tau_e \alpha + \tau_e \tau_e + \tau_u \tau_e}. \quad (\text{D.1})$$

The sign of Equation (D.1) is determined by its numerator, and we can conclude that  $\lim_{\tau_u \rightarrow 0} \frac{d\alpha}{d\tau_u} > 0$ .

### b. Return volatility

1. Return volatility is given by

$$\sigma(\tilde{v} - \tilde{p}) = \sqrt{\text{Var}(\tilde{v} - \tilde{p})},$$

where

$$\text{Var}(\tilde{v} - \tilde{p}) = \frac{(1 - a_v)^2}{\tau_v} + a_u^2. \quad (\text{D.2})$$

Using Equation (D.2), we can calculate the  $\lim_{\tau_u \rightarrow \infty} \text{Var}(\tilde{v} - \tilde{p}) = \frac{\tau_v}{(\tau_e + \tau_v + \tau_\epsilon)^2}$ .

2. Using Equation (D.2), we can calculate the  $\lim_{\tau_u \rightarrow 0} \text{Var}(\tilde{v} - \tilde{p}) = \frac{\tau_e^2 \tau_v}{(\tau_e \tau_v + \tau_e \tau_\epsilon)^2}$ .

### c. Trading volume

1. Trading volume is given by

$$\text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi}} \text{Var}[\gamma D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})], \quad (\text{D.3})$$

where  $\sigma_y^2 = \text{Var}[\gamma D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})]$  is given by (F.1). Taking the limit of this expression, we get

$$\lim_{\tau_u \rightarrow \infty} \text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi}} (\tau_e + \tau_\epsilon).$$

2. Taking the limit of (D.3), we get

$$\lim_{\tau_u \rightarrow 0} \text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi}} \tau_\epsilon.$$

## Appendix E. Proof of Proposition 4

### a. Price informativeness

1. Using Equation (13), we can derive

$$\begin{aligned} \frac{d\alpha}{d\tau_e} &= \frac{-(\tau_u \tau_\epsilon \alpha^3 + 2\sqrt{\tau_u} \tau_\epsilon \alpha^2 + \tau_\epsilon \alpha - \sqrt{\tau_u})}{3\tau_e \tau_u \tau_\epsilon \alpha^2 + 4\tau_e \sqrt{\tau_u} \tau_\epsilon \alpha + \tau_e \tau_\epsilon + \tau_u \tau_\epsilon} \\ &= \frac{\frac{\tau_u \tau_\epsilon \alpha}{\tau_e}}{3\tau_e \tau_u \tau_\epsilon \alpha^2 + 4\tau_e \sqrt{\tau_u} \tau_\epsilon \alpha + \tau_e \tau_\epsilon + \tau_u \tau_\epsilon} > 0, \end{aligned}$$

where the second equality arises from using (13).

### b. Return volatility

1. Using Equation (D.2), we can calculate

$$\begin{aligned} \lim_{\tau_e \rightarrow \infty} \text{Var}(\tilde{v} - \tilde{p}) &= \frac{(\alpha \sqrt{\tau_u} + 1)^2 (\tau_v \alpha^2 \tau_u + 2\tau_v \alpha \sqrt{\tau_u} + \tau_u + \tau_v)}{(\tau_v + \tau_\epsilon + \tau_u + \alpha^2 \tau_u \tau_v + \alpha^2 \tau_u \tau_\epsilon + 2\alpha \sqrt{\tau_u} \tau_v + 2\alpha \sqrt{\tau_u} \tau_\epsilon)^2}. \end{aligned}$$

2. Using Equation (D.2), we can calculate the  $\lim_{\tau_e \rightarrow 0} \text{Var}(\tilde{v} - \tilde{p}) = \frac{\tau_u^2 \tau_v}{(\tau_u \tau_v + \tau_u \tau_\epsilon)^2}$ .

