Internet Appendix for
“Exchange Competition, Entry, and Welfare”

In this appendix, we collect several robustness extensions to the analysis of the liquidity determination stage of the game developed in the baseline model of the paper.

1 A model with SD at the second round

In this section we consider a variation of the baseline model presented in the paper, in which we assume that SD enter the market at the second round of the liquidity determination stage of the game (the proofs of the results involve minor variations from the ones in of the baseline model and we omit them for brevity). This captures the intuition that through technological services FD are quicker in accommodating liquidity traders’ demand shocks than SD. In this case, the market clearing conditions in periods 1 and 2 are given respectively by $x^L_1 + \mu x^{FD}_1 = 0$ and $x^L_2 + \mu (x^{FD}_2 - x^{FD}_1) + (1 - \mu)x^{SD}_2 = 0$ (see Figure 1 for the modified timeline). We restrict attention to linear equilibria where

\begin{align*}
p_1 &= -\tilde{\Lambda}_1 u_1, \quad (1a) \\
p_2 &= -\tilde{\Lambda}_2 u_2 + \tilde{\Lambda}_{21} u_1, \quad (1b)
\end{align*}

where we use $\sim$ to denote variables related to the model with SD entering at the second round.

\begin{align*}
\tilde{\Lambda}_1 &= 1 + (\gamma + \gamma^L)\mu \gamma \tau_u \tau_v \\
\tilde{\Lambda}_2 &= \frac{1}{(\gamma + \gamma^L)\tau_v} \\
\tilde{\Lambda}_{21} &= \left(\frac{\gamma^L \tau_v \tilde{\Lambda}_1 - 1}{\tilde{\Lambda}_2}\right) \tilde{\Lambda}_2.
\end{align*}

We obtain the following result:

**Proposition 1.** For $\mu \in (0,1]$, there exists a unique equilibrium in linear strategies in the stock market where SD enter at the second round, where

- Exchanges make costly entry decision; N enter.
- Dealers acquire FD technology.
- Platforms make technological capacity decisions ($\mu_i$).
- Liquidity traders receive $u_1$ and submit market order $x^{FD}_1$.
- FD submit limit order $\mu x^{FD}_1$.
- New cohort of liquidity traders receives $u_2$, observes $p_1$, and submits market order $x^{FD}_2$.
- FD submit limit order $\mu x^{FD}_2$.
- SD submit limit order $(1 - \mu)x^{SD}_2$.

Figure 1: Timeline in the model where SD enter at the second round.
Compared to the baseline case, \( \tilde{\Lambda}_1 > \Lambda_1, \Lambda_1 < \tilde{\Lambda}_2 < \Lambda_2 \), and \( |\tilde{\Lambda}_{21}| > |\Lambda_{21}| \).

Thus, SD entry at the second round reduces (increases) the competitive pressure faced by FD at the first (second) round, explaining the decrease (increase) in first (second) period liquidity. Comparing dealers’ payoffs across the two models (maintaining the convention of using \( \tilde{~} \) to denote the variables obtained in the RO case), we obtain the following result.

**Proposition 2.** \( \tilde{CE}^{FD} > \tilde{CE}^{SD} \), and SD have a higher payoff when entering in the second round, whereas the result for FD is ambiguous: \( \tilde{CE}^{SD} > CE^{SD} \), and \( \tilde{CE}^{FD} \gtrless CE^{FD} \).

As in the baseline model, more access to the liquidity supply market has value for dealers. In the baseline model, in the first round FD supply liquidity anticipating the possibility to rebalance their position at the second round. This heightens the competitive pressure they exert on SD compared to the model studied in this section, explaining why \( \tilde{CE}^{SD} > CE^{SD} \). The payoff comparison for FD is less clear cut. Indeed, compared to the baseline model, liquidity is lower (higher) at the first (second) round. We define the demand for technological services as \( \tilde{\phi}(\mu) = \tilde{CE}^{FD} - \tilde{CE}^{SD} \).

**Proposition 3.** In the model where SD enter at the second round, \( \tilde{\phi}(\mu) \) is decreasing in \( \mu \).

Our numerical simulations yield the following results: we obtain that entry is excessive at all \( \tau \).

Additionally, conduct (fee) regulation yields \( N^{CO} = 1 \) and is welfare superior to structural (entry) regulation.

## 2 A model with “Committed” SD

In the liquidity provision model of Section 3, only FD can supply liquidity to second period traders. This is a convenient assumption which however fails to recognize that in actual markets liquidity provision is ensured by a multitude of market participants. In this section we check how the introduction of a mass of “committed” SD, present in the market at each trading round, can alter the risk-sharing properties of the market, affecting exchanges’ technology supply, and the welfare ranking among different forms of regulatory intervention.

More in detail, we assume that a mass \( \epsilon \) of “committed” SD is in the market at each trading round, and cannot choose to become FD—so that a total mass of \( 2\epsilon \) SD is unable to become FD. As a consequence, at the first round liquidity is supplied by \( \mu \) FD, and a mass

\[
\epsilon + 1 - \epsilon - \mu = 1 - \mu
\]

of SD. At the second round, instead, we have a total mass \( \mu \) FD and \( \epsilon \) SD of liquidity suppliers (see Figure 2 for the modified timeline of the model).

\footnote{In the extended set of simulations we obtain that entry is insufficient for low risk aversion, and intermediate \( f \).}
Exchanges make costly entry decision; N enter.

Liquidity traders receive $u_1$ and submit market order $x_1^L$.

FD submit limit order $\mu x_1^{FD}$.

SD submit limit order $(1-\mu)x_1^{SD}$.

Platforms make technological capacity decisions ($\mu_i$).

New cohort of liquidity traders receives $u_2$, observes $p_1$, and submits market order $x_2^L$.

FD submit limit order $\mu x_2^{FD}$.

Committed SD submit limit order $\epsilon x_2^{SD}$.

Dealers acquire FD technology.

Figure 2: Timeline in the model with committed SD.

With committed SD the market clearing equations in Section 2.4 are replaced by: $\mu x_1^{FD} + (1-\mu)x_1^{SD} + x_1^L = 0$, and $(x_2^{FD} - x_1^{FD})\mu + \epsilon x_2^{SD} + x_2^L = 0$, where $x_2^{SD}$ denotes the position of a committed dealer at the second round (see (7)). Thus, denoting by $\tilde{\Lambda}_t$, $\tilde{\Lambda}_{21}$, $\tilde{a}_t$, and $\tilde{b}$ the coefficients of the linear equilibrium with committed dealers we prove the following result:

**Proposition 4.** For $\mu \in [0,1]$, there exists a unique equilibrium in linear strategies in the stock market where a mass $\epsilon$ of SD is in the market at both rounds. At equilibrium, the sign of the comparative statics effect of $\mu$, as well as the ranking between liquidity traders’ hedging aggressiveness is preserved, while

1. $\tilde{\Lambda}_t$, $|\tilde{a}_t|$ are respectively decreasing and increasing in $\epsilon$.

