

(In)efficiency in Information Acquisition and Aggregation through Prices*

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Abstract

We study inefficiency in the acquisition of private information before trading in financial markets. As the cost of information declines, traders over-invest in information acquisition and trade too much on their private information. Generically, no policy exists based on the price of the financial asset and the individual trade volume inducing efficiency in both information acquisition and trading. Such an impossibility result turns into a possibility one when information acquisition is verifiable or when taxes can be made contingent on the aggregate volume of trade. When only ad-valorem taxes are available, they should not be used.

Keywords: information acquisition, aggregation through prices, information externalities, team efficiency.

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1 Introduction

Improvements in information technology are reducing the cost of acquiring and processing information. However, there are concerns about the social value of this bounty, in particular when it comes to the information acquired and used in financial trading. Policy proposals range from putting “sand in the wheels” of financial markets as a way of limiting speculative trading (the Tobin tax, which is an *ad-valorem* tax on transactions, is a prominent example) to subsidizing information acquisition because of its potential value as a public good. In this paper, we develop a tractable framework to study both the positive and normative issues of interest. The model is designed to perform welfare analysis in economies with endogenous private information and aggregation through prices where traders compete in demand schedules (generalized limit orders).¹ In particular, we characterize the sources of inefficiency in the collection of private information prior to trading and relate them to possible inefficiencies in the limit orders that traders submit given their available information.

Our model is a linear-quadratic-Gaussian market microstructure in which a unit-mass continuum of traders compete by submitting a collection of generalized limit orders (equivalently, a demand schedule). The traders face uncertainty about the asset’s value and the price-elastic supply of the asset. The asset supply may come from liquidity traders in a stock market, the central bank in a liquidity auction, the Treasury in a bond auction, or the regulator in the market for pollution permits. Before submitting their demand schedules, each trader collects a private signal about the asset’s value whose noise is endogenous and correlated across the traders. Such a correlation may originate, for example, in the traders paying attention to common sources of information, with source-specific noise. Importantly, such a correlation, in addition to being realistic, has major implications for the (in)efficiency of the equilibrium acquisition and usage of information, as we discuss further on.

Our first main result is that, except in very special cases, absent policy interventions, the market does not use the information it collects efficiently. The inefficiency originates in the interaction between two externalities. First, traders do not account for the fact that a collective change in demand schedules may induce a change in the information contained in the equilibrium price, which in turn affects other agents’ ability to align their trades with the asset’s value (a familiar learning externality, which has been extensively investigated in the literature). Second, and more interesting, traders do not account for how their orders affect the equilibrium market-clearing price and thereby the asset holding and ultimately the consumption by other agents (a pecuniary externality). We isolate the pecuniary externality by considering a fictitious environment where traders are naive, in that they do not learn from prices, but are endowed with an exogenous public signal whose precision is the same as the one contained in the equilibrium market-clearing price. In such a fictitious environment, traders fail to account for the fact that variations in their demands driven by

¹Competition in demand schedules is common in financial markets where limit orders are prevalent (e.g., in NYSE, more than 50% of trades are through limit orders, whereas market orders represent about 32% of total trading volume; see Li, Ye, and Zheng 2023). Other markets where limit orders are pervasive include central bank liquidity auctions, Treasury auctions, and pollution rights markets.

idiosyncratic noise in their private signals may trigger changes in the consumption by other agents which are not justified by fundamentals. Importantly, contrary to the case where the traders submit market orders (that is, price-inelastic demands), the inefficiency does not originate in the change in the agents' expenditures for given asset holdings but in the fact that other traders' asset holdings change due to the dependence of the limit orders on the equilibrium market-clearing price. Such a pecuniary externality is present only because information is incomplete. However, it is fundamentally different from the familiar learning externality described above. In particular, while the learning externality makes traders under-react to private information, the pecuniary externality makes them over-react.

The knife-edge case in which the two externalities cancel each other out obtains when the equilibrium demand schedules are perfectly inelastic. When the equilibrium schedules are downward sloping, the pecuniary externality dominates and the equilibrium trades feature excessive sensitivity to private information. When, instead, the equilibrium schedules are upward sloping, the learning externality dominates and the sensitivity of the equilibrium limit orders to the traders' private information is inefficiently low. Interestingly, as the precision of the traders' private information grows (for example, because of reductions in the cost of acquiring and processing information due to technological progress), the pecuniary externality gains weight in relation to the learning externality.²

We show that, no matter whether traders over- or under-respond to their private information, the aforementioned inefficiencies in the equilibrium usage of information (equivalently, in trading) can be corrected using a non-linear tax-subsidy contingent on the equilibrium price of the asset and the individual volume of trade. More precisely, a linear-quadratic tax on the volume of trade paired with an ad-valorem tax on the dollar amount paid induces traders to submit efficient limit orders.

Our second main result is that inducing the traders to trade efficiently does not guarantee that they collect private information efficiently before submitting the orders. In particular, traders over-invest in information acquisition when the efficient schedules are downward sloping, and under-invest in information acquisition when they are upward sloping. In other words, when, in the laissez-faire equilibrium, the pecuniary externality prevails over the learning externality so that the traders over-respond to their private information, forcing the traders to trade efficiently induces them to over-invest in the acquisition of private information. When, instead, the learning externality prevails over the pecuniary externality so that traders under-respond to private information, forcing the traders to trade efficiently induces them to under-invest in the acquisition of private information. The inefficiencies in the collection of information thus parallel those in the usage of information. Importantly, to uncover these results, one needs to allow the noise in the traders' private information to be correlated among the traders. If such a noise were completely independent, under the efficient demand schedules, the only effect of a variation in the precision of the traders' private information on

²Provided that the noise in the traders' information is not too large, when the precision of the traders' information is relatively low, the learning externality dominates and the demand schedules are upward sloping, whereas the opposite is true (i.e., the pecuniary externality dominates and the demand schedules are downward sloping) for high levels of precision.

welfare would be through the change in the dispersion of individual trades around the average trade. However, under the efficient demand schedules, the private and the social value of reducing such a dispersion coincide, so efficiency in the usage of information implies efficiency in the acquisition of information. It is common practice in the literature to assume that the noise in the agents' private signals is independent across agents. This assumption is made for tractability. In reality, the information the agents receive typically comes from common sources that are subject to noise at the source level. The attention agents allocate to such sources thus affects their exposure to both idiosyncratic and correlated noise. When this is the case, efficiency in trading does not guarantee efficiency in information acquisition.

We also show that, if traders could be trusted to submit the efficient demand schedules, (an unrealistic hypothetical), then an *ad-valorem* tax on the dollar amount paid would induce the efficient collection of private information.

Next, we show that, absent any policy intervention, as information technology makes the collection of information cheaper, the economy eventually enters into a regime of over-investment in information acquisition and excessive trading on private information. This is accompanied with inefficiently high price volatility, market depth, and price informativeness. In other words, the secular trend of improvement in information technology may have the undesirable effect of enticing over-investment in information acquisition and over-reaction to it in the trading of financial assets.³ It follows that policies that at the same time place “sand in the wheels” of financial markets and subsidize information collection are potentially welfare reducing.

Motivated by the results described above, we then turn to the question of what policy interventions induce efficiency in both the acquisition and usage of information. We show that, generically, there exist no taxes/subsidies contingent on the price of the asset and on the volume of individual trades that can induce efficiency in both the acquisition and the usage of information. This impossibility result, however, can be overturned by conditioning the tax/subsidy on the expenditure on information acquisition, when the latter is verifiable, or, when the latter is not verifiable, on the aggregate volume of trade. In the former case, efficiency in trading can be induced with standard taxes that depend only on the price paid and on the individual volume of trade, whereas efficiency in information acquisition can be induced through a separate tax/subsidy that depends on the amount of information purchased. In the latter case (i.e., when information acquisition is not verifiable), conditioning the marginal tax rate on the aggregate volume of trade provides the planner with flexibility in the way it realigns the private incentives for trading with the social ones. This extra flexibility in turn permits the planner to also realign the private benefits of acquiring more precise private information to their social counterparts.

Finally, we show that if the government is restricted to using simple *ad-valorem* taxes (that is,

³See, for example, Nordhaus 2015 on the sharp decline in the cost of computation (and therefore of information processing). See also Gao and Huang 2020 and Goldstein, S. Yang, and Zuo 2020 for the effects of the dissemination of corporate disclosures over the internet on the production of information by corporate outsiders.

taxes that are linear in individual expenditures on asset purchases), it is better not to use them. The result is striking given that ad-valorem taxes are typically proposed by advocates of policy interventions in financial markets.⁴ The reason is that such taxes fail to change the sensitivity of the equilibrium trades to private information and/or the value the traders assign to collecting private information. Such taxes only change the sensitivity of the equilibrium schedules to the price. However, in the laissez-faire equilibrium, for given precision of private information and sensitivity of the schedules to the private signals, the sensitivity of the equilibrium schedules to the price is welfare-maximizing. As a result, these taxes only bring the equilibrium farther away from the efficient allocation and hence are welfare decreasing. For the policy maker to improve over the laissez-faire equilibrium it is essential to use more sophisticated policies that are non-linear in individual expenditures on asset purchases, and with a marginal tax rate that depends on the aggregate volume of trade so as to induce efficiency not only in the submission of the limit orders but also in the collection of information.

Related literature The paper is related to several strands of the literature. The first is the literature investigating the sources of inefficiency in the equilibrium usage of information. See, among others, Palfrey 1985, Vives 1988, Angeletos and Pavan 2007, Amador and Weill 2010, and Vives 2017. Among these works, the closest are Amador and Weill 2010 and Vives 2017. Both these papers study inefficiency in information aggregation when traders submit demand schedules.⁵ In these papers, the agents' private information is exogenous. In Amador and Weill 2010, traders always under-respond to their private information, whereas this is not the case in Vives 2017. None of these papers studies the sources of inefficiency in the collection of private information, relate them to the interaction between pecuniary and learning externalities, and investigate policy corrections, which are the key contributions of our paper.

The second strand is the literature on information acquisition in markets. See Diamond and Verrecchia 1981 and Verrecchia 1982 for earlier contributions. More recently, Peress 2010 examines the trade-off between risk sharing and information production, whereas Manzano and Vives 2011 study information acquisition in markets with correlated noise, while Kacperczyk, Van Nieuwerburgh, and Veldkamp 2016 study information acquisition in markets with multiple risky assets. Dávila and Parlatore 2021 study the effect of trading costs on information aggregation and acquisition. Mondria, Vives, and L. Yang 2021 study a model where traders have to exert effort (pay attention) to reduce noise in the interpretation of the information content of the price. None of these papers studies inefficiency in information acquisition and how it relates to inefficiency in trading.⁶ Vives 1988 shows

⁴Several EU countries use ad-valorem taxes for shares transactions (Belgium, France, Hungary, Italy, Portugal, and Spain) with the promise of switching to the common EU FTT once introduced. In Asia, ad valorem taxes are used by India, Malaysia, Pakistan, Thailand, Hong Kong, and Singapore. China has a stamp duty and so does Brazil (see Dowd 2020).

⁵See also Kyle 1989, Vives 2011, and Rostek and Weretka 2012 for related models of strategic competition in schedules.

⁶See also the literature on the Grossman-Stiglitz paradox, namely on the lack of incentives to acquire information

that, in a Cournot economy in which a continuum of privately-informed traders with conditionally independent private signals submit market orders, both the decentralized acquisition of information and the equilibrium trades are efficient. In the present paper, we show that the same result extends to economies in which the information collected in equilibrium is subject to correlated noise, provided that the traders are restricted to submitting market orders instead of richer supply/demand functions. When traders submit market orders, neither the pecuniary externality nor the learning externality of conditioning on prices are present. As a result, the planner cannot improve on the laissez-faire equilibrium if it is equally restricted to using only market orders. However, welfare can be higher under limit orders even if trading and information acquisition are inefficient.

Efficiency in the usage of information implies efficiency in information acquisition in the macro business-cycle economies considered in Angeletos, Iovino, and La'O 2020. In these economies, prices imperfectly aggregate information, as in our paper, but agents have access to complete markets that permit them to fully insure against idiosyncratic consumption risk. In contrast, in our economy, markets are incomplete, in the sense that traders consume the returns to their own investments; in such economies, policies that correct inefficiencies in the usage of information need not induce efficiency in the collection of information. Colombo, Femminis, and Pavan 2022 consider an economy in which production is affected by investment spillovers. They show, among other things, that simple state-invariant subsidies to technology adoption induce efficiency in production when information is exogenous but not when it is endogenous. In the latter case, more sophisticated Pigouvian-like taxes where the marginal rates depend on aggregate output and on the aggregate investment in the new technology are necessary to induce efficiency in both the usage and the acquisition of information. That paper, however, abstracts from information aggregation through prices, which is the focus of the present paper. Colombo, Femminis, and Pavan 2014 show that efficiency in actions need not imply efficiency in information acquisition when individual payoffs depend on the dispersion of individual actions around the average action. In the present paper, we show that, even in the absence of such externalities, efficiency in usage does not imply efficiency in information acquisition when agents compete in schedules. The relation between efficiency in information usage and in information acquisition is also investigated in Angeletos and Sastry 2022 and Hebert and La'O 2022. The first paper considers economies in which markets are complete, whereas the second one an abstract linear-quadratic game as in Angeletos and Pavan 2007. In both papers, like in ours, agents learn from the behavior of others. However, neither of these two papers analyzes the welfare implications of such a learning originating from the agents submitting price-contingent schedules. There are no pecuniary externalities in either of these papers, whereas such externalities play a key role in our analysis. As explained above, these externalities naturally arise when agents submit demand schedules. Importantly, neither Angeletos and Sastry 2022 nor Hebert and La'O

when prices are fully revealing (see Grossman and Stiglitz 1980, and Vives 2014 for a potential resolution of the paradox). Related is also the literature on strategic complementarity/substitutability in information acquisition (see, among others, Ganguli and L. Yang 2009, Hellwig and Veldkamp 2009, Manzano and Vives 2011, Myatt and Wallace 2012, and Pavan and Tirole 2022).