### c. Trading volume

1. Taking the limit of (D.3), we get

$$\lim_{\tau_e \rightarrow \infty} \text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi}} \tau_\epsilon.$$

2. Taking the limit of (D.3), we get

$$\lim_{\tau_e \rightarrow 0} \text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2}{\pi}} \tau_\epsilon.$$

## Appendix F. Certainty Equivalent $W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e)$ in Section 5.2

At date 0, trader  $i$  chooses  $\tau_{u,i}$  and  $\tau_{e,i}$  to maximize  $E[-e^{-\gamma D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})} (\tilde{v} - \tilde{p}) + \gamma C(\tau_{u,i}, \tau_{e,i})]$ . We use the formula

$$E[e^{-xy}] = \frac{1}{\sqrt{(1 + \sigma_{xy})^2 - \sigma_x^2 \sigma_y^2}},$$

where  $x \sim N(0, \sigma_x^2)$ ,  $y \sim N(0, \sigma_y^2)$  and  $\text{Cov}(x, y) = \sigma_{xy}$ .

Define

$$x = \tilde{v} - \tilde{p}, y = \gamma D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i}).$$

The certainty equivalent is

$$\begin{aligned} W(\tau_{u,i}, \tau_{e,i}; \tau_u, \tau_e) &= -\frac{1}{\gamma} \ln(-U_i), \\ &= \frac{1}{2\gamma} \ln \left[ (1 + \sigma_{v-p, \gamma D_i})^2 - \sigma_{v-p}^2 \sigma_{\gamma D_i}^2 \right] \\ &\quad - C(\tau_{u,i}, \tau_{e,i}). \end{aligned}$$

Very intuitive: the first covariance term  $(1 + \sigma_{v-p, \gamma D_i})^2$  is the benefit of beating the market, and the second term  $\sigma_{v-p}^2 \sigma_{\gamma D_i}^2$  of variance products is the cost of excess trading. The relevant terms are given by

$$\begin{aligned} \sigma_x^2 &= \frac{(1 - a_v)^2}{\tau_v} + a_u^2, \\ \sigma_y^2 &= \frac{\left( \frac{(\beta_{s,i} + \beta_{p,i} - a_v)^2}{\tau_v} + \left( \beta_{p,i} \alpha + \frac{\beta_{p,i}}{\sqrt{\tau_{u,i}}} - a_u \right)^2 + \frac{\beta_{s,i}^2}{\tau_\epsilon} + \frac{\beta_{p,i}^2}{\tau_{e,i}} \right)}{V(v|s_i, s_{p,i})^2}, \\ \sigma_{xy} &= \frac{\frac{(1-a_v)(\beta_{s,i} + \beta_{p,i} - a_v)}{\tau_v} - a_u \left( \beta_{p,i} \alpha + \frac{\beta_{p,i}}{\sqrt{\tau_{u,i}}} - a_u \right)}{V(v|s_i, s_{p,i})}, \end{aligned} \quad (\text{F.1})$$

where  $a_v$  and  $a_u$  are the price coefficients in Proposition 1 and  $\beta_{s,i}$  and  $\beta_{p,i}$  are the coefficients of the conditional expectation

$$E(\tilde{v}|s_i, s_{p,i}) = \beta_{s,i} \tilde{s}_i + \beta_{p,i} \tilde{s}_{p,i},$$

which are given by Equation (11).

## Appendix G. Proof of Proposition 5

Under cost function  $C(\tau_{u,i}) = \kappa_u \tau_{u,i}$ , we can compute

$$\lim_{\tau_u \rightarrow 0} \frac{d\phi(\tau_u)}{d\tau_u} = \frac{-4\tau_v^3 \tau_\epsilon - 3\tau_e \tau_v^3 + 4\tau_v^2 \tau_\epsilon^2 - 2\tau_e \tau_v^2 \tau_\epsilon}{4\gamma \tau_\epsilon \tau_e (\tau_v^2 + \tau_v \tau_\epsilon + \tau_\epsilon^2)^2} + 4\tau_v \tau_\epsilon^3 + 4\tau_e \tau_v \tau_\epsilon^2 + 8\tau_\epsilon^4 + 7\tau_e \tau_\epsilon^3.$$

Thus, if condition (i) of Proposition 5 is satisfied, then  $\lim_{\tau_u \rightarrow 0} \frac{d\phi(\tau_u)}{d\tau_u} > 0$ .