2. $|\tilde{\Lambda}_{21}|$ and $\tilde{b}$ are decreasing in $\epsilon$.

**Proof.** To obtain the equilibrium of the liquidity provision model in this case, we start by assuming that equilibrium prices are as follows:

$$p_1 = -\tilde{\Lambda}_1 u_1 \quad (3a)$$
$$p_2 = -\tilde{\Lambda}_2 u_2 + \tilde{\Lambda}_{21} u_1 \quad (3b)$$

where $\tilde{\Lambda}_{21}$, $\tilde{\Lambda}_t$, $t \in \{1,2\}$ denote the price coefficients to be determined at equilibrium. The market clearing conditions are then given by:

$$\mu x_1^{FD} + (1-\mu)x_1^{SD} + x_1^L = 0 \quad (4a)$$
$$ (x_2^{FD} - x_1^{FD})\mu + \epsilon x_2^{SD} + x_2^L = 0. \quad (4b)$$

We now obtain the equilibrium strategies of market participants. For a second period liquidity trader, (A.5) holds:

$$x_2^L = \tilde{a}_2 u_2 + \tilde{b} u_1, \quad (5)$$

where

$$\tilde{a}_2 = \gamma^L r_v \tilde{\Lambda}_2 - 1 \quad (6a)$$
$$\tilde{b} = -\gamma^L r_v \tilde{\Lambda}_{21} \quad (6b)$$
denote the coefficient of a second period liquidity trader in the current version of the liquidity provision model. Based on the arguments developed in the proof of Proposition 1, for a SD who is in the market at round \( t \), we have:

\[
x^{SD}_t = -\gamma \tau_v p_t,
\]

(7)

whereas for a FD at the second round,

\[
x^{FD}_2 = -\gamma \tau_v p_2.
\]

(8)

For a first period liquidity trader (A.16) holds:

\[
x^L_1 = \tilde{a}_1 u_1
\]

(9a)

\[
\tilde{a}_1 = \gamma^L \tau_v \tilde{\Lambda}_1 - 1.
\]

(9b)

Finally, for a FD at the first trading round (A.22) holds.

Solving the first period market clearing condition (4a) for \(-x^{FD}_1\) and replacing it in the second period market clearing condition (4b) yields

\[
\mu x^{FD}_2 + (1 - \mu) x^{SD}_1 + x^L_1 + x^L_2 + \epsilon x^{SD}_2 = 0.
\]

(10)

We can now replace SD, FD and liquidity traders’ strategies from (7), (8), (9a) in the market clearing condition (10), and identify the second period price equilibrium coefficients:

\[
p_2 = \frac{1 - \mu}{(\epsilon + \mu) \gamma \tau_v} u_1 + \frac{\tilde{a}_2}{(\epsilon + \mu) \gamma \tau_v} u_2.
\]

(11)

Finally, using (6a), (6b), and (9b) in the expressions for \( \tilde{\Lambda}_2 \) and \( \tilde{\Lambda}_{21} \) in (11), we identify the second period price coefficients, obtaining:

\[
\tilde{\Lambda}_2 = \frac{1}{((\epsilon + \mu) \gamma + \gamma^L) \tau_v} \quad (12a)
\]

\[
\tilde{\Lambda}_{21} = -(1 - ((1 - \mu) \gamma + \gamma^L) \tau_v \tilde{\Lambda}_1) \tilde{\Lambda}_2.
\]

(12b)

Therefore, the model with a fixed mass of SD at both rounds induces a lower \( \tilde{\Lambda}_2 \), as one would expect.

The analysis of the first round does not change compared to the baseline model, since in that case too a mass \( \mu \) of SD is in the market. To determine \( \tilde{\Lambda}_1 \) we replace the strategies of liquidity providers (FD and SD), and liquidity traders in the first period market clearing equation (4a), solve for \( p_1 \), and identify \( \tilde{\Lambda}_1 \) which, according to (A.23) is given by:

\[
\tilde{\Lambda}_1 = \left( 1 + \frac{\mu \gamma^L \tau_u}{\Lambda_2 + \mu \gamma \tau_u} \right) \frac{\gamma + \gamma^L}{\tau_v}^{-1} \frac{1}{\tau_v}
\]

(13)

We only need to keep in mind the change in \( \tilde{\Lambda}_2 \), which implies the following closed form expression for the first period price impact:

\[
\tilde{\Lambda}_1 = \frac{1 + \gamma \mu \tau u_v (\gamma^L + \gamma (\mu + \epsilon))}{(\gamma + \gamma^L + \gamma \mu \tau_u v (\gamma + 2 \gamma^L) (\gamma^L + \gamma (\mu + \epsilon))) \tau_v} > 0.
\]

(14)

Note that since \( \tilde{\Lambda}_2 \) is decreasing in \( \epsilon \), based on (14), and (12b) it follows that \( \tilde{\Lambda}_1 \) is also decreasing in \( \epsilon \), and differentiating \( \tilde{\Lambda}_{21} \):

\[
\frac{\partial \tilde{\Lambda}_{21}}{\partial \epsilon} = \frac{\gamma^2 \mu (\gamma + \gamma^L) \tau_v + \gamma \mu \tau_u^2 \tau_v (\gamma + 2 \gamma^L) (\gamma \mu + \gamma^L) (\gamma^L + \gamma (\mu + \epsilon))^2 + 2 \gamma \mu \tau_u (\gamma + 2 \gamma^L) (\gamma^L + \gamma (\mu + \epsilon))) (\gamma^L + \gamma (\mu + \epsilon)))}{(\gamma^L + \gamma (\mu + \epsilon))^2 (\gamma + \gamma^L + \gamma \mu \tau_u \tau_v (\gamma + 2 \gamma^L) (\gamma^L + \gamma (\mu + \epsilon)))^2} > 0,
\]
implying that $|\tilde{A}_{21}|$, and $\tilde{b}$ are decreasing in $\epsilon$.

Furthermore, since $\tilde{A}_t$ is decreasing in $\epsilon$, due to (6a) and (9b), $|\tilde{a}_t|$ is increasing in $\epsilon$, and since by inspection $\tilde{A}_1 < \tilde{A}_2$, $|\tilde{a}_1| > |\tilde{a}_2|$, this concludes the proof of Proposition 4.

Committed dealers improve the risk bearing capacity of the market, increasing its liquidity at both rounds, and leading traders to hedge a larger fraction of their endowment shock. As second period traders face heightened competition in speculating against the propagated endowment shock, a larger $\epsilon$ reduces their response to $u_1$ ($\hat{b}$).