2022 identify whether agents over- or under-invest in information acquisition, whether they over- or under-respond to their private information and to the market-clearing price, and which policies can correct inefficiency in information acquisition and trading, which is the focus of our analysis.

The third strand is the recent literature on the impact of technological progress on the collection of information and its usage in financial markets. Farboodi, Matray, and Veldkamp 2018 show that the growth of big data, combined with the size distribution of firms, can lead to a decline in price informativeness for smaller firms. Peress 2005 shows that a declining cost of information collection is outweighed by a parallel decline in the cost of entry to financial markets and the interaction between the two can explain several empirical anomalies. Malikov 2019 shows that falling information costs can actually contribute to a rise in passive investment by reducing the cost of, and therefore the returns to, stock picking. Several papers (see, among others, Azarmsa 2019, Mihet 2018, and Kacperczyk, Nosal, and Stevens 2019) show that technological progress that facilitates the collection of information can lead to increasing levels of inequality. Unlike most of the work in this literature, we focus on the normative implications of technological improvements in the collection of information.

A fourth strand is the literature building on Tobin 1978's proposal to put sand-in-the-wheels on foreign exchange transactions as a way to curb volatility and speculation. Similar interventions have been advocated for financial markets (e.g. Stiglitz 1989 and L. H. Summers and V. P. Summers 1989). High volumes of speculation (particularly in the short-term) and/or "noise trading" are typically assumed to be detrimental to welfare. However, some theoretical work shows that a tax on financial transactions may increase price volatility and lower market depth and welfare (see, among others, Kupiec 1996, Sørensen 2017, and Song and Zhang 2005). Subrahmanyam 1998 and Dow and Rahi 2000 show that a quadratic transaction tax may have ambiguous effects on speculators' profits and on the welfare of other traders. Umlauf 1993, Colliard and Hoffmann 2017, and Deng, Liu, and Wei 2018 document a negative impact of transaction taxes on trading volume and an ambiguous impact of the same taxes on price volatility and market depth. Using transaction data from the Italian Stock Exchange, Cipriani, Guarino, and Uthemann 2022 estimate a model with price elastic informed and noise traders to assess the effects of a transaction tax on informed and noise traders. They find that the tax reduces trading activity and price volatility, but also reduces price informativeness for most stocks. Our paper contributes to this literature by showing that simple ad-valorem taxes are welfare reducing and that efficiency in both the usage and the acquisition of information requires conditioning tax bills on the expenditure on information acquisition (when the latter is verifiable) or on the aggregate volume of trade (when information acquisition is not verifiable, for example because it originates in attention).

Finally, in this paper, we assume that higher investments in information acquisition can reduce the agents' exposure to correlated noise in information. Recent work by Woodford 2012a, Woodford 2012b, and Nimark and Sundaresan 2019 shows that rational inattention can also explain the agents' exposure to correlated noise, and that the equilibrium of a rationally-inattentive economy shares several features with those of an economy in which the agents' use of information is "biased" in the

sense of prospect theory. Particularly related in this respect is Frydman and Jin 2022, which shows how rational inattention can lead to endogenous bias in valuation, and that the noise in perception is closely linked to the bias in perception. Our paper shares with this literature the property that investments in information acquisition also affect the agents’ exposure to correlated noise, something that, from the perspective of an outside observer, may look like a bias in decision making.

Organization. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 compares the equilibrium to the efficient usage of information, identifies the sources of the inefficiency, and shows how certain taxes/subsidies may restore efficiency in trading. Section 4 identifies inefficiencies in information acquisition and discusses policy corrections. Section 5 concludes. All proofs are in the Appendix at the end of the document (The proofs are self-contained but expanded derivations can be found in the Supplementary Material).

2 Model

In this section, we describe the environment, as well as the traders’ problem of choosing a demand schedule and the private information to acquire prior to trading.

2.1 Environment

The market is populated by a continuum of traders, indexed by $i \in [0, 1]$, trading a homogenous and perfectly divisible asset. Let x_i denote the quantity of the asset demanded by trader i and $\tilde{x} = \int_0^1 x_i di$ the traders’ aggregate demand. Each trader i ’s payoff from purchasing x_i units of the asset at price p is given by $\pi_i \equiv (\theta - p)x_i - \lambda x_i^2/2$, where λ is a positive scalar, and where $\theta \sim N(0, \sigma_\theta^2)$. The variable θ proxies for the traders’ gross common value from purchasing the asset, whereas the term $\lambda x_i^2/2$ is a quadratic trading or adjustment cost whose role is to induce imperfectly elastic demands.⁷

Traders face an exogenous inverse asset supply $p = \alpha - u + \beta \tilde{x}$, where α and β are positive scalars, and where $u \sim N(0, \sigma_u^2)$ is an aggregate shock.⁸ Such a supply may originate from the combination of various operations of liquidity traders such as pension or index funds trading the asset as part of broader market portfolios, along with the operations of large liquidity suppliers such as central banks trading the asset as part of their liquidity programs. The planner believes the costs of such a supply to be equal to $(\alpha - u)\tilde{x} + \beta \tilde{x}^2/2$. This specification permits us to equivalently interpret the supply of the asset as coming from a “representative supplier” with payoff $p\tilde{x} - (\alpha - u)\tilde{x} - \beta \tilde{x}^2/2$. In this case, the term $\alpha - u$ proxies for the opportunity cost for the representative supplier of unloading the asset, and $\beta \tilde{x}^2/2$ for a quadratic trading or adjustment cost. Importantly, both the traders and the planner treat such a supply as exogenous to their own operations.

⁷See also Vives 2011 and Rostek and Weretka 2012 for examples of models with a quadratic adjustment cost.

⁸As usual, the role of this shock is to prevent the price from being fully revealing of the information the traders collectively possess.

To simplify the derivation of the equilibrium formulas, we assume that the variables θ and u are independently distributed. The results, however, extend to the case where they are imperfectly correlated. For notational purposes, given any Gaussian random variable h with variance σ_h^2 , we denote by $\tau_h \equiv 1/\sigma_h^2$ the variable's precision.

The traders do not know θ . They privately collect information about θ prior to submitting their demand schedules, but also condition the latter on the information that the market-clearing price contains about θ (that is, they account for the fact that the equilibrium price imperfectly aggregates the traders' dispersed information about θ).

Formally, we assume that each trader observes a signal $s_i \equiv \theta + \epsilon_i$, where $\epsilon_i \equiv f(y_i)(\eta + e_i)$ is a combination of idiosyncratic and correlated noise. Precisely, the noise variable $\eta \sim N(0, \sigma_\eta^2)$ is perfectly correlated among the traders whereas the variables $e_i \sim N(0, \sigma_e^2)$ are i.i.d. among the traders. The variables $(\theta, u, \eta, (e_i)_{i \in [0,1]})$ are jointly independent. The exposure of trader i to the noise variables (η, e_i) is a decreasing function f of trader i 's effort $y_i \in \mathbb{R}_+$. Depending on the context, y_i can be interpreted either as the amount of information acquired by the trader, or the attention the latter allocates to exogenous sources of information. The cost of y_i is given by a differentiable function $\mathcal{C}(y_i)$, with $\mathcal{C}'(y_i), \mathcal{C}''(y_i) > 0$ for all $y_i > 0$.

The idea behind the above information structure is that traders learn from a variety of information sources differing in their noises and in the extent to which such noises are correlated among the traders.⁹ To maintain the analysis simple, we assume that the information received from such sources is summarized in a uni-dimensional statistics and that the marginal effect of effort on the reduction of the influence of both noises is the same, with the function f taking the form $f(y) = y^{-1/2}$. Such an assumption allows us to express the precision

$$\tau_\epsilon(y) \equiv \frac{\tau_e \tau_\eta}{\tau_e + \tau_\eta} y \tag{1}$$

of the combined noise term ϵ as a linear function of y . The analysis below is facilitated by the uni-dimensionality of the traders' information-acquisition strategies. However, the key insights extend to richer specifications in which $y_i = (y_i^\eta, y_i^e)$, with y_i^η and y_i^e parametrizing the traders' exposure to common and idiosyncratic noise, respectively.

Timing. At $t = 0$, the traders simultaneously make their information-acquisition decisions $(y_i)_{i \in [0,1]}$. At $t = 1$, the traders observe their private signals $(s_i)_{i \in [0,1]}$. At $t = 2$, the traders simultaneously submit their demand schedules. At $t = 3$, the market clears, the equilibrium price is determined, the equilibrium trades are implemented, and payoffs are realized.

Remark. While our analysis is motivated by trading in financial markets, the model can also be applied to many other environments. For example, a regulator may be supplying pollution permits to firms that need them to produce. Assuming that each unit of output requires a permit and that each permit has a unit price of p , we then have that π_i is firm i 's profits net of its expenditure on permits, $p\tilde{x}$ is the total revenue the regulator obtains from the sale of the permits, and $(\alpha - u)\tilde{x} + \beta\tilde{x}^2/2$ is

⁹The correlation in the noise may in turn reflect error at the "source" level as, e.g., in Myatt and Wallace 2012.

the cost to the regulator of allocating \tilde{x} permits, with such a cost accounting for the environmental impact of pollution. The uncertainty the firms face over the inverse supply function of pollution permits then reflects the uncertainty over the regulator's tolerance for pollution.

2.2 Traders' problem

Given $I_i \equiv (y_i, s_i)$, trader i chooses a demand schedule that maximizes, for each price p , the trader's expected payoff

$$\mathbb{E} \left[(\theta - p) x_i - \lambda \frac{x_i^2}{2} \mid I_i, p \right]$$

taking into account how the price p co-moves with the traders' fundamental value θ , the supply shock u , and the common noise η in the traders' information. The solution to this problem is the demand schedule given by

$$X(p; I_i) = \frac{1}{\lambda} (\mathbb{E}[\theta \mid I_i, p] - p) \tag{2}$$

where $\mathbb{E}[\theta \mid I_i, p]$ denotes the trader's expectation of θ given the quality of the trader's information, as proxied by y_i , the realization s_i of the trader's private signal, and the price p .¹⁰

At $t = 0$, each trader $i \in [0, 1]$ then selects y_i to maximize the expected profit

$$\mathbb{E} \left[\left(\theta - p - \frac{\lambda}{2} X(p; I_i) \right) X(p; I_i) \right] - \mathcal{C}(y_i)$$

where the expectation is over (s_i, θ, p) , given y_i . Following the pertinent literature, we focus on equilibria and on team-efficient allocations (defined below) in which the market-clearing price p is an affine function of all aggregate variables (θ, u, η) , and where all agents acquire information of the same quality (equivalently, pay the same attention to all relevant sources), and follow the same rule to map their private information into the demand schedules.

3 Inefficiency in trading

In this section, we fix the precision of the traders' private information τ_ϵ , as defined in (1) — equivalently, we fix the traders' information acquisition activity y_i and assume that $y_i = y$ for all i . We first solve for the traders' equilibrium demand schedules and then compare them to their efficient counterparts. The analysis permits us to identify inefficiency in the usage of information and policies alleviating the inefficiency. Because y is held fixed, to ease the notation, we drop it from the arguments of many of the functions we introduce below when there is no risk of confusion.

¹⁰Our linear-quadratic model is close to the standard CARA-Normal one, except for the fact that, in the latter, the denominator of the asset demand is the product of the traders' constant risk aversion coefficient and the conditional variance of the asset value.

3.1 Equilibrium usage of information

In any symmetric equilibrium in which the price is an affine function of (θ, u, η) , each trader's demand schedule is an affine function of her private signal s_i and the price p . That is,

$$x_i = X(p; I_i) = a^* s_i + \hat{b}^* - \hat{c}^* p$$

for some scalars $(a^*, \hat{b}^*, \hat{c}^*)$ that depend on the exogenous parameters of the model, as well as on the quality $y_i = y$ of the agents' information.¹¹ Aggregating across traders, we then have that the aggregate demand is equal to $\tilde{x} = \int x_i di = a^*(\theta + f(y)\eta) + \hat{b}^* - \hat{c}^* p$. As usual, the idiosyncratic errors in the traders' signals wash out in the aggregate demand.¹² However, the agents' information-acquisition activity (parametrized by y) impacts the aggregate demand through its effect on the traders' exposure to common noise η . This property has important implications for the positive and normative results we discuss below. Letting

$$z \equiv \theta + \underbrace{f(y)\eta - \frac{u}{\beta a^*}}_{\omega}, \quad (3)$$

we then have that the equilibrium price must satisfy

$$p = \frac{\alpha + \beta \hat{b}^*}{1 + \beta \hat{c}^*} + \frac{\beta a^*}{1 + \beta \hat{c}^*} z. \quad (4)$$

The information about θ contained in the market-clearing price is thus the same as the one contained in the endogenous public signal z whose noise $\omega \equiv f(y)\eta - u/\beta a^*$ is a combination of the common noise η in the traders' private information and the shock u to the supply of the asset. Given y and the sensitivity a^* of the traders' demand schedules to their private information s_i , we then have that the precision of the noise ω in the endogenous signal z contained in the price is equal to $\tau_\omega(a^*)$, with the function $\tau_\omega(a)$ given by

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{\beta^2 a^2 \tau_u + y \tau_\eta}. \quad (5)$$

For any τ_ω , let

$$K(\tau_\omega) \equiv \tau_\epsilon y \tau_\eta (y \tau_\eta - \tau_\omega) \quad (6)$$

and

$$\Lambda(\tau_\omega) \equiv y^2 \tau_\eta^2 (\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega \tau_\epsilon (\tau_\theta + 2y \tau_\eta), \quad (7)$$

¹¹The reason why we denote the sensitivity \hat{c}^* of the equilibrium demand schedules to the price and the constant term \hat{b}^* in the equilibrium demand schedules with the $\hat{}$ symbol is that, in the Appendix, we use the notation $x_i = a^* s_i + \hat{b}^* + \hat{c}^* z$ to denote the induced trades (the volume of the asset purchased/sold by each trader i) as a function of the trader's private information and the endogenous signal z contained in the equilibrium price. We do not use $\hat{}$ for the sensitivity a^* of the equilibrium demand schedules to the traders' private information s_i because that sensitivity is the same no matter whether one looks at the submitted demand schedules or the induced trades.