We can also compute

$$\begin{aligned}\lim_{\tau_u \rightarrow 0} \phi(\tau_u) &= \frac{\tau_v - \tau_\varepsilon}{2\gamma(\tau_v^2 + \tau_v\tau_\varepsilon + \tau_\varepsilon^2)} - \kappa_u \text{ and} \\ \lim_{\tau_u \rightarrow \infty} \phi(\tau_u) &= -\kappa_u.\end{aligned}$$

Now define

$$\begin{aligned}\underline{\kappa}_u &= \max\left\{0, \frac{\tau_v - \tau_\varepsilon}{2\gamma(\tau_v^2 + \tau_v\tau_\varepsilon + \tau_\varepsilon^2)}\right\} \text{ and} \\ \overline{\kappa}_u &= \max_{\tau_u} [\phi(\tau_u) + \kappa_u].\end{aligned}$$

Note that  $\overline{\kappa}_u$  is actually independent of  $\kappa_u$  because  $\phi(\tau_u) + \kappa_u$  is effectively the first-order derivative of investors' sophistication-acquisition benefit gross sophistication-acquisition cost. When  $\kappa_u \in (\underline{\kappa}_u, \overline{\kappa}_u)$ ,  $\phi(\tau_u)$  crosses zero at least twice so that at least three equilibrium sophistication levels exist (corner level 0 and two interior levels). QED.

## Appendix H. Proof of Proposition 6

Under cost function  $C(\tau_{e,i}) = \kappa_e \tau_{e,i}$ , we can compute

$$\frac{d\varphi(\tau_e)}{d\tau_e} = -\frac{\tau_v(\tau_v + 2\tau_\varepsilon + 2\tau_e)}{2\gamma(\tau_v^2 + \tau_v\tau_\varepsilon + \tau_v\tau_e + \tau_\varepsilon^2 + 2\tau_\varepsilon\tau_e + \tau_e^2)^2} < 0.$$

Hence,  $\varphi(\tau_e)$  is always downward sloping. We can also compute

$$\lim_{\tau_e \rightarrow 0} \varphi(\tau_e) = -\frac{2\gamma\kappa_e\tau_v^2 + 2\gamma\kappa_e\tau_v\tau_\varepsilon - \tau_v + 2\gamma\kappa_e\tau_\varepsilon^2}{2\gamma(\tau_v^2 + \tau_v\tau_\varepsilon + \tau_\varepsilon^2)} \text{ and}$$

$$\lim_{\tau_e \rightarrow \infty} \varphi(\tau_e) = -\kappa_e.$$

Thus, for small enough  $\kappa_e$ , then  $\lim_{\tau_e \rightarrow 0} \varphi(\tau_e) > 0$ , and there is a unique symmetric equilibrium with  $\tau_e > 0$  given by

$$\begin{aligned}\tau_{e,i}^* &= \tau_e^* \\ &= -\frac{\gamma^{(1/2)}\kappa_e^{(1/2)}\tau_v - (-\tau_v(3\gamma\kappa_e\tau_v - 2))^{(1/2)} + 2\gamma^{(1/2)}\kappa_e^{(1/2)}\tau_\varepsilon}{2\gamma^{(1/2)}\kappa_e^{(1/2)}}.\end{aligned}$$

Otherwise, there is a unique symmetric equilibrium with  $\tau_{e,i}^* = \tau_e^* = 0$ . QED.