To measure the impact on the welfare of market participants and the market for technological services, we appropriately replace the equilibrium coefficients with their tilde-ed counterparts in the expressions for the market participants’ payoffs (see (6a), (6b), (8a), and (8b)):

$$CE_{1SD}^{SD} = \frac{\gamma}{2} \ln \left( 1 + \frac{\tilde{A}_{21}^2 \tau_v}{\tau_u} \right), \quad (15a)$$

$$CE_{1FD} = \frac{\gamma}{2} \left( \ln \left( 1 + \frac{\tilde{A}_{21}^2 \tau_v}{\tau_u} \right) + \left( \frac{\tilde{A}_{21} + \tilde{A}_1}{\tilde{A}_2} \right)^2 \right) + \ln \left( 1 + \frac{\tilde{A}_{21}^2 \tau_v}{\tau_u} \right), \quad (15b)$$

where we note that, in view of Proposition 4, $CE_{1SD}^{SD}$ is decreasing in $\mu$, and since

$$\frac{\partial}{\partial \mu} \frac{\tilde{A}_{21} + \tilde{A}_1}{\tilde{A}_2} = - \frac{\gamma \gamma L \tau_v (\gamma + 2\gamma L + \gamma \epsilon) (2\gamma \mu + \gamma L + \gamma \epsilon)}{(\gamma + \gamma L + \gamma \mu \tau_v (\gamma + 2\gamma L) (\gamma L + \gamma (\mu + \epsilon)))^2} < 0,$$

$CE_{1FD}$ is also decreasing in $\mu$. Also, we have

$$CE_{1L}^{L} = \frac{\gamma L}{2} \ln \left( 1 + \frac{\text{Var}[E[v - p_1|p_1]]}{\text{Var}[v - p_1|p_1]} \right) + \frac{2 \text{Cov}[p_1, u_1]}{\gamma L}, \quad (16a)$$

$$CE_{2L}^{L} = \frac{\gamma L}{2} \ln \left( 1 + \frac{\text{Var}[E[v - p_2|p_1,p_2]]}{\text{Var}[v - p_2|p_1,p_2]} \right) + \frac{2 \text{Cov}[p_2, u_2|p_1]}{\gamma L} + \frac{\text{Var}[E[v - p_2|p_1]]}{\text{Var}[v]} - \left( \frac{\text{Cov}[p_2, u_1]}{\gamma L} \right)^2, \quad (16b)$$

and a substitution similar to the one made in the case $\epsilon = 0$ allows to express traders’ payoffs in terms of the equilibrium coefficients and conclude that they $CE_{1L}^{L}$ is increasing in $\mu$.

Finally, since the strategy of a second round SD, bar the time index, is identical to that of a SD at the first round, we have:

$$CE_{2SD}^{SD} = \frac{\gamma}{2} \ln \left( 1 + \frac{\text{Var}[p_2|p_1]}{\text{Var}[v]} \right) = \frac{\gamma}{2} \ln \left( 1 + \frac{\tilde{A}_{21}^2 \tau_v}{\tau_u} \right), \quad (17)$$

which, in view of Proposition 4, is decreasing in $\mu$.

Comparing (15b) with (15a) we see that $CE_{1FD} > CE_{1SD}^{SD}$, and, consistently with what we have done in the baseline case, we define the demand for technological services as the difference between the payoff of a FD and the one of a first period SD:

$$\phi(\mu) = CE_{1FD} - CE_{1SD}^{SD}.$$

Differentiating the above expression with respect to $\mu$ yields $\partial \phi / \partial \mu > 0$.

Finally, defining the inverse demand for technological services as the payoff difference between FD and first period dealers: $\phi(\mu) \equiv CE_{1FD} - CE_{1SD}^{SD}$, we also obtain the following result:
Corollary 1. With committed SD:

1. The comparative statics effect of $\mu$ on dealers’ and traders’ payoffs, and the inverse demand for technological services, are as in Propositions 2, 3, and Corollary 5. Furthermore, the payoff of second period committed dealers is decreasing in $\mu$.

2. Standard dealers’ payoffs are decreasing in $\epsilon$.

Committed dealers heighten competition in the provision of liquidity, explaining the second part of Corollary 1. Numerical simulations show that an increase in $\epsilon$ increases the payoff of liquidity traders at both rounds as well as that of FD. The former effect is in line with the improved liquidity provision enjoyed by liquidity traders. The intuition for the latter is that besides the competitive effect, committed dealers also improve FD ability to share risk when they retrade at the second period. This also explains why in our simulations, the demand for technological services can be non-monotone in $\epsilon$, as shown in Figure 3.

More in detail, the demand for technological services can increase (decrease) with a higher $\epsilon$ in the low (high) payoff volatility scenario. This is consistent with the fact that an increase in $\tau_v$ leads first period traders to hedge a larger portion of their endowment, increasing FD risk exposure, and thereby increasing the value of technological services to this class of liquidity providers. Indeed, based on (9b) and (14), $\frac{\partial \bar{a}_1}{\partial \tau_v} < 0$.

Our simulations also confirm that as in Numerical Result 1, gross welfare: $GW(\mu) = \mu(CE^{FD} - CE_1^{SD}) + CE_1^{SD} + \epsilon CE_2^{SD} + CE_1^L + CE_2^L$, is increasing in $\mu$. Furthermore, we also find that $GW(\mu)$ is increasing in $\epsilon$.

We can now use the model to rank market outcomes against the different welfare benchmarks introduced in Section 5.

Numerical Result 1. With committed dealers, for $\epsilon$ small, we obtain Numerical Result 2. Otherwise:

1. With high payoff volatility ($\tau_v = 3$), $\pi^M(\mu^{FB}) < 0$, and the general ranking result of Proposition 8 applies.

2. With low payoff volatility ($\tau_v = 25$), the monopoly profit function is larger and flatter than with $\epsilon = 0$. We have that $\pi^M(\mu^{FB}) > 0$, and:

\[
\gamma = 0.5, \gamma^L = 0.25, \tau_u = 100, \tau_v = 3
\]

\[
\gamma = 0.5, \gamma^L = 0.25, \tau_u = 100, \tau_v = 25
\]

Figure 3: Comparative statics effect of an increase in $\epsilon$ on the demand for technological services.

We have run our numerical simulations assuming $\epsilon \in \{0.01, 0.03, 0.05\}$, and standard risk aversion. When $\epsilon \in \{0.01, 0.03\}$ and $\tau_v = 25$, insufficient entry obtains for intermediate values of $f$, as in the baseline model. When $\epsilon = 0.05$, only excessive entry obtains.
(a) $\mu^{ST} = \mu^{UST}$, and $\mu^{CO} > \mu^{CFE} \geq \max\{\mu^{FB}, \mu^{ST}\}$.

(b) Entry regulation can yield a higher welfare than fee regulation when fixed costs are small. Furthermore, at all solutions $N$ and $\mu$ are decreasing in $f$.