¹²This is because we make the convention that the analog of the strong law of large numbers holds for a continuum of independent random variables with uniformly bounded variances. The last property holds as long as the y_i 's have a common lower bound strictly larger than 0.

and, for any a , let \hat{C} and \hat{B} be the functions given by

$$\hat{C}(a) \equiv -\frac{\tau_\omega(a)y\tau_\eta(1-\lambda a-\beta a)-\lambda a\tau_\theta\tau_\omega(a)-\beta ay\tau_\eta\tau_\theta}{\lambda\beta ay\tau_\eta(\tau_\omega(a)+\tau_\theta-\tau_\theta\tau_\omega(a))+\beta\tau_\omega(a)y\tau_\eta}, \quad (8)$$

and

$$\hat{B}(a) \equiv \frac{\alpha}{\beta+\lambda} \left(\lambda\hat{C}(a) - 1 \right). \quad (9)$$

We then have the following result:

Proposition 1 (equilibrium trading). *Suppose $y_i = y$ for all i , with y exogenous. There exists a unique symmetric equilibrium. The sensitivity of the traders' equilibrium demand schedules to their private information, a^* , is given by the unique real root to the equation*

$$a^* = \frac{1}{\lambda} \frac{K(\tau_\omega(a^*))}{\Lambda(\tau_\omega(a^*))}, \quad (10)$$

and is such that $0 < a^* < 1/\lambda$. Given a^* , the equilibrium values of the other two parameters \hat{c}^* and \hat{b}^* defining the equilibrium demand schedules are given by the functions (8) and (9).

Fixing the quality of the traders' private information y , the equilibrium demand schedules thus solve a familiar fixed-point problem in which the traders correctly account for the information contained in the market-clearing price, and the latter is consistent with the submitted demand schedules. As anticipated above, the novelty relative to previous work is the presence of common noise in the traders' information, η , which is present in both the aggregate demand schedule and the market-clearing price.

3.2 Efficient usage of information

To isolate the inefficiencies in the equilibrium usage of information, we first identify the welfare losses (relative to the full-information benchmark) under any symmetric profile of affine demand schedules. We then characterize the demand schedules that minimize these losses (equivalently, that maximize ex-ante welfare) over the relevant class. The comparison between the equilibrium and the efficient schedules permits us to identify the inefficiency in equilibrium trading. By considering a fictitious environment in which traders do not learn from prices, we then identify the pecuniary externalities that, jointly with the familiar learning externalities that are present when agents learn from prices, are responsible for the inefficiency. Finally, we show how the interaction between the two externalities relates to the slope of the demand schedules. We discuss policies correcting the inefficiency in trading at the end of the section.

3.2.1 Welfare losses

Ex-post welfare is given by

$$W \equiv \int_0^1 \left(\theta x_i - \frac{\lambda}{2} x_i^2 \right) di - \left(\alpha - u + \beta \frac{\tilde{x}}{2} \right) \tilde{x}.$$

The integral term is the total payoff that the traders derive from purchasing the asset. The remaining term is the supply cost. The traders' payoffs are net of the expenses they incur to purchase the asset. These expenses do not appear in the welfare function because they are a zero-sum transfer between the traders and the relevant asset suppliers and all agents' payoffs are linear in consumption.

It is easy to see that the trades that maximize ex-post welfare are given by $x_i = x^o$ for all i , with

$$x^o \equiv (\theta + u - \alpha) / (\beta + \lambda). \quad (11)$$

When traders know θ , these trades coincide with those sustained in equilibrium, which is a manifestation of the First Welfare Theorem.¹³

Next, let W^o denote welfare under the first-best allocation, and $WL \equiv \mathbb{E}[W^o] - \mathbb{E}[W]$ denote the ex-ante expected welfare losses that arise when the traders purchase the asset in a quantity different from the first-best level, due to imperfect information. Under any strategy profile for the agents in which $X(p; I_i)$ is affine in s_i and p , the welfare losses can be expressed as follows (the derivations are in the Appendix):

$$WL = \frac{1}{2}(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2]. \quad (12)$$

The term $\mathbb{E}[(\tilde{x} - x^o)^2]$ captures the losses due to the discrepancy between the aggregate level of trade \tilde{x} and its first-best counterpart, x^o . The term $\mathbb{E}[(x_i - \tilde{x})^2]$, instead, captures the losses due to the dispersion of the individual trades around the average level.

3.2.2 Efficient demand schedules

Consistently with the rest of the literature (see, among others, Vives 1988, Angeletos and Pavan 2007, Amador and Weill 2012, and Vives 2017), we define the efficient use of information as the demand schedule that minimizes the ex-ante welfare losses over the set of demand schedules that are affine in the private signals and the price. While the welfare definition accounts for the costs of supplying the asset, the optimization is over the traders' demand schedules, respecting the exogeneity of the supply of the asset. This definition permits us to isolate the inefficiencies in the traders' usage of information. Accordingly, we say that $(a^T, \hat{b}^T, \hat{c}^T)$ identifies the efficient use of information if, and only if, when all traders submit the demand schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$, the welfare losses are as small as under any other affine schedule $x_i = a' s_i + \hat{b}' - \hat{c}' p$.¹⁴

Lemma 1 (efficiency of demands for given sensitivity to private information). *For any sensitivity a of the demand schedules to the traders' private information, the values of \hat{c} and \hat{b} in the demand schedules that minimize the welfare losses are given by the same functions (8) and (9) that define the equilibrium usage of information.*

Inefficiencies in trading, if present, are thus entirely due to the sensitivity of the equilibrium demand schedules to private information. Given such a sensitivity, the response of the equilibrium

¹³Clearly, the theorem does not require that the traders know u . In fact, it does not even require that they know θ . It suffices that they have no way of learning about their payoffs beyond what they know prior to trading.

¹⁴Again, we use the symbol $\hat{\cdot}$ to distinguish the efficient demand schedules from the efficient trades.

schedules to the price and the unconditional level of trade are efficient. Using Lemma 1, we can express the welfare losses as a function $WL(a, \tau_\omega(a))$ of the sensitivity of the traders' schedules to their private information and the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price (the expression for $WL(a, \tau_\omega(a))$ is in the Appendix – proof of Proposition 2). The efficient level of a , which we denote by a^T , is thus the value of a that minimizes $WL(a, \tau_\omega(a))$. Let

$$\Delta(a) \equiv -\frac{\tau_\epsilon \beta^2 y^4 \tau_\eta^4 \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y \tau_\eta}\right)^2}{\lambda^2 (\beta^2 a^2 \tau_u + y \tau_\eta)^2 (\tau_\omega(a) + \tau_\theta)}$$

and

$$\Xi(a) \equiv \frac{y \tau_\epsilon \tau_\eta^2 \beta (\tau_\omega(a) + \tau_\theta)}{\lambda \tau_e}.$$

We then have the following result:

Proposition 2 (efficient trading). *Suppose that $y_i = y$ for all i , with y exogenous. The planner's problem has a unique solution. The sensitivity a^T of the traders' demand schedules to their private information is implicitly given by the solution to*

$$a^T = \frac{1}{\lambda} \frac{K(\tau_\omega(a^T))}{\Lambda(\tau_\omega(a^T)) + \Xi(a^T) + \Delta(a^T)} \quad (13)$$

and is such that $0 < a^T < 1/\lambda$. Given a^T , the other two parameters defining the efficient demand schedules, \hat{c}^T and \hat{b}^T , are given by the same functions in (8) and (9) that describe the corresponding coefficients under the equilibrium usage of information.

When, for any a , \hat{b} and \hat{c} are set optimally, the welfare losses $WL(a, \tau_\omega(a))$ are a convex function of a reaching a minimum at $a = a^T$, with $0 < a^T < 1/\lambda$. Note that the equation (13) that determines the value of a^T differs from the one in (10) yielding the equilibrium value of a^* only by the two terms $\Delta(a)$ and $\Xi(a)$ in the denominator of the right-hand side of (13). The term $\Delta(a)$ is essentially a scaling of

$$\frac{\partial WL(a, \tau_\omega(a))}{\partial \tau_\omega(a)} \frac{\partial \tau_\omega(a)}{\partial a}.$$

Therefore, this term can be thought of as a proxy for the familiar *learning externality* originating in the fact that traders do not internalize that the sensitivity of their demand schedules to their private information determines the informativeness of the equilibrium price and hence the possibility for other traders to use the price as an endogenous signal for θ when choosing how many shares to purchase. This term is always negative reflecting the under-response of the equilibrium demand schedules to private information. Essentially, traders do not consider that responding more to their private information leads to a more informative price and hence to more efficient trades. The social planner, instead, internalizes this effect and asks that the traders respond more to their private information.

The term $\Xi(a)$, instead, is a *pecuniary externality*. When the traders respond to their private signals, they do not internalize that variations in their demands due to noise in their signals impact other traders' asset holdings through the dependence of other traders' demands on the price. Being

noise-driven, such variations are not justified in the planner's eyes. The planner thus asks that the traders respond less to their private signals to reduce the welfare losses of such noise-driven variations. The term $\Xi(a)$ is thus always positive, reflecting the over-response of the equilibrium trades to private information.

Importantly, both externalities arise because of the following properties: (1) incomplete information, (2) traders submit demand schedules, and (3) markets are incomplete. Clearly, when information is complete, the First Welfare Theorem applies. Similarly, when the traders' demands do not condition on the price, there is nothing the planner can do to improve upon the traders' ability to tell apart variations in their expectations of θ driven by the fundamental value of the asset from those driven by noise; ex-ante welfare is below the complete information level, but there is no inefficiency in the equilibrium usage of information (see Section 2 in the Supplement for a formal proof of this result, as well as the discussion in Subsection 3.2.4). Finally, when markets are complete, traders can fully insure against ex-post idiosyncratic variations in their consumption due to interim idiosyncratic variations in their perceptions of the fundamental value of the asset at the trading stage; again, the Welfare Theorems then guarantee efficiency of the equilibrium trades.

3.2.3 Fictitious environment

To shed more light on the two externalities introduced above, consider a *fictitious environment* in which the traders are naive in that they do not recognize the information contained in the market-clearing price. Such a benchmark is similar in spirit to the (fully) cursed equilibrium of Eyster and Rabin 2005. The reason for considering such an environment is that it permits us to isolate the pecuniary externality by shutting down the more familiar learning externality. To facilitate the comparison to the true economy, assume that, in this fictitious environment, each trader, in addition to observing the private signal

$$s_i = \theta + \underbrace{f(y)(\eta + e_i)}_{\equiv \epsilon_i}$$

as in the true economy, also observes an exogenous public signal

$$z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$$

whose structure is the same as the one contained in the market-clearing price, but with the endogenous noise $-u/\beta a$ replaced by the exogenous one χ , with the latter drawn from a Normal distribution with mean zero and variance τ_χ^{-1} independently of all other variables (this shock is the same for all traders). Let $\tau_\zeta \equiv y\tau_\eta\tau_\chi/(\tau_\chi + y\tau_\eta)$ denote the precision of the total noise $\zeta \equiv f(y)\eta + \chi$ in the exogenous signal z . As we show in the Appendix, in the cursed equilibrium of this fictitious economy, traders submit affine demand schedules $x_i = a_{exo}^* s_i + \hat{b}_{exo}^* + \hat{c}_{exo}^* z - \hat{d}_{exo}^* p$, where the sensitivity of the traders' demands to their private information is given by

$$a_{exo}^* = \frac{1}{\lambda} \frac{K(\tau_\zeta)}{\Lambda(\tau_\zeta)}, \quad (14)$$

with the functions $K(\cdot)$ and $\Lambda(\cdot)$ as defined in (6) and (7), respectively. Note that the formula in (14) is similar to the one in (10) in the true economy, except for the fact that the precision $\tau_\omega(a)$ of the endogenous public signal contained in the market-clearing price is replaced by the precision τ_ζ of the exogenous public signal about θ .