## Appendix I. Proof of Proposition 7

Using Bayes' rule, we can compute investors' beliefs given by Equations (20) and (21). In a symmetric equilibrium with  $\tau_{u,i} = \tau_u$  and  $\tau_{e,i} = \tau_e$ , all investors have the same conditional variance  $\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$ . Thus, inserting these two expressions into the demand (5), we can compute the expression of  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$ , which is, in turn, inserted into (18) to compute the equilibrium price as a function of  $\tilde{v}$ ,  $\tilde{u}$ , and  $\tilde{q}$ . Comparing coefficients with the conjectured price function (19), we can form a system of equations to determine the three unknown price coefficients  $a_v$ ,  $a_u$ , and  $a_q$  in Proposition 7. Inserting the expressions of  $a$ 's into  $\alpha \equiv \frac{a_u}{a_v}$  and simplifying yields the cubic (22) that determines the value of  $\alpha$ . The discriminant

of the cubic (22) is negative. Thus, there exists a unique real root, which establishes the uniqueness of a financial market equilibrium.

Return predictiveness  $m$  is given by

$$m = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\frac{a_v(1-a_v)}{\tau_v} - \frac{a_u^2}{\tau_u} - \frac{a_q^2}{\tau_q}}{\frac{a_v^2}{\tau_v} + \frac{a_u^2}{\tau_u} + \frac{a_q^2}{\tau_q}}.$$

The sign of  $m$  is determined by a cubic equation  $Q(\tau_q) = q_3\tau_q^3 + q_2\tau_q^2 + q_1\tau_q + q_0$ . Because the discriminant of this cubic equation is positive, there are three real solutions to  $Q(\tau_q) = 0$ . Also, the  $\lim_{\tau_q \rightarrow \infty} m > 0$ . Hence, denoting  $\hat{\tau}_q$  as the maximum real root to  $Q(\tau_q) = 0$ , we can conclude that  $m > 0$  for any  $\tau_q > \hat{\tau}_q$ . QED.

## Appendix J. Proof of Proposition 8

See proof of proposition 1 in Ganguli and Yang (2009) for price coefficients  $a_v$  and  $a_q$  and the properties of information acquisition. For the SUB-equilibrium,

$$\begin{aligned}m &= \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\frac{a_v(1-a_v)}{\tau_v} - \frac{a_q^2}{\tau_q}}{\frac{a_v^2}{\tau_v} + \frac{a_q^2}{\tau_q}} \\ &= -\frac{\gamma\tau_v(2\gamma\tau_\eta + \gamma\tau_q - \tau_q(\gamma^2 - 4\tau_\eta\tau_\varepsilon)^{1/2})}{(2\tau_\eta)(\beta^4\tau_q^3 + 2\beta^4\tau_q^2\tau_\eta + \beta^4\tau_q\tau_\eta^2 + 2\beta^2\tau_q^2\tau_\varepsilon + \tau_v\beta^2\tau_q^2 + 2\beta^2\tau_q\tau_\eta\tau_\varepsilon + 2\tau_v\beta\gamma\tau_q + \tau_v\gamma^2 + \tau_q\tau_\varepsilon^2)} < 0.\end{aligned}$$

For the COM-equilibrium,

$$\begin{aligned}m &= \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\frac{a_v(1-a_v)}{\tau_v} - \frac{a_q^2}{\tau_q}}{\frac{a_v^2}{\tau_v} + \frac{a_q^2}{\tau_q}} \\ &= -\frac{\gamma\tau_v(2\gamma\tau_\eta + \gamma\tau_q + \tau_q(\gamma^2 - 4\tau_\eta\tau_\varepsilon)^{1/2})}{(2\tau_\eta)(\beta^4\tau_q^3 + 2\beta^4\tau_q^2\tau_\eta + \beta^4\tau_q\tau_\eta^2 + 2\beta^2\tau_q^2\tau_\varepsilon + \tau_v\beta^2\tau_q^2 + 2\beta^2\tau_q\tau_\eta\tau_\varepsilon + 2\tau_v\beta\gamma\tau_q + \tau_v\gamma^2 + \tau_q\tau_\varepsilon^2)} < 0.\end{aligned}$$