In Figure 4 we present a case in which, for a small entry cost, entry regulation yields a higher welfare than fee regulation, a result that is at odds with Proposition 8. The reason for this finding is as follows. The presence of committed dealers boosts the demand for technological services, making the monopoly solution “closer” to the first best (with $\epsilon > 0$, $\pi^M(\mu^{FB})$ can be positive). In this case, increasing welfare via fee regulation, requires the planner to set $\mu$ very high (much higher than at FB), substantially depressing industry profits for mild liquidity gains. Thus, for a small entry cost, controlling $\mu$ by choosing $N$ can be better. Summarizing:

- When $\mu^M$ and $\mu^{FB}$ are far apart ($\pi^M(\mu^{FB}) < 0$), a very high $N$ is needed to increase capacity via entry, which is very expensive in terms of fixed costs (see Figure 5, panel (a)), and it is optimal to control $\mu$ to induce $N = 1$.

- When $\mu^M$ and $\mu^{FB}$ are closer ($\pi^M(\mu^{FB}) > 0$), increasing welfare with CO substantially depresses industry profits for mild liquidity gains. In this case, it may be better to control $\mu$ by choosing $N$ (see Figure 5, panel (b)).
Figure 5: The figure illustrates the effectiveness of alternative regulatory tools to approach $\mu^{FB}$. In panel (a) $\epsilon = 0$, $\pi^M(\mu^{FB}) < 0$, and fee regulation dominates. In panel (b) $\epsilon > 0$, and entry regulation can dominate.

3 A model with free-entry into market making

In this section, we suppose that there exists a mass $M = 1$ of potential dealers and that each one of them assigns a value $K > 0$ to the alternative of staying out of the intermediation industry. In this way, the mass of dealers who enter the industry $\hat{\mu} \leq 1$ is given by the solution to the following equation:

$$CE^SD(\mu(\hat{\mu}); \hat{\mu}) = K,$$

where $CE^SD$ denotes the equilibrium certainty equivalent of a SD when $\hat{\mu}$ dealers enter the market and $\mu(\hat{\mu})$ of them become FD. We thus assume that timewise, dealers’ entry decisions take place before the capacity determination and liquidity provision stages of the game. Hence, at equilibrium, a potential dealer must anticipate the mass of agents who become dealers ($\hat{\mu}$), as well as the mapping between the latter and the mass of those who choose to become FD ($\mu(\hat{\mu})$). The figure below displays the timeline of events.
Potential dealers decide whether to enter the intermediation industry; \( \hat{\mu} \in (0, 1] \) enter.

- Exchanges make costly entry decision; \( N \) enter.
- Dealers acquire FD technology.
- Platforms make technological capacity decisions (\( \mu(\hat{\mu}) \)).

Liquidity traders receive \( u_1 \) and submit market order \( x^L_1 \).
- FD submit limit order \( \mu(\hat{\mu})x^{FD}_1 \).
- SD submit limit order \( (\hat{\mu} - \mu(\hat{\mu}))x^{SP}_1 \).

Dealers acquire FD technology.
- Platforms make technological capacity decisions (\( \mu_i(\hat{\mu}) \)).

New cohort of liquidity traders receives \( u_2 \), observes \( p_1 \), and submits market order \( x^L_2 \).
- FD submit limit order \( \mu(\hat{\mu})x^{FD}_2 \).

Figure 6: The timeline.

We determine the equilibrium via a backward induction argument, breaking down its calculation in three steps which lead to three equilibrium conditions:

1. **Equilibrium conditions implied by the liquidity provision stage of the game.** We start by assuming that \( \hat{\mu} \) dealers enter prior to the liquidity provision stage of the game, and that \( \mu(\hat{\mu}) \) of them become FD (so that the mass of SD is given by \( \hat{\mu} - \mu(\hat{\mu}) \)). Then, the sequence of market clearing equations reads as follows:

\[
\begin{align*}
\mu(\hat{\mu})x^{FD}_1 + (\hat{\mu} - \mu(\hat{\mu}))x^{SD}_1 + x^L_1 &= 0 \\
\mu(\hat{\mu})(x^{FD}_2 - x^{FD}_1) + x^L_2 &= 0.
\end{align*}
\]

As market participants are price takers, the strategies we derived in the baseline case still apply:

\[
\begin{align*}
x^{FD}_2 &= -\gamma \tau v p_2 \\
x^{FD}_1 &= \gamma \frac{\Lambda^*_1 \tau u}{(\Lambda^*_2)^2} u_1 - \frac{\gamma}{(\Lambda^*_2)^2} p_1 \\
x^{SD}_1 &= -\gamma \tau v p_1 \\
x^L_2 &= (\gamma L \tau v \Lambda^*_2 - 1) u_2 - \gamma L \tau v \Lambda^*_2 u_1 \\
x^L_1 &= (\gamma L \tau v \Lambda^*_1 - 1) u_1,
\end{align*}
\]

where the starred variables denote the equilibrium values of the price coefficients. Applying the same steps to solve for the equilibrium prices that we used for the baseline model, we obtain the following expressions:

\[
\begin{align*}
p^*_1 &= -\Lambda^*_1 u_1 \\
p^*_2 &= -\Lambda^*_2 u_2 + \Lambda^*_2 u_1,
\end{align*}
\]
where

\[
\Lambda_1^* = \frac{1 + \gamma \mu \tau_v (\gamma \mu \hat{\mu} + \gamma^L)}{\tau_v (\gamma \mu \hat{\mu} + \gamma^L)(\gamma \hat{\mu} + 2 \gamma^L) + \gamma \hat{\mu} + \gamma^L} \\
\Lambda_2^* = \frac{1}{\tau_v (\gamma \mu \hat{\mu} + \gamma^L)} \\
\Lambda_{21}^* = -\frac{\gamma \mu \hat{\mu} (1 + \tau_v (\gamma \mu \hat{\mu} + \gamma^L)^2)}{\tau_v (\gamma \mu \hat{\mu} + \gamma^L)(\gamma \mu \hat{\mu} + \gamma^L)(\gamma \hat{\mu} + 2 \gamma^L) + \gamma \hat{\mu} + \gamma^L}.
\]

(23)  
(24)  
(25)

2. Equilibrium condition implied by the capacity determination stage of the game. We can now replace (23)-(25) into the certainty equivalents of market participants, and define the demand for technological services as follows:

\[
\phi^*(\mu(\hat{\mu}); \hat{\mu}) = CE^{FD} - CE^{SD}.
\]

(26)

A platform’s profit function is then given by

\[
\pi^*(\mu(\hat{\mu}); \hat{\mu}) = (\phi^*(\mu(\hat{\mu}); \hat{\mu})) - c \frac{\mu(\hat{\mu})}{N} - f.
\]

(27)

Depending on the platform competition assumptions, this step yields a different equilibrium functional relationship \(\mu(\hat{\mu})\). For example, with a monopolist, the equilibrium mapping \(\mu(\hat{\mu})\) comes from the FOC of the profit maximization problem:

\[
\phi^*(\mu(\hat{\mu}); \hat{\mu}) = \left(1 + \frac{\partial \phi^*}{\partial \mu} \frac{\mu}{\phi^*}\right)^{-1} c.
\]

(28)

When \(N\) platforms are in the market, the relationship obtains from the equilibrium condition of the CFE problem. When instead the installed capacity is chosen by the planner, \(\mu(\hat{\mu})\) comes from the relevant FOC of the planner’s problem (plus the constraints that apply to each program).