Now suppose that, in this fictitious economy, the planner can control the sensitivity a of the traders' demands to their private information. However, given a , the planner must choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a and $(\hat{b}, \hat{c}, \hat{d})$ as between a_{exo}^* and $(\hat{b}_{exo}^*, \hat{c}_{exo}^*, \hat{d}_{exo}^*)$ in the cursed equilibrium.¹⁵ The level of a that maximizes ex-ante welfare is then equal to

$$a_{exo}^T = \frac{1}{\lambda} \frac{K(\tau_\zeta)}{\Lambda(\tau_\zeta) + \frac{y\tau_\epsilon\tau_\eta^2\beta(\tau_\zeta + \tau_\theta)}{\lambda\tau_e}}. \quad (15)$$

Again, the formula for a_{exo}^T is similar to the one for a^T defining the efficient sensitivity to private information in the true economy, except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ and the term $\Delta(a)$ in the denominator of the expression giving the socially-optimal level of a in the true economy is equal to zero, reflecting the fact that the planner recognizes that the agents do not learn from the price. Note that $y\tau_\epsilon\tau_\eta^2\beta(\tau_\zeta + \tau_\theta)/\lambda\tau_e$ has exactly the same form as the pecuniary externality $\Xi(a)$ in the true economy. Hence, in this fictitious economy, the cursed-equilibrium demand schedules unambiguously feature an excessively high sensitivity to private information: $a_{exo}^* > a_{exo}^T$. Furthermore, when the precision of the exogenous public signal in the cursed economy is the same as the precision of the endogenous public signal under the efficient demand schedules of the true economy (that is, when $\tau_\zeta = \tau_\omega(a^T)$), a_{exo}^T coincides with the solution to the equation $\partial WL(a_{exo}^T, \tau_\omega(a^T))/\partial a = 0$ and $a_{exo}^T < a^T$: in the true economy, the planner recognizes the value of increasing the precision of the endogenous signal contained in the market-clearing price and thus demands that traders respond more to their private information.

3.2.4 Sign of externalities and slope of demand schedules

We now return to the economy in which both the traders and the planner account for the information contained in the market-clearing price. Whether the sensitivity of the equilibrium demand schedules to the traders' private information is excessively high or low (compared to the efficient level a^T) then depends on which of the two externalities described above prevails. Comparing (10) with (13), we have that the sign of $a^* - a^T$ equals the sign of $\Xi(a^T) + \Delta(a^T)$. When $\Xi(a^T) + \Delta(a^T) = 0$, the two externalities cancel each other out, the submitted schedules are price-inelastic (i.e., $\hat{c}^T = 0$)

¹⁵In the true economy, maintaining the same relationship between a and (\hat{b}, \hat{c}) is without loss of optimality for the planner (see Lemma 1 above). This need not be the case in the fictitious economy. However, imposing the restriction permits us to isolate the relevant effects.

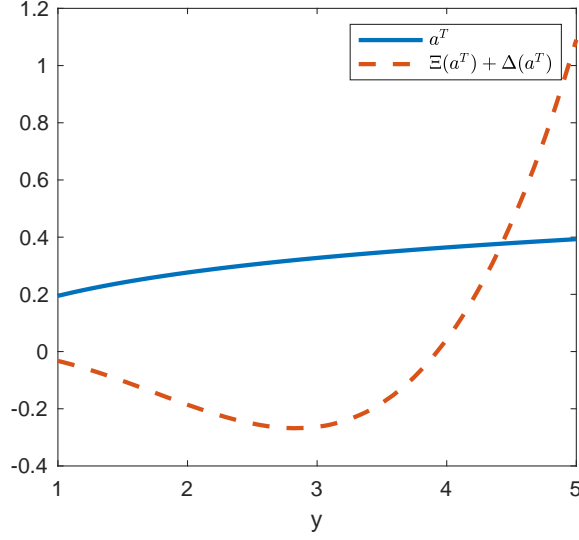


Figure 1: The blue solid line corresponds to a^T whereas the orange dashed line represents the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

and $a^* = a^T$. When, instead, $\Xi(a^T) + \Delta(a^T) > 0$, the pecuniary externality dominates, $\hat{c}^T > 0$ (the efficient demand schedules are downward sloping) and the equilibrium schedules feature an excessive response to the traders's private information. Finally, when $\Xi(a^T) + \Delta(a^T) < 0$, the learning externality dominates, $\hat{c}^T < 0$ (the efficient demand schedules are upward sloping) and the equilibrium response to private information is insufficiently low.

It is worth noting that if the traders were restricted to submitting market orders (like in a Cournot model), then the usage of information would be efficient since the two externalities would not be present (See the Supplement for a formal proof of this result).

Using simulations, it is possible to nail down the effect of variations in the quality y of the traders' private information on the two externalities and on the slope of the efficient demand schedules. Figure 1 depicts the sensitivity of the traders' efficient demand schedules to their private information a^T (solid blue curve) as well as the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ (dashed orange curve), as a function of the quality y of the traders' private information. As y increases, the efficient response a^T to the traders' private information increases, reflecting the higher value of responding to more accurate private information. Furthermore, because both Ξ and Δ increase with a^T , a higher y contributes to a higher value of $\Xi(a^T) + \Delta(a^T)$ via the indirect effect that y has on the two externalities through a^T . In addition, holding a^T fixed, we have that y has a direct effect on both $\Xi(a^T)$ and $\Delta(a^T)$. Whereas $\Xi(a^T)$ is increasing in y , $\Delta(a^T)$ is decreasing. Combining the direct with the indirect effects, we then have that $\Xi(a^T)$ unambiguously increases with y , whereas $\Delta(a^T)$ is non-monotonic in y . For small values of y , the sum of the two externalities is negative and decreasing in y , whereas, for sufficiently high values of y , the sum of the two externalities is positive

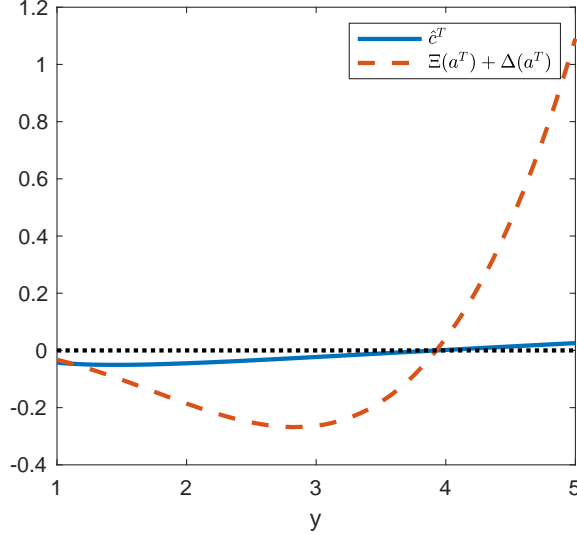


Figure 2: The blue solid line corresponds to \hat{c}^T whereas the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The two curves switch signs for the same value of y . The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

and increasing in y , as can be seen from Figure 1.

Next, we turn to the relationship between the two externalities and the slope of the efficient demand schedules, \hat{c}^T . Figure 2 depicts the sensitivity \hat{c}^T of the efficient demand schedules to the price (the solid blue curve) along with the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ (the orange dashed curve), as a function of the quality y of the traders' private information. The two curves switch sign for the same value of y . As explained above, when the traders possess high-quality private information (high values of y), the marginal value of generating additional information through the price is low and the pecuniary externality dominates over the learning externality, so that $\Xi(a^T) + \Delta(a^T)$ is positive and increasing in y . In this case, \hat{c}^T is positive meaning that the efficient demand schedules are downwards sloping, as they would be in an economy in which the fundamental value of the asset θ is known to the traders. When, instead, the traders possess low-quality private information, the learning externality dominates over the pecuniary externality so that $\Xi(a^T) + \Delta(a^T)$ is negative and first decreasing and then increasing in y . In this case, \hat{c}^T is negative meaning that the efficient demand schedules are upwards sloping, reflecting the high sensitivity of the traders' estimates of the fundamental value of the asset θ to the price, relatively to the sensitivity of the same estimates to their private information.

We conclude this subsection by highlighting the role that the common noise η in the traders' private information plays for the sign and magnitude of the two externalities identified above. Unsurprisingly, both a^* and a^T are increasing in the precision τ_η of the noise η , reflecting the fact that responding to private information is more valuable (both for the traders and for the planner) when it is affected less by correlated noise and hence more precise. Similarly, holding a fixed, we have

that the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price naturally increases with τ_η , reflecting the fact that the noise in the traders' signals becomes less correlated when τ_η increases and, as a result, washes out more at the aggregate level, making the price more informative, for given demand schedules. Furthermore, fixing a^T , the absolute value of both $\Xi(a^T)$ and $\Delta(a^T)$ increases with τ_η , reflecting the larger role that either externality plays when the noise in the private signals is less correlated. However, whereas the pecuniary externality $\Xi(a^T)$ increases with τ_η , the learning externality $\Delta(a^T)$ decreases with it. Combining all of the above effects, we then have that the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ can be non-monotonic in τ_η , depending on the other parameters' values.

3.3 Policies inducing efficient trading with exogenous information

Next, we discuss policies that correct the inefficiencies in the usage of information identified in the previous subsections, once again holding fixed the quality of the traders' information y for the time being.

Proposition 3 (policy inducing efficient trading). *Suppose that $y_i = y$ for all i , with y exogenous. There exists $\delta, t_p, t_0 \in \mathbb{R}$ such that the efficient use of information can be implemented with a policy that charges the traders a total tax bill equal to $T(x_i, p) = \frac{\delta}{2}x_i^2 - t_0x_i + t_p p x_i$ where t_0 , t_p and δ are functions of all parameters.*

The efficient use of information can thus be induced through a combination of a linear-quadratic tax $\frac{\delta}{2}x_i^2 - t_0x_i$ on the individual volume of trade (equivalently on the quantity of the asset purchased), along with a (more familiar) ad-valorem tax $t_p p x_i$. The role of δ is to manipulate the traders' adjustment cost (from λ to $\lambda + \delta$). This manipulation suffices to induce the traders to submit demand schedules whose sensitivity to their exogenous private information is equal to the efficient level a^T . The role of the linear ad-valorem tax is to guarantee that, once the sensitivity a coincides with the efficient level a^T , the sensitivity \hat{c} of the equilibrium demand schedules to the price coincides with the efficient level \hat{c}^T . In the absence of such a correction, the traders fail to submit the efficient demand schedules, even if they respond efficiently to their private information. Finally, the role of the linear tax t_0x_i on the individual volume of trade is to guarantee that the fixed part \hat{b} of the demand schedules (equivalently, the unconditional volume of trade) also coincides with its efficient counterpart \hat{b}^T .

The tax scheme of Proposition 3 induces the traders to submit the efficient limit orders. In principle, such a scheme is simple to implement, as it only requires conditioning taxes on variables (price and individual volume of trade) that are easy to observe. However, the scheme requires a non-linear dependence of the total tax bill on the quantity purchased. Such non-linearities, while conceptually straight-forward, are sometimes perceived as difficult to implement in practice. The question of interest is then whether a planner who is restricted to simple ad-valorem taxes such as

those often discussed in the policy debate can still improve upon the laissez-faire equilibrium by choosing t_p optimally.

Proposition 4. *[sub-optimality of ad-valorem taxes with exogenous information] Suppose that $y_i = y$ for all i , with y exogenous. If a planner is constrained to use ad-valorem taxes (that is, a policy that, given p and x_i , charges each trader a total tax bill equal to $T(p, x_i) = t_p p x_i$, for some $t_p \in \mathbb{R}$) then the optimal policy is such that $t_p = 0$.*

Hence, if the planner is constrained to using a tax that is linear in the expenditure $p x_i$ on the asset (equivalently, an ad-valorem tax), the optimal value of the tax is zero. This is because any such a policy fails to manipulate the relative importance that each trader attaches to his private information and the price in predicting the value of the asset. In other words, such a tax does not change the information contained in the equilibrium price. As a result, the equilibrium sensitivity to private information, a^* , is the same as in the laissez-faire equilibrium, no matter the value of t_p . Because, in the laissez-faire equilibrium, for any a , the other two elements of the equilibrium demand schedules, $\hat{b}(a)$ and $\hat{c}(a)$, are welfare-maximizing (see Lemma 1), any ad-valorem tax with $t_p \neq 0$, by changing \hat{b} and \hat{c} without changing a , reduces welfare.

4 Inefficiency in information acquisition and policy corrections

We now investigate how inefficiencies in information acquisition relate to inefficiencies in trading (equivalently, in information usage), and how the planner can correct them through appropriate policy interventions.

4.1 Inefficiency in information acquisition under efficient trading

Suppose that the traders can be trusted to submit the efficient limit orders; can they be trusted to collect private information efficiently? We first consider the case where efficiency in trading is exogenous and then the case in which it is induced through the policy in Proposition 3. In both cases, we find that the traders do not acquire information efficiently. We conclude by considering richer families of policy interventions which permits us to uncover both an impossibility and a couple of possibility results.

Proposition 5 (inefficiency in information acquisition under efficient trading). *Let y^T denote the socially optimal quality of private information and suppose that all traders submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). When $\hat{c}^T > 0$ (i.e., when the efficient demand schedules are downward sloping), the quality of private information acquired in equilibrium is higher than y^T , whereas the opposite is true when $\hat{c}^T < 0$ (i.e., when the efficient demand schedules are upward sloping).*

Recall from the previous section that, when $\hat{c}^T > 0$, traders over-respond to private information. Forcing them to respond less to their private information then induces them to over-invest in information acquisition. When, instead, $\hat{c}^T < 0$, traders under-respond to private information. Forcing them to trade efficiently then induces them to under-invest in information acquisition.