## Appendix K. Proof of Proposition 9

Using Bayes' rule, we can compute investors' beliefs. We can then insert the expressions for  $E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  and  $\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  into the demand  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_{p,i})$ , which has the same expression as Equation (5). This demand is, in turn, inserted into the market clearing condition to compute the equilibrium price as a function of  $\tilde{v}$  and  $\tilde{q}$ . Comparing coefficients with the conjectured price function (23), we can form a system of equations to determine the two unknown price coefficients  $a_v$  and  $a_q$  in Proposition 9. Inserting the expressions of  $a$ 's into  $\alpha \equiv \frac{a_q}{a_v}$  and simplifying yields a cubic equation of  $\alpha$ . The discriminant of the cubic is negative. Thus, there exists a unique real root given by  $\alpha = -\gamma/\tau_\varepsilon$ , which establishes the uniqueness of a financial market equilibrium. The true (econometrician's) distribution of prices is given by  $\tilde{p} = a_v\tilde{v} + a_q\tilde{q}$ , not  $\tilde{p} = a_v\rho\tilde{v} + a_v\sqrt{(1-\rho^2)}\tilde{q} + a_q\tilde{q}$ . The latter

is only under the beliefs of the investors, but not under the true distribution. Hence, return predictiveness  $m$  is given by

$$\begin{aligned} m &= \frac{\text{Cov}(\tilde{v} - \tilde{p}, \tilde{p})}{\text{Var}(\tilde{p})} = \frac{\frac{(1-a_v)a_v}{\tau_v} - \frac{a_q^2}{\tau_q}}{\frac{(a_v)^2}{\tau_v} + \frac{a_q^2}{\tau_q}} \\ &\quad \times \frac{\tau_v(\tau_v\gamma^4 + (1-\rho^2)\gamma^2\tau_q\tau_\epsilon^2 - (1-\rho)}{\tau_v\gamma^2\tau_q\tau_\epsilon - (1-\rho)\tau_q^2\tau_\epsilon^3} \\ &= -\frac{\tau_v(\tau_v\gamma^4 + (1-\rho^2)\gamma^2\tau_q\tau_\epsilon^2 - (1-\rho)}{(\tau_q\tau_\epsilon^2 + \tau_v\gamma^2)(\tau_v\gamma^2 + (1-\rho^2)\tau_q\tau_\epsilon^2 + \tau_q\tau_v\rho\tau_\epsilon)}. \end{aligned}$$

The limits of  $m$  are given by

$$\lim_{\gamma \rightarrow 0} m > 0 \text{ and } \lim_{\gamma \rightarrow \infty} m < 0.$$

Hence, if  $\gamma$  is sufficiently small, then  $m > 0$ , but if  $\gamma$  is sufficiently large, then  $m < 0$ . Price informativeness is given by

$$\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \frac{\tau_v\alpha^2 + \tau_q}{\alpha^2}.$$

Trading volume in this model is  $\int_0^1 |D(p; s_i, s_{p,i})| di$  and is given by

$$\text{Volume} = \frac{1}{\gamma} \sqrt{\frac{2(\gamma^2\tau_v + 2\tau_q\tau_\epsilon^2 - 2\rho\tau_q\tau_\epsilon^2 + \tau_q\tau_v\tau_\epsilon)}{\pi\tau_q\tau_v}}.$$

From this equation, it immediately follows that volume is decreasing in  $\rho$ . QED.