Note that in this case the gross welfare function obtains by substituting (23)-(25) into the payoff expressions for both dealers and liquidity traders, and computing

\[
GW^*(\mu(\hat{\mu}); \hat{\mu}) = \mu(\hat{\mu}) CE^{FD} + (\hat{\mu} - \mu(\hat{\mu})) CE^{SD} + (1 - \hat{\mu}) K + CE_1^L + CE_2^L
= \mu(\hat{\mu}) \phi^*(\mu(\hat{\mu})) + \hat{\mu} (CE^{SD} - K) + K + CE_1^L + CE_2^L,
\]

(29)

which yields the following expression for the planner’s objective function:

\[
\mathcal{P}(\mu(\hat{\mu}); \hat{\mu}) = GW^*(\mu(\hat{\mu}); \hat{\mu}) - c \mu(\hat{\mu}) - f N
= \pi^*(\mu(\hat{\mu}); \hat{\mu}) N + \psi^*(\mu(\hat{\mu}); \hat{\mu}),
\]

where in the first line of the above expression \(GW^*(\hat{\mu})\) is as in (28), whereas in the second line we have

\[
\psi^*(\mu(\hat{\mu}); \hat{\mu}) = \hat{\mu} (CE^{SD} - K) + K + CE_1^L + CE_2^L.
\]

(30)

3. Equilibrium condition implied by the entry decision of dealers. Finally, replacing the equilibrium mapping \(\mu(\hat{\mu})\) obtained at the previous step in dealers’ payoffs, we pin down the mass of dealers who enter the intermediation industry using (18), where

\[
CE^{SD}(\mu(\hat{\mu}); \hat{\mu}) = \frac{\gamma}{2} \ln \left(1 + \frac{(\Lambda_1^*)^2 \tau_v}{\tau_u}\right).
\]

(31)
This extension is more complicated than our baseline model. Indeed, the mapping \( \mu(\hat{\mu}) \) (which is obtained at the second step of the algorithm described above) becomes a crucial ingredient of equilibrium determination (i.e., the solution of (18)). Given that the condition yielding \( \mu(\hat{\mu}) \) is non-linear, we cannot rely on closed form solutions. Our numerical simulations confirm the findings of Numerical Result 2:

1. With low payoff volatility \( (\tau_v = 25) \), for intermediate values of \( f \), entry is insufficient.
2. With high payoff volatility \( (\tau_v = 3) \), entry is always excessive.

Additionally, in all our results \( N^{CO} = 1 \).

### 4 A model with asymmetric information

In this extension we assume that at the second round a liquidity trader receives an imperfect signal about the first period endowment shock: \( s_i = u_1 + \epsilon_i \), where \( \epsilon_i \sim N(0, \tau_v^{-1}) \), i.i.d. across traders and orthogonal to all the other random variables in the model. With this assumption, the second period information set of a liquidity trader becomes \( \Omega_2^L = \{u_2, s_i\} \).

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<td>Dealers</td>
<td>Liquidity traders receive ( u_1 ) and submit market order ( x_1^L ).</td>
<td>New cohort of liquidity traders receives ( u_2 ), observes ( s_i = u_1 + \epsilon_i ), and submits market order ( x_2^L ).</td>
<td>Asset liquidates.</td>
</tr>
<tr>
<td>make costly entry decision; ( N ) enter.</td>
<td>acquire FD technology.</td>
<td>FD submit limit order ( \mu x_1^{FD} ).</td>
<td>FD submit limit order ( \mu x_2^{FD} ).</td>
<td></td>
</tr>
<tr>
<td>Platforms make technological capacity decisions ( (\mu_i) ).</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Entry and capacity determination stage

Liquidity determination stage (virtual single platform)

Assuming prices are as in (A.1a), (A.1b) in the paper (as error terms in traders’ signals are idiosyncratic, the SLLN convention applies so that prices only depend on endowment shocks), his equilibrium strategy is given by

\[
X^L_2(u_1, s_i) = \gamma^L \frac{E[v - p_2|\Omega^L_2]}{\Var[v - p_2|\Omega^L_2]} - \frac{\Cov[v - p_2, v|\Omega^L_2]}{\Var[v - p_2|\Omega^L_2]},
\]

where:

\[
E[v - p_2|\Omega^L_2] = \Lambda_2 u_2 - \Lambda_2 \beta \tau_v s_i
\]

\[
\Var[v - p_2|\Omega^L_2] = 1 + \beta \Lambda_2^2 \tau_v
\]

and \( \beta = (\tau_u + \tau_v)^{-1} \). Substituting the above expressions into (32) and rearranging, we obtain:

\[
X^L_2(u_1, s_i) = (\gamma^L \Lambda_2 \tau_v - 1) \frac{1}{1 + \beta \Lambda_2^2 \tau_v} u_2 - \gamma^L \tau_v \frac{\beta \Lambda_2 \tau_v}{1 + \beta \Lambda_2^2 \tau_v} s_i.
\]

\[\text{(34)}\]
Asymmetric information impairs second period traders’ ability to anticipate \( u_1 \), leading them to adjust their strategies in two ways. First, they tune their hedging aggressiveness to the second period price reaction to \( u_1 \). The higher (in absolute value) is \( \Lambda_{21} \), the larger is the impact of \( u_1 \) on \( p_2 \), which signals a lower ability of FDs to absorb the second period endowment shock. Other things equal, this leads second period traders to limit their hedging activity. Second, since

\[
\left| \gamma^L \tau_e \frac{\beta \Lambda_{21} \tau_e}{1 + \beta \Lambda_{21}^2 \tau_e} \right| < \left| \gamma^L \tau_e \Lambda_{21} \right|
\]

they reduce their speculative activity on \( u_1 \).