In the special case in which $\hat{c}^T = 0$ (that is, when the efficient demand schedules are completely price-inelastic and hence can be implemented with market orders), in the absence of policy interventions, a trader endowed with information of quality y^T would trade efficiently. In this case, when information is endogenous, the trader acquires information of efficient quality y^T , even in the absence of policy interventions.

The above results hinge on the traders being exposed to correlated noise in their information sources, that is, on $\tau_\eta \in (0, +\infty)$. When $\tau_\eta = 0$, the noise in the agents' private signals is infinite, making the signals worthless both for the individual traders and for the planner. When, instead, $\tau_\eta \rightarrow +\infty$, the correlated noise in the agents' private signals disappears, in which case, fixing the agents' demand schedules, we have that the aggregate volume of trade is invariant to the quality of the traders' private information. This is the case considered in most of the previous literature. In this situation, holding the demand schedules fixed, we have that the only effect of an increase in the quality of the traders' private information on welfare is through the reduction in the dispersion of individual trades around the aggregate level of trade. Because this effect is weighted equally by the planner and by the individual traders, the private and the social value of information coincide, which guarantees that the information acquired in equilibrium is efficient.

Recall that, for small y , the learning externality dominates over the pecuniary externality and the efficient demand schedules are upward sloping, whereas, for large y , the pecuniary externality dominates and the efficient demand schedules are downward sloping. The above results thus suggest that, as technological progress makes information cheaper (that is, the cost of information acquisition decreases), the economy is likely to eventually enter into a regime of over-investment in information acquisition.

To further understand the implications of the above inefficiencies on asset-pricing variables, it is helpful to introduce the following:

Definition 1. [market quality variables] Let *market depth* be the inverse of the sensitivity of the price to the supply shock u : $MD \equiv (dp/du)^{-1} = 1 + \beta\hat{c}$. Let the *volatility of the price* be: $\sigma_p = (\text{Var}[p])^{\frac{1}{2}}$. Finally, let the *informativeness* of the price be the precision of the endogenous signal contained in the price: τ_ω .

Figure 3 shows how the above asset-pricing variables are affected by changes in the cost of information acquisition, both under the decentralized equilibrium of the laissez-faire economy and under the efficient allocation (where both the acquisition and usage of information coincide with the welfare-maximizing levels). The figure assumes a quadratic cost of information $C(y) = By^2/2$; a reduction in the cost of information corresponds to a reduction in the parameter B . As the cost of

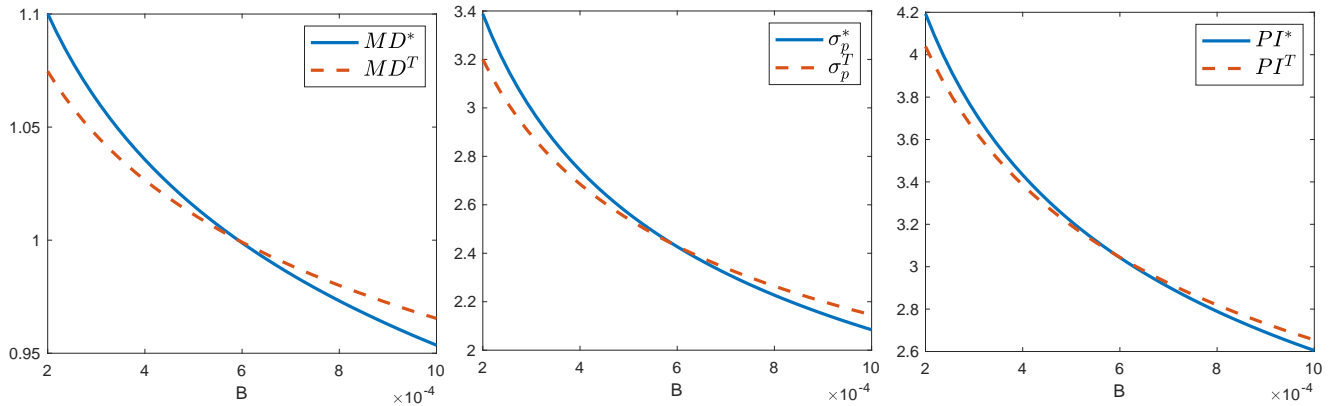


Figure 3: The first panel depicts market depth, the second one price volatility, and the third one price informativeness. In each panel, the blue solid curve corresponds to the laissez-faire economy, whereas the orange dashed curve corresponds to the solution to the planner’s problem. The x -axis in all three panels represents the scalar B that parametrizes the quadratic cost of information. The other parameter values are $\lambda = \beta = 1.3$, $\tau_e = 0.8$, $\tau_\eta = 0.6$, $\tau_\theta = 0.1$, and $\tau_u = 30$.

information decreases (moving from right to left along the x -axis) market depth, price volatility, and price informativeness all move from being inefficiently low to being inefficiently high.

We could also establish the following:

Numerical result: *When $\hat{c}^T < 0$ (i.e., when the efficient demand schedules are upward sloping), the equilibrium in the absence of policy interventions is such that the acquisition of private information, the sensitivity of the demand schedules to private information, price volatility, market depth, and price informativeness are all inefficiently low. The opposite is true when $\hat{c}^T > 0$ (i.e., when the efficient demand schedules are downward sloping). As the cost of acquiring information decreases, the economy moves from the first regime to the second.*

To obtain the result, we simulated the model 1,000 times drawing the parameters τ_u , τ_e , τ_η , τ_θ , λ , and β uniformly from 1 to 30. The cost function of acquiring information in the simulations is $C(y) = By^2/2$, with B drawn uniformly from 0 to 0.01. In all the simulations, the sign of \hat{c}^T , $y^* - y^T$, and $a^* - a^T$ is the same. Figure 4 illustrates the relationship between the cost of information, parametrized by B , the inefficiency $y^* - y^T$ in the acquisition of information, the slope \hat{c}^T of the efficient demand schedules, and the inefficiency $a^* - a^T$ of the equilibrium limit orders under one of these simulations.

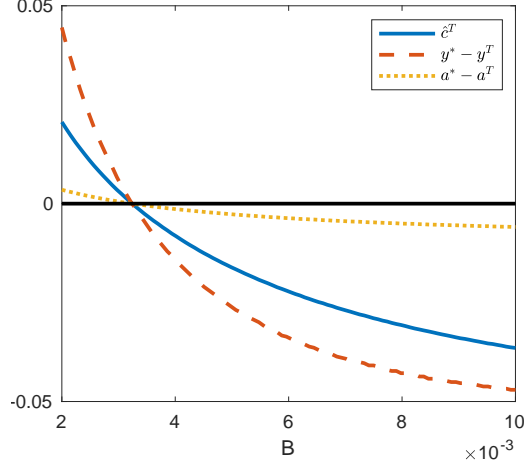


Figure 4: The blue solid curve represents the slope of the efficient demand curve. The orange dashed curve represents the inefficiency in information acquisition, where a positive number means inefficiently high acquisition, and a negative number means inefficiently low acquisition. The yellow dotted curve represents the inefficiency in the sensitivity of the demand schedules to private information, where a positive number means inefficiently high sensitivity, and a negative number means inefficiently low sensitivity. The x -axis is the value of B that parametrizes the cost of information $C(y) = BY^2/2$. The parameter values in the simulations are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, and $\tau_u = 30$.

4.2 Policy corrections

We now address the question of whether efficiency in information acquisition can be induced through an appropriate policy design. We start by considering the problem of a planner who trusts the traders to submit the efficient demand schedules and then consider the more relevant problem of a planner who does not trust the traders and hence seeks to induce efficiency in both information acquisition and trading through the same policy.

Proposition 6. [*policy inducing efficiency in information acquisition under efficient trading*] Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the quality of information is y^T . Suppose the planner trusts the traders to submit the efficient demand schedules. The planner can then induce the traders to collect the efficient information y^T by charging them a total tax bill equal to $T(p, x_i) = \hat{t}_p p x_i$, with $\hat{t}_p > 0$ if $\hat{c}^T > 0$ (downward-sloping demands) and $\hat{t}_p < 0$ if $\hat{c}^T < 0$ (upward-sloping demands).

Hence, a planner who trusts the traders to submit the efficient demand schedules can induce the traders to collect information efficiently by using a simple “ad valorem” tax on total asset purchases similar to those discussed in the policy debate.

Next, consider the more realistic case of a planner who does not trust the market to submit the efficient limit orders and hence seeks to design a policy that induces efficiency in both information

acquisition and trading. In the previous section, we showed that, when the quality of information is exogenous, efficiency in trading can be induced through a combination of a linear-quadratic tax on the individual volume of trade paired with an ad-valorem tax (both rebated in a lump-sum manner, if desired). Based on other results in the literature, one may conjecture that the same policy mix also induces efficiency in information acquisition. The next result shows that this is not the case. If the planner were to use the tax $T(p, x_i)$ in Proposition 3 (applied to $y = y^T$) that induces the traders to submit the efficient demand schedules when information is exogenous and such that $y_i = y^T$ for all i , then the traders would respond by acquiring information of quality different from y^T and by submitting demand schedules different from the efficient ones. More generally, the proposition shows that there exists no policy measurable in the individual volume of trade and in the price of the financial asset that induces efficiency in both trading and information acquisition.

Proposition 7 (impossibility to induce efficiency in both information acquisition and trading with standard policies). *Generically (i.e., with the exception of a set of parameters of zero Lebesgue measure), there exists no policy $T(x_i, p)$ that induces efficiency in both information acquisition and trading.*

The result is established in the Appendix by showing that any policy that induces the traders to submit the efficient demand schedules once they collect the efficient amount of private information y^T must coincide with the one in Proposition 3 (applied to $y = y^T$), except for terms that play no role for incentives. However, any such a policy induces the traders to misperceive the value of their private information (around the efficient level y^T) and hence induces them to collect an inefficient amount of private information.

We conclude with two possibility results. The first one establishes that, when information acquisition is verifiable, efficiency in both information acquisition and trading can be obtained by conditioning the total tax bill on the expenditure on information acquisition. The second result establishes that, when information acquisition is not verifiable (e.g., because it reflects the attention paid to various exogenous sources), then efficiency in both acquisition and trading can be obtained by conditioning the marginal tax rate on the aggregate volume of trade.

Proposition 8 (policy inducing efficiency in both information acquisition and trading when acquisition is verifiable). *Suppose that the acquisition of private information is verifiable. There exists $A \in \mathbb{R}$ such that efficiency in both information acquisition and trading can be induced through the policy*

$$T^{tot}(x_i, p, y_i) = \frac{\delta}{2}x_i^2 - t_0x_i + t_p p x_i - A y_i$$

where (δ, t_p, t_0) are as in Proposition 3 for $y = y^T$.

Simulations show that $A < 0$ when $\hat{c}^T > 0$ whereas $A > 0$ when $\hat{c}^T < 0$. That is, expenditures on information acquisition are taxed when the efficient demand schedules are downward sloping and

subsidized when they are upward sloping, reflecting the fact that, under the policy of Proposition 3, agents over-invest in information acquisition in the former case and underinvest in the latter.

Proposition 9 (policy inducing efficiency in both information acquisition and trading when acquisition is non-verifiable). *Suppose that information acquisition is not verifiable. There exist $\delta^*, t_{\tilde{x}}^*, t_0^*, t_p^* \in \mathbb{R}$ such that efficiency in both information acquisition and trading can be induced through a (linear-quadratic) tax bill of the form*

$$T^*(x_i, \tilde{x}, p) = \frac{\delta^*}{2} x_i^2 + (t_{\tilde{x}}^* \tilde{x} - t_0^*) x_i + t_p^* p x_i$$

in which the marginal tax rate $\partial T^*(x_i, \tilde{x}, p) / \partial x_i$ depends on the aggregate volume of trade \tilde{x} .

The dependency of the marginal tax rate on the aggregate volume of trade \tilde{x} is essential to induce efficiency in both information acquisition and trading. As we show in the Appendix, with this type of policies, the planner can equalize the expected marginal tax rate

$$\frac{\partial}{\partial x_i} \mathbb{E}[T^*(x_i, \tilde{x}, p) | x_i, p; y_i, y^T] \Big|_{y_i=y^T; x_i=a^T s_i + \hat{b}^T - \hat{c}^T p}$$

of each individual trader who acquires information of quality $y_i = y^T$ and then submits the efficient demand schedule $a^T s_i + \hat{b}^T - \hat{c}^T p$ with the discrepancy

$$\mathbb{E}[\theta | x_i, p; y_i, y^T] - p - \lambda x_i \Big|_{y_i=y^T; x_i=a^T s_i + \hat{b}^T - \hat{c}^T p}$$

between the marginal benefit and the marginal cost of expanding the individual volume of trade around the efficient level $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$. Eliminating such a discrepancy is essential to induce efficiency in trading. Importantly, the new contingency provides the planner with flexibility on how to eliminate such a discrepancy. When, instead, the policy depends only on x_i and p , there exists a unique way of eliminating such a discrepancy, as shown in the proof of Proposition 7. The extra flexibility in turn can be used to realign the marginal private value of more precise private information to its social counterpart, something that is not possible when the policy depends only on x_i and p . In the proof in the Appendix, we also show that, when information acquisition is not verifiable, the policy that implements efficiency in both information acquisition and trading is in fact unique up to terms that do not matter for incentives.

As explained in the previous section, the taxes discussed in the policy debate are typically ad-valorem (that is, linear in the individual expenditures $p x_i$). When information is exogenous, we showed in Proposition 4 that such taxes reduce welfare. The next proposition shows that this is the case also when information is endogenous.