## Appendix L. Proof of Proposition 10

See proof of proposition 2 in Eyster et al. (2019). Price informativeness is given by

$$\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \frac{\tau_\epsilon(\tau_v + N\tau_\epsilon)}{\tau_v + \tau_\epsilon + N\tau_\epsilon}. \quad \text{QED.}$$

## Appendix M. Proof of Proposition 11

Using Bayes' rule, we can compute investors' beliefs. Inserting the expressions for  $E(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  and  $\text{Var}(\tilde{v}|\tilde{s}_i, \tilde{s}_{p,i})$  into the demand (24), we can compute the expression of  $D(\tilde{p}; \tilde{s}_i, \tilde{s}_p)$ , which is, in turn, inserted into the market clearing condition to compute the equilibrium price as a function of  $\tilde{v}$ ,  $\tilde{x}$ , and  $\tilde{q}$ . Comparing coefficients with the conjectured price function (25), we can form a system of equations to determine the three unknown price coefficients  $a_v$ ,  $a_x$ , and  $a_q$  in Proposition 11. Inserting the expressions of  $a$ 's into  $\alpha \equiv \frac{a_q}{a_v}$ ,  $\theta \equiv \frac{a_x}{a_v}$  and simplifying yields the cubic equation for  $\alpha$  and  $\theta = -\alpha$ . QED.

## Appendix N. Proof of Corollary 2

We can obtain the unknown price coefficients by taking the limit of the price coefficients derived in Proposition 11. The asset price is isomorphic to the standard REE equilibrium, and therefore, there is no momentum  $m < 0$ . The expression for momentum is the same as in the proof of Proposition 9 with  $\rho = 1$ . QED.

## Appendix O. Endogenous Cursedness

### O.1. Setting

We analyze cursed traders based on Eyster et al. (2019). We modify our setup in Section 3.1 in four aspects. First, we shut down the idiosyncratic sophistication level choice in our model by setting  $\tau_e \rightarrow \infty$ . Second, we assume there is a finite number of traders  $N$ . Third, the risky asset pays date 2 value  $\tilde{v} + \tilde{\zeta}$ , where  $\tilde{\zeta} \sim N(0, \tau_\zeta^{-1})$  is an unlearnable risk, so the private signal is only about the payoff component  $\tilde{v}$ . Fourth, investors' behavior lies between rationality and full cursedness. Traders maximize the following utility:

$$\max_{D_i} E[-e^{-\gamma D_i(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i, \tilde{s}_p]^{1-\chi_i} E[-e^{-\gamma D_i(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i, \tilde{s}_u]^{\chi_i}, \quad (\text{O.1})$$

where  $\chi_i \in [0, 1]$  is trader  $i$ 's cursedness. In (O.1), we add one modification to Eyster et al. (2019) by allowing the cursed self conditions on information  $s_u$ , where

$$\tilde{s}_u = \tilde{s}_p + \tilde{u}, \quad (\text{O.2})$$

where  $s_p$  is the best signal extracted from the price, and  $\tilde{u}$  is a common error. This common error is comparable to the common error in our setup. With these modifications, the price of the risky asset is given by

$$\tilde{p} = a_v \sum_{i=1}^N \tilde{s}_i + a_u \tilde{u}.$$

Let  $D_i^* = D(\tilde{s}_i, \tilde{p}, \tilde{u})$  be the solution to (O.1). We then insert this demand into (O.1) and compute the indirect value function at date 1:

$$\begin{aligned} U(\tilde{s}_i, \tilde{p}, \tilde{u}; \chi_i) &= E[-e^{-\gamma D_i^*(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i, \tilde{s}_p]^{1-\chi_i} \\ &\quad \times E[-e^{-\gamma D_i^*(\tilde{v} + \tilde{\zeta} - \tilde{p})} | \tilde{s}_i, \tilde{s}_u]^{\chi_i}. \end{aligned} \quad (\text{O.3})$$