The expressions for all other traders’ strategies at both trading rounds do not change compared to the baseline model, so that we can replace (34) together with (A.10), (A.12), and (A.16) into (A.8) and solve for the second period equilibrium price, obtaining

\[
p_2 = -\Lambda_2 u_2 + \Lambda_{21} u_1,
\]

where

\[
\Lambda_2 = -\left( \gamma^L \Lambda_2 \tau_v - 1 \right) \frac{1}{(1 + \beta \Lambda_{21}^2 \tau_v) \mu \gamma \tau_v} \quad (35a)
\]

\[
\Lambda_{21} = \frac{((1 - \mu) \gamma + \gamma^L) \Lambda_1 \tau_v - 1}{((1 - \beta \tau_e) \gamma^L \Lambda_2 \tau_v - 1) \mu \gamma \tau_v} \left( (\gamma^L \Lambda_2 \tau_v - 1) \right).
\]

Depending on the precision of second period traders’ signal, we can divide the analysis into three cases:

1. **Perfect signal.** This is what happens when \( \tau_e \to \infty \), so that \( \beta \to 0 \), and \( \beta \tau_e \to 1 \). In this case (35a) and (35b), respectively tend to

\[
\Lambda_2 = -\gamma^L \Lambda_2 \tau_v - 1 \quad (36a)
\]

\[
\Lambda_{21} = \left( (1 - \mu) \gamma + \gamma^L \right) \Lambda_1 \tau_v - 1 \Lambda_2, \quad (36b)
\]

as in the baseline case. Thus, in this case, the equilibrium is the same as in our baseline model.

2. **Imprecise signal:** \( 0 < \tau_e < \infty \). In this case, solving (35a) for \( \Lambda_2 \) and rearranging yields:

\[
\Lambda_2 = \frac{1}{(\gamma^L + \mu \gamma (1 + \beta \Lambda_{21}^2 \tau_v))) \tau_v} \quad (37)
\]

Substituting the first period strategies of FD, SD, and liquidity traders (respectively, (A.22), (A.12), and (A.16)) in the first period market clearing equation (A.7a) and solving for the equilibrium price yields

\[
p_1 = -\Lambda_1 u_1,
\]

where

\[
\Lambda_1 = \frac{\Lambda_2^2 - \mu \gamma \tau_u \Lambda_{21}}{\mu \gamma \tau_u + (\gamma + \gamma^L) \Lambda_{21}^2 \tau_v}. \quad (38)
\]

Finally, replacing (35b) in (38) and solving for \( \Lambda_1 \) yields

\[
\Lambda_1 = \frac{(\gamma^L \Lambda_2 \tau_v - 1) \tau_u + (1 + (\beta \tau_e - 1) \gamma^L \Lambda_2 \tau_v) \Lambda_2^2 \tau_v}{((\gamma + \gamma^L)(1 - \gamma^L \Lambda_2 \tau_v) (\tau_u + \Lambda_2^2 \tau_v) + (\mu \gamma \tau_u + (\gamma + \gamma^L) \Lambda_{21}^2 \tau_v) \beta \Lambda_2 \gamma^L \tau_e \tau_v) \tau_v}. \quad (39)
\]
We can now replace \((39)\) into \((35b)\), and successively replace the resulting expression at the RHS of \((37)\), obtaining a mapping

\[
\lambda_2 = g(\lambda_2).
\]

The fixed points of the above mapping correspond to the roots of a 7-th degree equation \(G(\lambda_2) = \lambda_2 - g(\lambda_2) = 0\), which can be written as follows:

\[
G(\lambda_2) = \Lambda_2^5q_1 + \Lambda_2^5q_2 + \Lambda_2^5q_3 + \Lambda_2^5q_4 + \Lambda_2^5q_5 + \Lambda_2^5q_6 + \Lambda_2q_7 + q_8 = 0, \tag{40}
\]

where \(q_1, q_3, q_5, q_7 > 0\), while \(q_2, q_4, q_6, q_8 < 0\) as can be seen from the explicit expressions for \(q_i, i \in \{1, 2, \ldots, 8\}\):

\[
q_1 = (\gamma L^2)^2\tau_{\epsilon}^2(\gamma^2\gamma L(1 + 2\mu)\tau_{u\epsilon}^2\tau_{\epsilon} + \gamma^3\mu\gamma L^2\tau_{u\epsilon} + \gamma^3\tau_{u\epsilon}^2\tau_{\epsilon} + \gamma\tau_{u(\gamma L)^2(2 + \mu)\tau_{u\epsilon} + \mu}) + \gamma\mu\tau_{\epsilon} + (\gamma L^3)^2\tau_{\epsilon}^2\tau_{u\epsilon} > 0 \tag{41a}
\]

\[
q_2 = -\gamma L^3\tau_{u\epsilon}(\gamma^2\gamma L\tau_{u\epsilon}\tau_{\epsilon}(4\mu\tau_{u\epsilon} + 4\mu\tau_{\epsilon} + 3\tau_{u\epsilon} + 2\tau_{\epsilon}) + 2\gamma^3\mu\tau_{u\epsilon}\tau_{\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon}) + 2\gamma(\mu(\tau_{u\epsilon} + \tau_{\epsilon})(1 + (\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon}) + (\gamma L)^2\tau_{u\epsilon}(3\tau_{u\epsilon} + 2\tau_{\epsilon})) + (\gamma L)^3\tau_{u\epsilon}\tau_{\epsilon}(3\tau_{u\epsilon} + 2\tau_{\epsilon})) < 0 \tag{41b}
\]

\[
q_3 = \tau_{\epsilon}^2(\gamma^2\gamma L)\tau_{u\epsilon}(\tau_{u\epsilon}^2(2(\gamma L)^2(1 + \mu)(1 - \mu)\tau_{u\epsilon}\tau_{\epsilon} + 2\mu + 3) + 2(\gamma L)^2(1 + 2\mu)\tau_{u\epsilon}^3\tau_{\epsilon} + 4(1 + \mu)\tau_{u\epsilon}\tau_{\epsilon} + (1 + 2\mu)\tau_{\epsilon}^2) + \gamma^3\mu\tau_{u\epsilon}\tau_{\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon}) + 2(\gamma L)^2\tau_{u\epsilon}^3\tau_{\epsilon} + 2\tau_{u\epsilon}\tau_{\epsilon} + \tau_{\epsilon}^2) + \gamma(\mu(2(\gamma L)^4\tau_{u\epsilon}^2\tau_{\epsilon}^2 + 3(\gamma L)^2\tau_{u\epsilon}^2\tau_{\epsilon} + 4(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + (\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + \tau_{u\epsilon} + \tau_{\epsilon}) + 2(\gamma L)^2\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})(\tau_{u\epsilon}(2(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + 3) + \tau_{\epsilon})) + (\gamma L)^3\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})(\tau_{u\epsilon}(2(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + 3) + \tau_{\epsilon})) > 0 \tag{41c}
\]