Proposition 10 (sub-optimality of ad-valorem taxes with endogenous information). *Suppose that information is endogenous and that the planner is restricted to using ad-valorem taxes of the form $T(x_i, p) = t_p p x_i$, for some $t_p \in \mathbb{R}$. Then the optimal t_p is zero.*

The intuition for the result is similar to the one for Proposition 4. Ad-valorem taxes do not affect the equilibrium sensitivity to private information. They also do not affect the value that each trader

assigns to increasing the precision of his private information. These taxes only affect the sensitivity of the equilibrium limit orders to the price and the unconditional volume of trade. However, given y and a , the sensitivity of the equilibrium schedules to the price and the unconditional volume of trade (the parameters \hat{c} and \hat{b} in the limit orders) are efficient under the laissez-faire allocation. As a result, these taxes only bring the equilibrium allocation further away from the efficient one, and hence reduce welfare.¹⁶

Propositions 4 and 10 have important implications for the debate on how to tax transactions in financial markets. They show that, no matter whether information is exogenous or endogenous, the policies that are typically proposed do more harm than good. The policy maker should instead consider more sophisticated taxes that are non-linear in the purchases of the asset and that condition marginal tax rates on the aggregate volume of trade.

In a similar vein, it is often suggested that governments can improve over the laissez-faire equilibrium by manipulating prices through asset purchases. One can show that such policies are also welfare detrimental. The reason is essentially the same as for ad-valorem taxes.

5 Conclusions

We identify inefficiency in the trading of financial assets and relate it to the information that traders collect privately before submitting their orders. We show that, if the traders' private information were exogenous, inefficiency in trading could be corrected with a combination of ad-valorem taxes with non-linear subsidies/taxes on the individual volume of asset purchases. However, when information is endogenous, there exist no policy measurable in the price of the financial asset and in the volume of individual trades that induces efficiency in both trading and information acquisition. The above impossibility result can be turned into a possibility one by conditioning the total tax bill on individual expenditures on information acquisition (when the latter are verifiable), or by conditioning the marginal tax rate on the aggregate volume of trade. In practice, authorities typically consider only simple linear *ad-valorem* taxes. We find that, if these taxes are the only instrument available, they should not be used.

In future work, it would be interesting to extend the analysis to a broader class of economies in which financial decisions interact with real decisions, and in which agents exchange multiple assets over multiple periods.

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¹⁶See Dávila and Walther 2021 for a study of corrective taxation in environments where the Government's interventions are limited because some agents cannot be taxed, or certain activities cannot be regulated.

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6 Appendix

Proof of Proposition 1. As explained in the main text, when the traders submit affine demand schedules with parameters (a, \hat{b}, \hat{c}) , the market-clearing price can be expressed as follows

$$p = \frac{\alpha + \beta \hat{b}}{1 + \beta \hat{c}} + \frac{\beta a}{1 + \beta \hat{c}} z,$$

where $z \equiv \theta + \omega$ is the endogenous public signal contained in p , with noise $\omega \equiv f(y)\eta - u/(\beta a)$ of precision $\tau_\omega(a) \equiv (\beta^2 a^2 y \tau_u \tau_\eta) / (\beta^2 a^2 \tau_u + y \tau_\eta)$.¹⁷ In turn, this implies that the equilibrium trades $x_i = a s_i + \hat{b} - \hat{c} p$ can be expressed as affine functions $x_i = a s_i + b + c z$ of the traders’ exogenous private information s_i and the endogenous public information z contained in the market-clearing price, with

$$b = \hat{b} - \hat{c} \frac{\alpha + \beta \hat{b}}{1 + \beta \hat{c}} \quad (16)$$

and

$$c = -\frac{\beta a \hat{c}}{1 + \beta \hat{c}}. \quad (17)$$

For each vector (a, \hat{b}, \hat{c}) describing the demand schedules, there exists a unique vector (a, b, c) describing the equilibrium trades and vice versa. Hereafter, we find it more convenient to characterize the equilibrium use of information in terms of the vector (a, b, c) describing the equilibrium trades. Replacing $\tilde{x} = \int x_i di = (a + c)z + \frac{u}{\beta} + b$ into the expression for the inverse aggregate supply function $p = \alpha - u + \beta \tilde{x}$, we then have that the equilibrium price can be expressed as follows:

$$p = \alpha + \beta b + \beta(a + c)z. \quad (18)$$

Using standard projection formulas, we then have that

$$\mathbb{E}[\theta | I_i, p] = \mathbb{E}[\theta | s_i, z] = \gamma_1(\tau_\omega(a)) s_i + \gamma_2(\tau_\omega(a)) z$$

where, for any τ_ω ,

$$\gamma_1(\tau_\omega) \equiv \frac{\tau_\epsilon y \tau_\eta (y \tau_\eta - \tau_\omega)}{y^2 \tau_\eta^2 (\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega \tau_\epsilon (\tau_\theta + 2y \tau_\eta)} \quad (19)$$

and

$$\gamma_2(\tau_\omega) \equiv \left(1 - \gamma_1(\tau_\omega) \frac{\tau_\theta + y \tau_\eta}{y \tau_\eta}\right) \frac{\tau_\omega}{\tau_\omega + \tau_\theta}. \quad (20)$$

Optimality requires that the equilibrium trades satisfy $x_i = \frac{1}{\lambda} (\mathbb{E}[\theta | s_i, z] - p)$ which, together with the results above, is equivalent to

$$x_i = \frac{1}{\lambda} [\gamma_1(\tau_\omega(a)) s_i - (\alpha + \beta b) + (\gamma_2(\tau_\omega(a)) - \beta(a + c)) z].$$

The sensitivity a^* of the equilibrium demand schedules to the traders’ private information must thus

¹⁷To derive $\tau_\omega(a)$ we use the fact that $f(y) = 1/\sqrt{y}$.

satisfy $a = \gamma_1(\tau_\omega(a))/\lambda$, which is equivalent to equation (10) in the main text. The sensitivity of the equilibrium trades to the endogenous public signal must satisfy

$$c = \frac{1}{\beta + \lambda} \left[\left(1 - \lambda a \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a \right]. \quad (21)$$

The constant b in the equilibrium trades must satisfy

$$b = -\frac{\alpha}{\beta + \lambda}. \quad (22)$$

Inverting the relationship between b and \hat{b} and c and \hat{c} using (16) and (17), we conclude that, given a^* , the values of \hat{c}^* and \hat{b}^* are given by the functions (8) and (9), as claimed in the proposition.

To complete the proof, it thus suffices to show that equation (10) admits a unique solution and that such a solution satisfies $0 < a^* < 1/\lambda$. To see this, note that this equation is equivalent to

$$0 = \lambda\beta^2\tau_u a^3 [y^3\tau_\eta^3 + y^2\tau_\eta^2(\tau_\epsilon + \tau_\theta) - y\tau_\eta\tau_\epsilon(\tau_\theta + 2y\tau_\eta)] + \lambda a y^3\tau_\eta^3(\tau_\epsilon + \tau_\theta) - \tau_\epsilon y^3\tau_\eta^3. \quad (23)$$

In a cubic equation of the form $Ax^3 + Bx^2 + Cx + D = 0$, if

$$\Delta \equiv 18ABCD - 4B^3D + B^2C^2 - 4AC^3 - 27A^2D^2 < 0,$$

the equation has a unique real root. In our case, $B = 0$ and $C > 0$ and, as a result, $\Delta = -4AC^3 - 27A^2D^2$. Furthermore, using the fact that $\tau_\epsilon \equiv y\tau_\epsilon\tau_\eta/(\tau_\epsilon + \tau_\eta)$, we have that

$$\begin{aligned} A &= \lambda\beta^2\tau_u (y^3\tau_\eta^3 + y^2\tau_\eta^2(\tau_\epsilon + \tau_\theta) - y\tau_\eta\tau_\epsilon(\tau_\theta + 2y\tau_\eta)) \propto y\tau_\eta (y^2\tau_\eta^2 + y\tau_\eta\tau_\theta - \tau_\epsilon\tau_\theta - \tau_\epsilon y\tau_\eta) \\ &\propto (\tau_\theta + y\tau_\eta)(y\tau_\eta - \tau_\epsilon) \propto y\tau_\eta - \frac{y\tau_\epsilon\tau_\eta}{\tau_\epsilon + \tau_\eta} \propto \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} > 0. \end{aligned}$$

Therefore $\Delta < 0$ and hence the above cubic equation has a unique real root. Furthermore, because D is negative, the unique real root is positive. Replacing $a = 1/\lambda$ into the cubic equation, we have that

$$\begin{aligned} &\beta^2\tau_u \frac{1}{\lambda^2} (y^3\tau_\eta^3 + y^2\tau_\eta^2(\tau_\epsilon + \tau_\theta) - y\tau_\eta\tau_\epsilon(\tau_\theta + 2y\tau_\eta)) + y^3\tau_\eta^3(\tau_\epsilon + \tau_\theta) - \tau_\epsilon y^3\tau_\eta^3 \\ &= \beta^2\tau_u \frac{y\tau_\eta}{\lambda^2} (y^2\tau_\eta^2 + y\tau_\eta\tau_\theta - \tau_\epsilon\tau_\theta - \tau_\epsilon y\tau_\eta) + y^3\tau_\eta^3\tau_\theta > 0. \end{aligned}$$

This implies that $0 < a^* < 1/\lambda$. Q.E.D.

Proof of Lemma 1. As explained in the proof of Proposition 1, when the traders submit demand schedules of the form $x_i = as_i + \hat{b} - \hat{c}p$, for some (a, \hat{b}, \hat{c}) , the trades induced by market clearing can be expressed as $x_i = as_i + b + cz$, with the values of b and c given by (16) and (17). Using the fact that $\tilde{x} = a(\theta + f(y)\eta) + b + cz$, we thus have that ex-ante welfare can be expressed as follows:

$$\mathbb{E}[W] = \mathbb{E} \left[(\theta - \alpha + u)(a(\theta + f(y)\eta) + b + cz) - \frac{\beta}{2}(a(\theta + f(y)\eta) + b + cz)^2 - \frac{\lambda}{2} \int_0^1 (as_i + b + cz)^2 di \right].$$

Note that, given a , $\mathbb{E}[W]$ is concave in b and c . For any a , the optimal values of b and c are thus given by the FOCs $\partial\mathbb{E}[W]/\partial b = 0$ and $\partial\mathbb{E}[W]/\partial c = 0$. The values given by (21) and (22) solve these equations. Using (16) and (17) to go from the optimal trades to the demand schedules that

implement them, we thus conclude that, for any choice of a^T , the optimal values of \hat{c}^T and \hat{b}^T are given by the functions (8) and (9), as claimed. Q.E.D.

Proof of Proposition 2. Using Lemma 1, one can show that the welfare losses can be expressed as a function

$$\begin{aligned}
WL(a, \tau_\omega(a)) &= \frac{\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} \right]^2}{2(\beta + \lambda)\tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}}{2(\beta + \lambda)y\tau_\eta} \\
&\quad + \frac{\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} \right]^2}{2(\beta + \lambda)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}
\end{aligned} \tag{24}$$

of a and $\tau_\omega(a)$. The socially optimal level of a must solve $dWL(a, \tau_\omega(a))/da = 0$ which yields the condition in the proposition. One can also verify that, at $a = 1/\lambda$, $dWL(a, \tau_\omega(a))/da > 0$, whereas, at $a = 0$, $dWL(a, \tau_\omega(a))/da < 0$. Hence a^T must satisfy $0 < a^T < 1/\lambda$, as claimed. Q.E.D.

Derivation of Conditions (14) and (15). In the cursed economy, each trader receives a private signal $s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv \epsilon_i}$ and a public signal $z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$, and believes p to be orthogonal to (θ, η) . Following steps similar to those leading to Proposition 1, we have that $\mathbb{E}[\theta|s_i, z] = \gamma_1 s_i + \gamma_2 z$, where

$$\gamma_1 \equiv \frac{\tau_\epsilon y \tau_\eta (y \tau_\eta - \tau_\zeta)}{y^2 \tau_\eta^2 (\tau_\zeta + \tau_\epsilon + \tau_\theta) - \tau_\zeta \tau_\epsilon (\tau_\theta + 2y \tau_\eta)} \quad \text{and} \quad \gamma_2 \equiv \left(1 - \gamma_1 \frac{\tau_\theta + y \tau_\eta}{y \tau_\eta} \right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}.$$

Because the cursed-equilibrium demand schedules $x_i = a_{exo}^* s_i + \hat{b}_{exo}^* + \hat{c}_{exo}^* z - \hat{d}_{exo}^* p$ must satisfy $x_i = (\mathbb{E}[\theta|s_i, z] - p)/\lambda$, we have that $a_{exo}^* = \gamma_1/\lambda$, $\hat{b}_{exo}^* = 0$, $\hat{c}_{exo}^* = \gamma_2/\lambda$, and $\hat{d}_{exo}^* = 1/\lambda$. Using the formula for γ_1 above, we have that the formula for a_{exo}^* is equivalent to the one in (14) in the main text.

Now suppose that, given a , the planner is constrained to choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a and $(\hat{b}, \hat{c}, \hat{d})$ as between a_{exo}^* and $(\hat{b}_{exo}^*, \hat{c}_{exo}^*, \hat{d}_{exo}^*)$ in the cursed equilibrium. Following steps similar to those in the proof of Proposition 2, we then have that the value of a that minimizes the welfare must satisfy condition (15) in the main text. Q.E.D.