We then go back to date 0 to determine the optimal level of cursedness. Specifically, as in our baseline model and the bounded-rationality literature, we use the objective probability to compute the well-being. We consider a symmetric equilibrium. Suppose that all traders have a cursed level of  $\chi$ , and trader  $i$  has a level  $\chi_i$ . We use  $C(1 - \chi_i)$  to denote the cost of decreasing cursedness. For instance,  $C(1 - \chi_i) = k \times (1 - \chi_i)$ , where  $k > 0$  is a constant. Then, at date 0, trader  $i$  chooses  $\chi_i$  to maximize:

$$B(\chi_i; \chi) = E[U(\tilde{s}_i, \tilde{p}, \tilde{u}; \chi_i)] - C(1 - \chi_i). \quad (\text{O.4})$$

### O.2. Solution

Asset holdings are given by

$$D_i = \frac{(1 - \chi_i)E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u] - \tilde{p}}{\gamma[(1 - \chi_i)\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i \text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]]}.$$

Using Bayes' rule, we can compute investors' beliefs. We can then insert the expressions for  $E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p]$ ,  $E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]$ ,  $\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p]$ , and  $\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]$  into the demand  $D_i$ . This demand is, in turn, inserted into the market-clearing condition to compute the equilibrium price as a function of  $\tilde{v}$  and  $\tilde{u}$ . Comparing coefficients with the conjectured price function,

we can form a system of equations to determine the two unknown price coefficients  $a_v$  and  $a_u$ . Finally, we can plug the demand into the objective function to calculate the expected utility at date 0:

$$B(\chi_i; \chi) = -\exp \left\{ -\frac{[(1-\chi_i)(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] - \tilde{p}) + \chi_i(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u] - \tilde{p})]^2}{[(1-\chi_i)\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]]} \right\} - C(1-\chi_i).$$

We evaluate  $B(\chi_i; \chi)$  using the formula:

$$E(e^{-y^2}) = \frac{1}{\sqrt{1+2\sigma_y^2}} \exp \left\{ -\frac{\bar{y}^2}{1+2\sigma_y^2} \right\},$$

where

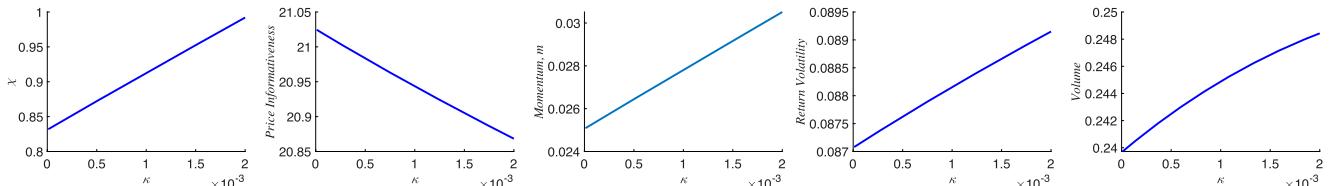
$$y \equiv -\frac{[(1-\chi_i)(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] - \tilde{p}) + \chi_i(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u] - \tilde{p})]}{[(1-\chi_i)\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]]^{1/2}}$$

with  $\bar{y} = 0$  and

$$\begin{aligned} \sigma_y^2 &= \frac{[(1-\chi_i)^2(\text{Var}(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] - \tilde{p})) + \chi_i^2(\text{Var}(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u] - \tilde{p}))]}{[(1-\chi_i)\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]]} + \\ &\quad + \frac{2(1-\chi_i)\chi_i\text{Cov}(E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] - \tilde{p}), E[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u] - \tilde{p})}{[(1-\chi_i)\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_p] + \chi_i\text{Var}[\tilde{v} + \tilde{\zeta}|\tilde{s}_i, \tilde{s}_u]]}. \end{aligned}$$

We can then calculate all the expectations and variances and take the FOC with respect to  $\chi_i$ . We have solved this model numerically. We have run numerical simulations for a wide range of parameter values, and we always find a unique symmetric equilibrium for the choice of cursedness. Figure O.1 plots the result for a typical parameter configuration:  $\tau_v = \tau_e = \tau_c = \tau_u = 25$ ,  $N = 10$ , and  $\gamma = 2$ . Our simulation results suggest that this setting with endogenous cursedness and a common noise fails to generate complementarities in the choice of  $\chi$ .