\[
q_4 = -\tau_{\epsilon}^2(\gamma^2\gamma L\tau_{u\epsilon}(\tau_{u\epsilon}^2(1(1 + \mu)(1 - \mu)\tau_{u\epsilon}\tau_{\epsilon} + 2\mu + 3) + 2(\gamma L)^2(1 + 2\mu)\tau_{u\epsilon}^3(4(1 + \mu) - \mu^2) + 2(\gamma L)^2(4\mu + 3)\tau_{u\epsilon}^3\tau_{\epsilon} + \tau_{\epsilon}^2) + 2\gamma^3\gamma^2\mu\tau_{u\epsilon}\tau_{\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})((1 - \mu)\tau_{\epsilon} + 2\tau_{u\epsilon}) + 2\gamma(\gamma L^2\tau_{u\epsilon}^2(\gamma L^2(8 + \mu)\tau_{u\epsilon}\tau_{\epsilon} + 2\mu + 1) + 2(\gamma L)^2(\mu + 3)\tau_{u\epsilon}^3\tau_{\epsilon} + 2\tau_{u\epsilon}\tau_{\epsilon}(1 + (\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + \mu) + 2\tau_{\epsilon}^2)(\gamma L)^2(\tau_{u\epsilon} + \tau_{\epsilon})(2(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon}(3\tau_{u\epsilon} + \tau_{\epsilon}) + \tau_{u\epsilon} + \tau_{\epsilon})) < 0 \tag{41d}
\]

\[
q_5 = \tau_{u\epsilon}\tau_{\epsilon}(\gamma^2\gamma L\tau_{u\epsilon}(2\tau_{u\epsilon}^2((\gamma L)^2(1 + \mu)(1 - \mu)\tau_{u\epsilon}\tau_{\epsilon} + 2\mu + 3) + \tau_{u\epsilon}\tau_{\epsilon}((\gamma L)^2(1 - \mu^2)\tau_{u\epsilon}\tau_{\epsilon} + 6\mu + 10) + (\gamma L)^2(1 + 2\mu)\tau_{u\epsilon}^3\tau_{\epsilon} + 2(2 + \mu)\tau_{\epsilon}^2) + \gamma^3\mu\tau_{u\epsilon}\tau_{\epsilon}(\tau_{u\epsilon}^2((\gamma L)^2(\mu - 1)\tau_{u\epsilon}\tau_{\epsilon} + 2) + 2\tau_{u\epsilon}\tau_{\epsilon}(2 + (\gamma L)^2(2(1 - \mu)\tau_{u\epsilon}\tau_{\epsilon} + \tau_{u\epsilon}^2((\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + 2)) + \gamma(\tau_{u\epsilon} + \tau_{\epsilon})((\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + (\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon} + 3) + 2) + 2(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon}(\gamma L)^2(\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon} + 6) + 4\tau_{\epsilon})) + (\gamma L)^3\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})(\tau_{u\epsilon}(\gamma L)^2(\gamma L^2\tau_{u\epsilon}\tau_{\epsilon} + \tau_{u\epsilon}(\gamma L)^2(\tau_{u\epsilon} + \tau_{\epsilon} + 6) + 4\tau_{\epsilon})) > 0 \tag{41e}
\]

\[
q_6 = -\tau_{u\epsilon}\tau_{\epsilon}(\gamma^2\tau_{u\epsilon}^2(2 + (\gamma L)^2(3 - \mu^2)\tau_{u\epsilon}\tau_{\epsilon}) + \tau_{u\epsilon}^2((\gamma L)^2(4\mu + 3)\tau_{u\epsilon}\tau_{\epsilon} + 2) + 2\tau_{u\epsilon}\tau_{\epsilon}(2 + (\gamma L)^2(3 - \mu)\tau_{u\epsilon}\tau_{\epsilon}) + 2\gamma^3\gamma L\mu\tau_{u\epsilon}\tau_{\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})((1 - \mu)\tau_{\epsilon} + \tau_{u\epsilon}) + 2\gamma L^4(\tau_{u\epsilon} + \tau_{\epsilon})((\tau_{u\epsilon}(2(\tau_{u\epsilon} + \tau_{\epsilon}) + (1 - \mu)\tau_{\epsilon} + (1 + \mu)\tau_{u\epsilon})((\gamma L)^2\tau_{u\epsilon} + 2 + \mu) + 2\tau_{\epsilon}) + (\gamma L)^2(\tau_{u\epsilon} + \tau_{\epsilon})^2(2 + 3(\gamma L)^2\tau_{u\epsilon}\tau_{\epsilon})) < 0 \tag{41f}
\]

\[
q_7 = \tau_{u\epsilon}^2(\tau_{u\epsilon} + \tau_{\epsilon})(\gamma^2\gamma L\tau_{u\epsilon}((3 + 2\mu)\tau_{u\epsilon} + 3\tau_{\epsilon}) + \gamma^3\mu\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon}) + \gamma((6 + \mu)\tau_{u\epsilon} + (6 - \mu)\tau_{\epsilon})(\gamma L)^2\tau_{u\epsilon} + \mu) + 3(\gamma L)^3\tau_{u\epsilon}(\tau_{u\epsilon} + \tau_{\epsilon})) > 0 \tag{41g}
\]
\[ q_8 = -(\gamma + \gamma^L)^2(\tau_u + \tau_\epsilon)^2 \tau_u < 0. \]  

(41h)

By DesCartes rule of sign, (40) has at most 7 positive real roots. Furthermore, we have:

\[ G(0) = -(\gamma + \gamma^L)^2(\tau_u + \tau_\epsilon)^2 \tau_u < 0 \]  

(42a)

\[ G(((\mu \gamma + \gamma^L)\tau_v)^{-1}) = \frac{\mu^3 \gamma^3 (1 + (\gamma^L + \mu \gamma)^2 \tau_u \tau_\epsilon)^2 (\tau_u + \tau_\epsilon)}{((\mu \gamma + \gamma^L)^2 \tau_v)^7} > 0, \]  

(42b)

implying that at the liquidity provision stage of the game, for given \( \mu \), an equilibrium exists where \( \Lambda_2 \) is lower than its corresponding value when \( \tau_\epsilon \to \infty \) (recall that when second period traders perfectly observe \( u_1 \), we have \( \Lambda_2 = ((\mu \gamma + \gamma^L)\tau_v)^{-1} \)).

Figure 7: First and second period illiquidity, assuming second period traders observe a noisy signal of \( u_1 \). Parameter values are as follows: \( \mu = 0.1, \gamma = 0.5, \gamma^L = 0.25, \tau_v = 3, \tau_u = 100, \) and \( \tau_\epsilon \in \{1, 2, \ldots, 100\} \). In all simulations, the equilibrium is unique.

Equilibrium existence with asymmetric information, when all traders are rational players (i.e., without noise traders) may seem puzzling. Indeed Bhattacharya and Spiegel (1991), show that the fear of trading at an informational disadvantage may lead agents to back out of a trade, compromising equilibrium existence. Notice, however, that in the context we analyze asymmetric information is on \( u_1 \) and not on the value of the payoff, which eliminates the lemon’s problem.