Proof of Proposition 3. Following the same steps as in the proof of Proposition 1, we have that the equilibrium value of a under the proposed policy is the unique solution to

$$a = \frac{1}{\lambda + \delta} \frac{K(\tau_\omega(a))}{\Lambda(\tau_\omega(a))}$$

whereas the values of b and c describing the equilibrium trades $x_i = a s_i + b + c z$ are given by

$$b = \frac{t_0 - (1 + t_p)\alpha}{\lambda + \delta + (1 + t_p)\beta} \quad \text{and} \quad c = \frac{\gamma_2(\tau_\omega(a)) - (1 + t_p)\beta a}{\lambda + \delta + (1 + t_p)\beta}.$$

Hence, the equilibrium trades under the proposed policy coincide with the efficient trades $x_i = a^T s_i + b^T + c^T z$ if and only if

$$\delta = \frac{\lambda(\Xi(a^T) + \Delta(a^T))}{y^2 \tau_\eta^2 (\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T) \tau_\epsilon (\tau_\theta + 2y \tau_\eta)},$$

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda} \left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a \right] - \beta a^T}{\beta \left\{ \frac{1}{\beta+\lambda} \left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a \right] + a^T \right\}}$$

and

$$t_0 = (1 + t_p)\alpha - \frac{\alpha [\lambda + \delta + (1 + t_p)\beta]}{\beta + \lambda}.$$

Q.E.D.

Proof of Proposition 4. Following the same steps as in the proof of Proposition 1, one can show that, under the proposed policy, the equilibrium trades are given by $x_i = as_i + b + cz$ where a is given by the same value as in Proposition 1 whereas

$$b = -(1 + t_p) \frac{\alpha}{(1 + t_p)\beta + \lambda} \quad (25)$$

and

$$c = \frac{1}{\beta(1 + t_p) + \lambda} \left[\left(1 - \lambda a \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - (1 + t_p)\beta a \right]. \quad (26)$$

Hence, any ad-valorem tax $t_p \neq 0$ induces the same sensitivity of the equilibrium trades to private information as in the laissez-faire equilibrium but different values of b and c . Because, given a^* , the values of b and c (equivalently, of \hat{b} and \hat{c}) in the laissez-faire economy maximize welfare, as shown in Lemma (1), we conclude that any policy $t_p \neq 0$ results in strictly lower welfare than $t_p = 0$. Q.E.D.

Proof of Proposition 5 Let y^T denote the socially optimal precision of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the precision of private information is y^T . Next, let $\mathbb{E}[W^T; \bar{y}]$ denote ex-ante gross welfare when all traders acquire information of quality \bar{y} but then submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Such a welfare function is gross of the costs of information acquisition. Finally, let $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ denote the ex-ante gross profit of a trader acquiring information of quality y_i when all other traders acquire information of quality \bar{y} , and all traders, including i , submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ mentioned above). The payoff is again gross of the cost of information acquisition. We start by establishing the following result:

Lemma 2 *Let y^T denote the socially optimal quality of private information and suppose that all traders submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). When $\hat{c}^T > 0$ (i.e., when the pecuniary externality dominates over the learning externality so that the efficient demand schedules are downward sloping), for any \bar{y} ,*

$$\left. \frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; y_i, \bar{y}] \right|_{y_i = \bar{y}} > \frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}]$$

whereas the opposite inequality holds when $\hat{c}^T < 0$ (i.e., when the learning externality dominates over the pecuniary externality and, as a result, the efficient demand schedules are upward sloping).

Proof of Lemma 6. First observe that, for any (\bar{y}, y_i) ,

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E} \left[(\theta - p(\theta, u, \eta; \bar{y})) X_i(\theta, u, \eta, e_i; \bar{y}, y_i) - \frac{\lambda}{2} X_i^2(\theta, u, \eta, e_i; \bar{y}, y_i) \right],$$

where

$$p(\theta, u, \eta; \bar{y}) = \alpha + \beta b^T + \beta(a^T + c^T)z(\theta, u, \eta; \bar{y})$$

and

$$X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T z(\theta, u, \eta; \bar{y}),$$

where b^T and c^T are the coefficients obtained from $(a^T, \hat{b}^T, \hat{c}^T)$ using the functions (16) and (17), and where $z(\theta, u, \eta; \bar{y}) \equiv \theta + f(\bar{y})\eta - u/\beta a^T$, with $f(y) = 1/\sqrt{y}$.¹⁸ One can then show that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i] \Big|_{y_i=\bar{y}} = -f(\bar{y})f'(\bar{y})a^T \left[\lambda \frac{a^T}{\tau_e} + (\beta + \lambda)(a^T + c^T) \frac{1}{\tau_\eta} \right]. \quad (27)$$

Next observe that

$$\mathbb{E}[W^T; \bar{y}] = \mathbb{E}_{\theta, u, \eta} \left[(\theta - \alpha + u) \tilde{X}(\theta, u, \eta; \bar{y}) - \frac{\lambda}{2} \frac{(a^T f(\bar{y}))^2}{\tau_e} - \frac{\lambda + \beta}{2} \left(\tilde{X}(\theta, u, \eta; \bar{y}) \right)^2 \right]$$

from which we have that

$$\frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}] = -\frac{\lambda (a^T)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda + \beta) (a^T + c^T)^2 f'(\bar{y})f(\bar{y}) \frac{1}{\tau_n}. \quad (28)$$

Comparing (27) with (28), we thus have that, when $c^T < 0$,

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i] \Big|_{y_i=\bar{y}} > \frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}],$$

whereas the opposite inequality holds when $c^T > 0$. Finally, use Condition (17) to observe that $\hat{c}^T = -\frac{c^T}{\beta(a^T + c^T)}$ and Condition (21), along with the formula for $\tau_\omega(a)$, to observe that $a^T + c^T > 0$. Jointly, the last two conditions imply that $\text{sgn}(\hat{c}^T) = -\text{sgn}(c^T)$ which completes the proof of the lemma. ■

Next observe that $\mathbb{E}[\pi_i^T; \bar{y}, y_i]$ and $\mathbb{E}[W^T; \bar{y}]$ are globally concave in y_i and \bar{y} , respectively. Because $\mathbb{E}[\pi_i^T; \bar{y}, y_i]$ is strictly concave in y_i , in equilibrium, all traders acquire information of quality y^* such that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i] \Big|_{y_i=\bar{y}=y^*} = C'(y^*).$$

The socially-optimal quality of information satisfies

$$\frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}] \Big|_{\bar{y}=y^T} = C'(y^T).$$

Because $\mathbb{E}[W^T; \bar{y}]$ is strictly concave in \bar{y} , the result in Lemma 6 implies that, when $\hat{c}^T > 0$, $y^T < y^*$, whereas, when $\hat{c}^T < 0$, $y^T > y^*$. Q.E.D.

Proof of Proposition 6. Under the proposed policy, each trader i 's ex-ante gross expected payoff when all traders other than i collect information of quality \bar{y} , trader i collects information of quality

¹⁸Observe that the functions (16) and (17) do not depend on y and hence c^T and b^T do not depend on y .

y_i , and all traders (including i) submit the efficient demand schedules (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$) is equal to

$$\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] = \mathbb{E} \left[\theta x_i - (1 + \hat{t}_p) (\alpha - u + \beta \tilde{x}) x_i - \frac{\lambda}{2} x_i^2 \right]$$

with

$$x_i = X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T \left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T} \right)$$

and

$$\tilde{x} = X(\theta, u, \eta; \bar{y}) = a^T (\theta + f(\bar{y})\eta) + b^T + c^T \left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T} \right),$$

where b^T and c^T are obtained from \hat{b}^T and \hat{c}^T using (16) and (17).¹⁹ It follows that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] = \frac{\beta(1 + \hat{t}_p)(a^T + c^T)a^T}{2\tau_\eta y_i \sqrt{\bar{y} y_i}} + \frac{\lambda a^T}{2\tau_\eta y_i \sqrt{y_i}} \left(\frac{a^T}{\sqrt{y_i}} + \frac{c^T}{\sqrt{\bar{y}}} \right) + \frac{\lambda (a^T)^2}{2y_i^2 \tau_e}.$$

Because $\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] - \mathcal{C}(y_i)$ is concave in y_i , for $y_i = \bar{y} = y^T$ to be sustained in equilibrium, it is both necessary and sufficient that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] \Big|_{y_i = \bar{y} = y^T} = \mathcal{C}'(y^T)$$

which holds if and only if

$$\hat{t}_p = \frac{\gamma_2(\tau_\omega(a^T)) - \beta a^T}{\beta a^T}$$

where $\gamma_2(\tau_\omega)$ is the function defined in the proof of Proposition 1. Q.E.D.

Proof of Proposition 7. Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Given any policy $T(x_i, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$V(y^T, y_i) \equiv \sup_{g(\cdot)} \{ \mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] - \mathcal{C}(y_i) \}$$

where $g: \mathbb{R}^2 \rightarrow \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z , with

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \equiv \mathbb{E} \left[\theta g(s_i, z) - (\alpha - u + \beta \tilde{x}) g(s_i, z) - \frac{\lambda}{2} (g(s_i, z))^2 \right] - \mathbb{E} [T(g(s_i, z), \alpha - u + \beta \tilde{x})].$$

Note that in writing V , we used the fact that p and z are related by $p = \alpha + \beta b^T + \beta(a^T + c^T)z$.

For the policy $T(x_i, p)$ to implement the efficient acquisition and usage of information, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that $\mathbb{E}[\theta | s_i, z] = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z$, where γ_1 and γ_2 are the functions defined in the proof of Proposition 1, we thus have that, for the policy T to implement

¹⁹Note that we used the fact that $p = P(\theta, u, \eta; \bar{y}) = \alpha - u + \beta X(\theta, u, \eta; \bar{y})$.

the efficient trades, it must be that T is differentiable in x_i and satisfy

$$\begin{aligned} & \frac{\partial}{\partial x} T(a^T s_i + b^T + c^T z, \alpha + \beta b^T + \beta(a^T + c^T)z) \\ &= [\gamma_1(\tau_\omega(a^T)) - \lambda a^T] \frac{x - b^T}{a^T} \\ &+ \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - (\gamma_1(\tau_\omega(a^T)) - \lambda a^T) \frac{c^T}{a^T} \right] \frac{p - \alpha - \beta b^T}{\beta(a^T + c^T)} - (\alpha + \beta b^T + \lambda b^T) \end{aligned}$$

for all (s_i, z) . This means that $T(x_i, p)$ is a polynomial of second order of the form

$$T(x_i, p) = \frac{\delta}{2} x_i^2 + (t_p p - t_0) x_i + K(p), \quad (29)$$

for some vector (δ, t_p, t_0) and some function $K(p)$ which plays no role for incentives and which therefore we can disregard. In the proof of Proposition 3, we showed that there exists a unique vector (δ, t_p, t_0) that induces the traders to submit the efficient demand schedules when the precision of their private information is y^T (the vector in Proposition 3 applied to $y = y^T$). Thus, if a policy T induces efficiency in both information acquisition and information usage, it must be of the form in (29) with (δ, t_p, t_0) as in Proposition 3 applied to $y = y^T$. When the policy takes this form, for any y_i , the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = a s_i + b + c z$, for some (a, b, c) , where $z = \theta + f(y^T)\eta - u/\beta a^T$ is the endogenous signal contained in the price. This implies that

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] = M - \beta(1 + t_p)(a^T + c^T)a \frac{1}{\sqrt{y^T} \sqrt{y_i} \tau_\eta} - \frac{(\lambda + \delta)ca}{\sqrt{y^T} \sqrt{y_i} \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e},$$

where M is a function of all terms that do not interact with y_i . Using the envelope theorem, we then have that

$$\left. \frac{\partial}{\partial y_i} V(y^T, y_i) \right|_{y_i=y^T} = \frac{[\beta(1 + t_p) + \lambda + \delta](a^T + c^T)a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda + \delta)(a^T)^2}{2\tau_e (y^T)^2} - C'(y^T).$$

Because the efficient y^T solves

$$\frac{(\beta + \lambda)(a^T + c^T)^2}{2\tau_\eta (y^T)^2} + \frac{\lambda (a^T)^2}{2\tau_e (y^T)^2} = C'(y^T), \quad (30)$$

we have that, for the policy to implement the efficient acquisition of private information, it must be that

$$(a^T + c^T)\tau_e [(\beta + \lambda)c^T - (\beta t_p + \delta)a^T] = \delta (a^T)^2 \tau_\eta.$$

One can verify that the values of δ and t_p from Proposition 3 do not solve the above equation except for a non-generic set of parameters. Q.E.D.