**Figure O.1.** (Color online) Implications of  $\kappa$  for Cursedness Choice, Asset Prices, and Trading Volume



*Notes.* This figure plots cursedness choice ( $\chi$ ), price informativeness ( $\frac{1}{\text{Var}(\tilde{v} + \tilde{\zeta}|\tilde{p})}$ ), return volatility ( $\sigma(\tilde{v} + \tilde{\zeta} - \tilde{p})$ ), price momentum ( $m$ ), and trading volume against the marginal cost of decreasing cursedness  $\kappa$ . The parameters are set as follows:  $\tau_v = \tau_e = \tau_c = \tau_u = 25$ ,  $N = 10$ , and  $\gamma = 2$ .

## Endnotes

<sup>1</sup>As discussed by Guesnerie (1992), this comprehension is broadly justified in two ways: the “eductive” justification that relies on the understanding of the logic of the situation faced by economic agents and that is associated with mental activity of agents aiming at “forecasting the forecasts of others” and the “evolutive” justification that emphasizes the learning possibilities offered by the repetition of the situation and that is associated with the convergence of several versions of learning processes. See section 7.1 in Vives (2008).

<sup>2</sup>See section 5 of Eyster et al. (2019) for extensive evidence that people do not sufficiently heed the information content of others’ behavior and of financial markets. Addoum and Murfin (2017) document that equity market participants fail to account for information reflected in publicly posted loan prices.

<sup>3</sup>Recently, Angeletos and Sastry (2019) find a related result of strategic complementarity and multiplicity with rational inattention: when other agents pay less attention, prices are more confusing, and one agent finds it harder to pay attention in a setting of complexity.

<sup>4</sup>Noise is only added to the intercept  $c_0$  primarily for the sake of tractability. It is the simplest possible additive noise. Hence, we assume that self 1 observes the correct slope  $c_1$ . This assumption is analogous to the standard ordinary least squares regression analysis, in which an error term is only added to the intercept.

<sup>5</sup>For instance, in describing the speculative trading activities in the Chinese stock market, a recent *Financial Times* article states, “Speculative decisions to buy or sell, on the other hand, are driven by market technicals, investment fads... speculative investing tends to be driven by expected changes in the market consensus, rather than by expected shifts in economic growth prospects” (“Fundamentals simply do not matter in China’s stock markets,” *Financial Times*, January 14, 2020, <https://www.ft.com/content/2362a9a-03479-11ea-a6d3-9a26f8c3cba4>). Although this anecdote does not speak directly to the common error assumption, it is consistent with the notion that, in reality, investors trade on some form of sentiment (“investment fads” or “market consensus”). Anything that affects expected changes in the market consensus affects the information processed by prices. That is, if it is a positive shock (optimism) then there is a bias to interpret price changes as good news. See section 2 of Bodoh-Creed (2020) for a summary of psychology and empirical literature on how sentiment—induced by factors, such as weather, seasonal affective disorder, and sporting match outcomes—can influence the beliefs and decision making of retail and institutional investors as well as asset prices.

<sup>6</sup> Although, throughout the entire paper, we take a behavioral interpretation based on individual investors, we can alternatively interpret the two selves of our investors as the research department (self 0) and trading desk (self 1) of an investment institution. The trading desk is responsible for trading assets, and it relies on the institution's research department to have an understanding of how to generate information from prices. Research departments describe the procedure on how to extract the best signal from prices in the form of research reports, but trading desks add noise in comprehending the procedures in the reports.

<sup>7</sup> We have also solved our setup in Section 3.1 with a finite number of investors  $N$  to make it more comparable to Eyster et al. (2019). The main implications of the model are robust to a finite number of traders.

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