Additionally, the plots in Figure 7 show that first (second) period illiquidity decreases (increases) in \( \tau_\epsilon \). One possibility is that for \( \tau_\epsilon \) small (large) second period traders hedge and speculate less (more). Stronger liquidity traders’ speculation has a beneficial impact on FD risk bearing capacity at the first round, because it heightens their rebalancing opportunities. However, stronger second period hedging has instead a detrimental impact on FD second period risk bearing capacity, because it implies that they have to absorb more of second period traders’ endowment shock.

For low signal precision, asymmetric information can lead to a self-reinforcing loop between second period illiquidity and second period traders’ hedging. As the second period market becomes less liquid, FDs scale down their liquidity provision in the first period (because they anticipate lower risk bearing capacity at the second round), leading first period liquidity traders to hedge less. This, in turn, reduces the impact of \( u_1 \) on \( p_2 \), boosting second period traders’ hedging activity. As a result, we numerically find that for low values of \( \tau_\epsilon \), at most three equilibria can arise (see Figure 8). The equilibria can be ranked in terms of second period illiquidity, with the two extreme equilibria (the intermediate equilibrium) being stable (unstable) with respect to best response dynamic.
Figure 8: Asymmetric information and equilibrium multiplicity. Parameter values are as follows: \( \mu = 0.3, \gamma = 0.1, \gamma^L = 1, \tau_u = 0.5, \tau_u = 0.01, \) and \( \tau_e = 0.1. \) The red (green) gridlines in the figure are drawn at the value of \( \Lambda_2 \) of the baseline model (case with \( \tau_e \to 0 \)).

3. Uninformative signal. This is what happens when \( \tau_e \to 0. \) In this case, second period traders stop speculating on the first period endowment shock and we obtain a unique equilibrium, where

\[
\begin{align*}
\lim_{\tau_e \to 0} \Lambda_2 &= \frac{\tau_u(\gamma + \gamma^L)^2}{\tau_u (\gamma + \gamma^L)^2 (\gamma \mu + \gamma L) + \gamma \mu} \quad (43a) \\
\lim_{\tau_e \to 0} \Lambda_1 &= \frac{1}{(\gamma + \gamma^L) \tau_v} \quad (43b) \\
\lim_{\tau_e \to 0} \Lambda_{21} &= -\frac{1}{(\gamma + \gamma^L) \tau_v} \quad (43c)
\end{align*}
\]

Comparing (43a), and (43b) with the second and first period illiquidity of the baseline model (see, respectively, (A.19a) and (A.29)) shows that with an infinitely noisy signal, the second (first) period market becomes deeper (less liquid). Indeed, in this case, second period traders cut back on their hedging activity and cease speculating on \( u_1. \) The former effect lowers FD second period exposure to fundamental risk. The latter effect impairs their ability to rebalance their position at the second round.

Additionally, the impact of \( u_1 \) on \( p_1 \) mirrors that of \( u_1 \) on \( p_2: \Lambda_{21} = -\Lambda_1 \) (see (43b) and (43c)). Since second period traders do not speculate on the propagated imbalance, at the second round FD hold on to their position on the first period endowment shock, implying that the pressure this induces on prices does not revert before liquidation. This, in turn, has an important implication for
the expression of FD payoff. Using the expression we obtained for the baseline case (see (A.38)), we have

\[ CE^{FD} = \frac{\gamma}{2} \left( \ln \left( 1 + \frac{\Lambda_2^2 \tau_v}{\tau_u} \right) + \ln \left( 1 + \frac{\Lambda_1^2 \tau_v}{\tau_u} + \left( \frac{\Lambda_1 + \Lambda_2}{\Lambda_2} \right)^2 \right) \right) \]

(44)

\[ = \frac{\gamma}{2} \left( \ln \left( 1 + \frac{\Lambda_2^2 \tau_v}{\tau_u} \right) + \ln \left( 1 + \frac{\Lambda_1^2 \tau_v}{\tau_u} \right) \right). \]

Given that (see (A.33))

\[ CE^{SD} = \frac{\gamma}{2} \ln \left( 1 + \frac{\Lambda_2^2 \tau_v}{\tau_u} \right), \]

(45)

this implies that the only value added by the possibility of re-trading at the second round for FD is given by the additional compensation these agents receive for providing liquidity to second period traders:

\[ \phi(\mu) = CE^{FD} - CE^{SD} = \frac{\gamma}{2} \ln \left( 1 + \frac{\Lambda_2^2 \tau_u}{\tau_v} \right), \]

(46)

which, due to (43a), can be easily shown to be decreasing in \( \mu \). Note that because the second period market is deeper in this case compared to the case with a perfect signal on \( u_1 \), this also implies that an uninformative signal reduces the value of technological services.

Finally, the impossibility to speculate on the propagated endowment shock makes second period traders’ payoff similar to that of first period traders:

\[ CE^L_t = \frac{\gamma^L}{2} \ln \left( 1 + \frac{a_t^2 - 1}{(\gamma^L)^2 \tau_u \tau_v} \right), \]

(47)

where \( a_t = \gamma^L \Lambda_t \tau_v - 1, \) for \( t \in \{1, 2\} \).

As a final result of our analysis of this case, it is possible to show that

\[ g(\Lambda_2)\big|_{\Lambda_2 = \frac{\tau_u (\gamma + \gamma^L)^2}{r \tau_v (\gamma + \gamma^L)^2 (\gamma + \gamma^L) + \gamma}} < 0, \]

implying that, when \( 0 < \tau_e < \infty \), second period equilibrium illiquidity is straddled by the two limit cases of perfect and uninformative second period signal (see Figure 8). Other things equal, starting from a parametrization yielding 3 equilibria, as \( \mu \) or \( \gamma \) increase, a unique equilibrium obtains where \( \Lambda_2 \) tends to (43a). Conversely, as \( \gamma^L, \tau_u, \) or \( \tau_v \) increase, uniqueness obtains with \( \Lambda_2 \) tending to the value it has in the baseline model. This suggests that, to simplify the analysis and gauge the effect of asymmetric information, we can concentrate on the case of an uninformative second period signal.

With this in mind, we replicate the numerical simulations of our baseline model (i.e., using the parameter values of Table 2 in the paper), assuming that second period traders observe an uninformative signal. The result of our simulations confirm all our findings in Numerical result 2–except for the presence of insufficient entry–as well as the welfare superiority of price controls compared to entry regulation. Thus, when the signal is uninformative, with the parameter values of Table 2, entry is always excessive. Additionally, in this case too we find that \( N^{CO} = 1 \) and that fee regulation is always superior to entry regulation.
References