Proof of Proposition 8. When all other traders acquire information of precision y^T and submit the efficient demand schedules for information of quality y^T , the maximal payoff that trader i can obtain by acquiring information of precision y_i is equal to

$$\hat{V}(y^T, y_i) \equiv \sup_{a, b, c} \{ \mathbb{E}[\tilde{\pi}_i(y^T, y_i); a, b, c] - \mathcal{C}(y_i) + A y_i \},$$

where

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); a, b, c] \equiv M - \beta(1 + t_p)(a + c)a \frac{1}{\sqrt{y^T} \sqrt{y_i} \tau_\eta} - \frac{(\lambda + \delta)ca}{\sqrt{y^T} \sqrt{y_i} \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e},$$

where M collects all variables that do not interact with y_i . Note that, in writing $\hat{V}(y^T, y_i)$, we use the fact that, for any y_i , the trader's payoff is maximized by submitting an affine demand schedule which induces trades $x_i = as_i + b + cz$ that are affine in (s_i, z) , where $z = \theta + f(y^T)\eta - u/\beta a^T$ is the endogenous signal contained in the price. Using the envelope theorem, we have that

$$\frac{\partial}{\partial y_i} \hat{V}(y^T, y_i) \Big|_{y_i=y^T} = \frac{[\beta(1 + t_p) + \lambda + \delta] (a^T + c^T) a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda + \delta) (a^T)^2}{2\tau_e (y^T)^2} - C'(y^T) + A,$$

where we use the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to $a^T s_i + b^T + c^T z$. Using the fact that y^T satisfies Condition (30) along with Condition (17) to express c^T as a function of \hat{c}^T , we thus have that the proposed policy induces the efficient acquisition of private information only if

$$A = -\frac{(a^T)^2}{2\tau_\eta (y^T)^2} \left[\frac{\beta(\beta + \lambda)\hat{c}^T}{(1 + \beta\hat{c}^T)^2} + \frac{\beta t_p + \delta}{1 + \beta\hat{c}^T} \right] - \frac{\delta (a^T)^2}{2\tau_e (y^T)^2}.$$

One can verify numerically that the function $\hat{V}(y^T, y_i)$ is globally quasi-concave in y_i . We thus conclude that the proposed policy implements the efficient acquisition and usage of information. Q.E.D.

Proof of Proposition 9. Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ for quality y^T). Given any policy $T(x_i, \tilde{x}, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$\tilde{V}(y^T, y_i) \equiv \sup_{g(\cdot)} \{ \mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] - C(y_i) \}$$

where $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z , with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$, and where

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \equiv \mathbb{E} \left[\theta g(s_i, z) - (\alpha - u + \beta \tilde{x}) g(s_i, z) - \frac{\lambda}{2} (g(s_i, z))^2 \right] - \mathbb{E} [T(g(s_i, z), \tilde{x}, \alpha - u + \beta \tilde{x})].$$

For the policy $T(x_i, \tilde{x}, p)$ to induce efficiency in both information acquisition and usage, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that

$$\mathbb{E} [\theta | s_i, z; y_i, y^T] \Big|_{y_i=y^T} = \gamma_1(\tau_\omega(a^T)) s_i + \gamma_2(\tau_\omega(a^T)) z,$$

with the functions γ_1 and γ_2 as in Proposition 1, we thus have that T must be differentiable in x_i and satisfy

$$\begin{aligned} & \frac{\partial}{\partial x_i} \mathbb{E} [T(a^T s_i + b^T + c^T z, \tilde{x}, \alpha - u + \beta \tilde{x}) | s_i, z; y_i, y^T] \Big|_{y_i=y^T} \\ &= \gamma_1(\tau_\omega(a^T)) s_i + \gamma_2(\tau_\omega(a^T)) z - [\alpha + \beta b^T + \beta(a^T + c^T) z] - \lambda (a^T s_i + b^T + c^T z) \end{aligned}$$

for all (s_i, z) , where $\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$, with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$. Next recall from the proof of Proposition 7 that, when the individual trades efficiently,

$$\begin{aligned} & \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - [\alpha + \beta b^T + \beta(a^T + c^T)z] - \lambda(a^T s_i + b^T + c^T z) \\ &= [\gamma_1(\tau_\omega(a^T)) - \lambda a^T] \frac{x - b^T}{a^T} + \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - (\gamma_1(\tau_\omega(a^T)) - \lambda a^T) \frac{c^T}{a^T} \right] \frac{p - \alpha - \beta b^T}{\beta(a^T + c^T)} \\ & \quad - (\alpha + \beta b^T + \lambda b^T). \end{aligned}$$

This means that $T(x_i, \tilde{x}, p)$ must be a polynomial of second order of the form

$$T(x_i, \tilde{x}, p) = \frac{\delta'}{2} x_i^2 + (pt'_p - t'_0 + t_{\tilde{x}} \tilde{x}) x_i + K'(\tilde{x}, p), \quad (31)$$

for some vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$, where $K'(\tilde{x}, p)$ is a function that does not depend on x_i , plays no role for incentives, and hence can be disregarded. Furthermore, under any such a policy,

$$\begin{aligned} & \frac{\partial}{\partial x_i} \mathbb{E} [T(x_i, \tilde{x}, p) | s_i, p; y_i, y^T] \\ &= \delta' x_i + pt'_p - t'_0 + \frac{t_{\tilde{x}}}{\beta} (p - \alpha) + \frac{t_{\tilde{x}}}{\beta} A^\#(y_i, y^T) s_i + \frac{t_{\tilde{x}}}{\beta} B^\#(y_i, y^T) p + \frac{t_{\tilde{x}}}{\beta} C^\#(y_i, y^T), \end{aligned}$$

where $A^\#(y_i, y^T)$, $B^\#(y_i, y^T)$, and $C^\#(y_i, y^T)$ are the coefficients of the projection

$$\mathbb{E} [u | s_i, p; y_i, y^T] = A^\#(y_i, y^T) s_i + B^\#(y_i, y^T) p + C^\#(y_i, y^T)$$

of u on (s_i, p) . When trader i acquires information of quality $y_i = y^T$ and trades efficiently,

$$\frac{\partial}{\partial x_i} \mathbb{E} [T(x_i, \tilde{x}, p) | s_i, p; y^T, y^T] = \delta x_i + t_p p - t_0$$

where

$$\delta = \delta' + \frac{t_{\tilde{x}}}{\beta} \hat{A}^\#, \quad (32)$$

$$t_p = t'_p + t_{\tilde{x}} \frac{1 + \hat{B}^\#}{\beta}, \quad (33)$$

and

$$t_0 = t'_0 + t_{\tilde{x}} \frac{\alpha}{\beta} - \frac{t_{\tilde{x}}}{\beta} \hat{C}^\#, \quad (34)$$

with $\hat{A}^\# \equiv A^\#(y^T, y^T)/a^T$,

$$\hat{B}^\# \equiv \left[B^\#(y^T, y^T) - \frac{A^\#(y^T, y^T) c^T}{a^T \beta (a^T + c^T)} \right],$$

and

$$\hat{C}^\# \equiv C^\#(y^T, y^T) - \frac{A^\#(y^T, y^T) b^T}{a^T} + \frac{A^\#(y^T, y^T) c^T (\alpha + \beta b^T)}{a^T \beta (a^T + c^T)}.$$

In the proof of Proposition 3, we showed that, when agents acquire information of quality y^T , for them to trade efficiently, the values of (δ, t_p, t_0) must coincide with those in Proposition 3 (applied to $y = y^T$). Thus, for the above policy to induce efficiency in both information acquisition and information usage, it must be that the vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$ satisfies Conditions (32)-(34) with (δ, t_p, t_0) given by the values determined in Proposition 3 applied to $y = y^T$. Note that, for any $t_{\tilde{x}}$, there exists unique values of (δ', t'_p, t'_0) that solve the above three conditions. Abusing notation, denote these values by $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$.

Next, note that, when the policy takes the form in (31), for any y_i , the optimal choice of $g(\cdot)$ is

affine and hence can be written as $g(s_i, z) = as_i + b + cz$, for some (a, b, c) . This implies that

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] = M - [t_{\bar{x}} + \beta(1 + t'_p(t_{\bar{x}}))] \frac{a(a^T + c^T)}{\sqrt{y^T} \sqrt{y_i \tau_\eta}} - \frac{(\lambda + \delta)ca}{\sqrt{y^T} \sqrt{y_i \tau_\eta}} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e},$$

where M is a function of all variables that do not interact with y_i . Using the envelope theorem, we then have that

$$\frac{\partial}{\partial y_i} \tilde{V}(y^T, y_i) \Big|_{y_i = y^T} = \frac{[t_{\bar{x}} + \beta(1 + t'_p(t_{\bar{x}})) + \lambda + \delta](a^T + c^T)a^T}{2\tau_\eta(y^T)^2} + \frac{(\lambda + \delta)(a^T)^2}{2\tau_e(y^T)^2} - \mathcal{C}'(y^T).$$

Once again, in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades $a^T s_i + b^T + c^T z$. Finally, recall that y^T is defined by Condition (30). Hence, for the above policy to induce efficiency in information acquisition, it must be that

$$\frac{(\beta + \lambda)(a^T + c^T)^2}{\tau_\eta} + \frac{\lambda(a^T)^2}{\tau_e} = \frac{[t_{\bar{x}} + \beta(1 + t'_p(t_{\bar{x}})) + \lambda + \delta](a^T + c^T)a^T}{\tau_\eta} + \frac{(\lambda + \delta)(a^T)^2}{\tau_e}. \quad (35)$$

Using (33), we have that $t'_p(t_{\bar{x}}) = t_p - t_{\bar{x}} \left(1 + \hat{B}^\#\right) / \beta$, with t_p given by the unique value determined in Proposition 3 applied to $y = y^T$. Because the function $H : \mathbb{R} \rightarrow \mathbb{R}$ given by $H(t_{\bar{x}}) \equiv t_{\bar{x}} + \beta t'_p(t_{\bar{x}}) = \beta t_p - t_{\bar{x}} \hat{B}^\#$ is linear, there exists a (unique) value of $t_{\bar{x}}$ that solves (35). We conclude that the policy in (31) with $t_{\bar{x}}$ given by the unique solution to (35) and with (δ', t'_p, t'_0) given by the unique solution $(\delta'(t_{\bar{x}}), t'_p(t_{\bar{x}}), t'_0(t_{\bar{x}}))$ to Conditions (32)-(34) induces efficiency in both information acquisition and information usage. Q.E.D.

Proof of Proposition 10. We establish the result by showing that the precision of private information y^* acquired in equilibrium is invariant in t_p . Once this property is established, the proposition follows from what established in the proof of Proposition 4.

Fix t_p , and denote by y and (a, b, c) the precision of private information acquired in equilibrium and the parameters defining the equilibrium trades in the economy with ad-valorem tax equal to t_p . For any y_i , let $V^\#(y_i) \equiv \sup_{g(\cdot)} \left\{ \mathbb{E}[\pi_i^\#(y_i; g(\cdot))] - \mathcal{C}(y_i) \right\}$ denote the maximal payoff that trader i can obtain by selecting a precision of private information equal to y_i when all other agents acquire information of precision y and then submit the limit orders corresponding to the parameters (a, b, c) , where $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ the trader purchases as a function of s_i and the endogenous public signal z contained in the equilibrium price, with²⁰

$$\mathbb{E}[\pi_i^\#(y_i; g(\cdot))] \equiv \mathbb{E} \left[\theta g(s_i, z) - (1 + t_p) (\alpha + \beta b + \beta(a + c)z) g(s_i, z) - \frac{\lambda}{2} (g(s_i, z))^2 | y_i \right]$$

denoting the trader's expected payoff, gross of the information cost, when following the rule $g(\cdot)$ after acquiring information of precision y_i . Note that in writing $\mathbb{E}[\pi_i^\#(y_i; g(\cdot))]$ we used the fact that the equilibrium price is given by $p = \alpha + \beta b + \beta(a + c)z$ with $z = \theta + f(y)\eta - u/(\beta a)$. By the definition of equilibrium, if agent i acquires information of quality $y_i = y$, the limit orders that maximize his payoff must be the equilibrium ones (that is, the one corresponding to the coefficients (a, b, c)). The

²⁰As above, given (a, b, c) , the sensitivity of the equilibrium limit orders \hat{c} to the price and the constant \hat{b} in the equilibrium limit orders are obtained from (a, b, c) using (16) and (17).

envelope theorem then implies that

$$\frac{dV^\#(y)}{dy_i} = \frac{\beta(1+t_p)(a+c)a}{2\tau_\eta y^2} + \frac{\lambda a(a+c)}{2\tau_\eta y^2} + \frac{\lambda(a)^2}{2y^2\tau_e} - C'(y). \quad (36)$$

Hence, the equilibrium value of y must satisfy $dV^\#(y)/dy_i = 0$. Let $M^\#(t_p, a, c, y)$ denote the function defined by the right-hand-side of (36). Next, use the derivations in the proof of Proposition 4 to observe that, given (t_p, y) , the equilibrium values of (a, b, c) are given by (10), (25), and (26). From the implicit function theorem we then have that

$$\frac{dy}{dt_p} = - \frac{\frac{\partial M^\#(t_p, a, c, y)}{\partial t_p} + \frac{\partial M^\#(t_p, a, c, y)}{\partial c} \frac{\partial c}{\partial t_p}}{\frac{\partial M^\#(t_p, a, c, y)}{\partial y} + \frac{\partial M^\#(t_p, a, c, y)}{\partial a} \frac{\partial a}{\partial y} + \frac{\partial M^\#(t_p, a, c, y)}{\partial c} \frac{\partial c}{\partial y}},$$

where we used the fact that, given y , the equilibrium level of a is invariant in t_p . Note that $\partial c/\partial t_p$ is the derivative of the equilibrium level of c with respect to t_p , holding y constant, whereas $\partial a/\partial y$ and $\partial c/\partial y$ are the derivatives of the equilibrium levels of a and c with respect to y , holding t_p fixed. Because $\frac{\partial}{\partial t_p} M^\#(t_p, a, c, y) = (\beta(a+c)a)/2\tau_\eta y^2$, $\frac{\partial}{\partial c} M^\#(t_p, a, c, y) = [\beta(1+t_p) + \lambda] a/2\tau_\eta y^2$, and $\partial c/\partial t_p = -\beta(a+c)/[\beta(1+t_p) + \lambda]$, we conclude that $dy/dt_p = 0$, as claimed. Q.E.D.