Market opacity and fragility: Why liquidity evaporates when it is most needed*

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Abstract

We show that, consistent with empirical evidence, access to order flow information allows traders to supply liquidity via contrarian marketable orders. Lack of market transparency can make liquidity demand upward sloping, inducing strategic complementarity and multiple equilibria. Then an initial dearth of liquidity may degenerate into a liquidity rout (as in a flash crash) and traders faced with the largest cost of trading are those consuming more liquidity at equilibrium. An increase in order flow transparency and/or in the mass of dealers who are in the market at all times has a positive impact on total welfare.

Keywords: Liquidity fragility, flash crash, strategic complementarity, order flow transparency.

JEL Classification Numbers: G10, G12, G14

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Introduction

Concern for the stability and resilience of financial markets has recently revived, in the wake of the sizeable number of "flash events" and other disruptions that have occurred in recent years.[1] Disrupted markets impair policy makers' ability to implement stabilizing macroeconomic policies, which compromises their capacity to pursue their mandate.[2] The debate over the ultimate cause at the root of these episodes is still open. However, some consensus seems to have gathered around the hypothesis that they are related to the presumably overall liquidity and welfare enhancing process of "electronification" that has affected different types of securities' markets over the past two decades. Indeed there's a suspicion that this has occurred at the cost of increased fragility: small changes in market parameters may have large effects on liquidity.[3] At the same time, episodes of extreme market turbulence, where liquidity seems to inexplicably disappear and markets become somewhat inelastic have also occurred in the past. As the experience of the stock market crash of October 19, 1987 makes clear, (apparently) fundamentals-unrelated crashes have been a worrying, regular feature of financial markets.[4]

A unifying characteristic of these episodes seems to be the jamming of the "rationing" function of market illiquidity. In "normal" market conditions, traders perceive a lack of liquidity as a cost, while arbitrageurs and liquidity suppliers regard it as an opportunity. Thus, an illiquidity hike leads the former to limit their demand for immediacy, and the latter to increase their supply of liquidity (i.e., the demand for and supply of liquidity, are respectively decreasing and increasing in the illiquidity of the market). In normal conditions, then, an illiquidity hike leads the net demand for a security to abate, producing a stabilizing effect on the market. However, on occasions, a bout of illiquidity, which can hardly be construed as fundamentals-driven, has a destabilizing impact, and fosters a disorderly "run for the exit" that is conducive to a rout. In these cases, traders attempt to place orders *despite* the liquidity shortage, and arbitrageurs flee the market, foregoing profitable (but risky) opportunities. In such conditions, liquidity is fragile. What can account for such a dualistic feature of market illiquidity?

In this paper, we argue that lack of transparency about relevant market conditions is an

[1] A "flash event" is a situation in which market liquidity suddenly evaporates in conjunction with a rapid increase in liquidity demand and the occurrence of extreme price changes, in the absence of fundamentals news, over a short time interval. Flash events have hit different markets. Starting with the May 6, 2010 U.S. "flash-crash" (equity, centralized) where the Dow Jones Industrial Average dropped by 9% in the middle of the trading day, and partially recovered by the end of trading; moving to the October 15, 2014 Treasury Bond crash (bonds, mainly OTC), where the yield on the benchmark 10-year U.S. government bond, dipped 33 basis points to 1.86% and reversed to 2.13% by the end of the trading day; to end with the August 25, 2015 ETF market freeze (ETF and equity, centralized), during which more than a fifth of all U.S.-listed exchange traded funds and products were forced to stop trading. More evidence of flash events is provided by NANEX and Bank of International Settlements (2017).

[2] The fragility in the US Treasury market has attracted attention recently, increasing the odds of a financial accident. See, e.g. "Fed Frets About Shadow Banks and Eyes Treasury Liquidity in New Report," New York Times, November 4, 2022

[3] See Chapter 4 in Duffie et al. (2022).

[4] See https://en.wikipedia.org/wiki/List_of_stock_market_crashes_and_bear_markets and also Ian Domowitz's "Will the real market failure please stand up?" for an account of a 1962 flash-crash forerunner.

important ingredient in the answer to this question. In current markets, trading automation arguably creates informational frictions by hampering some traders' access to reliable and timely market information (Ding et al. (2014)), thus impairing their ability to potentially enhance the risk-bearing capacity of the market. Furthermore, participation of some liquidity suppliers is variable (for technical or regulatory reasons).[5] The upshot is that accessibility to market information is vital to trade. In less automated markets, impaired access to market information arose because of different reasons. For example, in the 80s, access to the NYSE trading floor was crucial to have a good view of market conditions, but obviously constrained by physical limitations. Importantly, such frictions seem to have a bearing on episodes of liquidity crashes. Several accounts of the August 24, 2015 "flash-crash," point to the fact that uncertainty over the price of ETF constituents contributed to a huge investors' sellout, and sidelined the actions of arbitrageurs, exacerbating the liquidity dry-up in some ETFs.[6]

We use a stylized model of liquidity provision to show that, access to order flow information allows traders to supply liquidity via marketable orders, thereby improving the risk-bearing capacity of the market.[7] This is consistent with empirical evidence.[8] Conversely, the absence of reliable order flow information limits the participation of non-standard liquidity providers, which can seriously dent the ability of a market to absorb risk, to the extent that, in extreme conditions, it can cause a market crash. We also find that both an increase in market transparency and/or in the participation of liquidity providers who are continuously in the market, has a positive effect on total welfare. However, the latter could have a negative impact on market stability when market transparency is low.

More in detail, we analyse a two-period (trading rounds) model of a market in which a

[5] Ding et al. (2014) argue that in the U.S. "[n]ot all market participants have equal access to trade and quote information. Both physical proximity to the exchange and the technology of the trading system contribute to the latency." In the EU the situation is possibly even worse, as testified by the lack of a consolidated tape in a market environment displaying an even higher degree of market fragmentation than in the US (see, e.g. European Commission progress update on action 14 of the capital markets union 2020 action plan. Action 14: Consolidated tape., see also EU faces a last-ditch challenge from exchanges over trading reforms, Financial Times, 18 April, 2023.).

[6] In the morning of August 24, 2015, the Dow dropped roughly 1,100 points in the first five minutes of trading, and trading in several stocks was halted due to unusual market turbulence. The ensuing lack of reliable price information allowed profitable, but risky, arbitrage opportunities to go unexploited, leading to a widening of spreads and a thinning of market depth. For example, during the event, the spread between the SPDR S&P500 (SPY) and the Guggenheim S&P 500 Equal Weight ETF (RSP), two very similar ETFs whose prices are normally in sync, at one point reached \$21 (see What The E-T-F Happened On August 24? Forbes, 28 August, 2015). In a similar vein, in their account of the May 10, 2010 "Flash Crash" Easley et al. (2011) state: "This generalized severe mismatch in liquidity was exacerbated by the withdrawal of liquidity by some electronic market makers and by uncertainty about, or delays in, market data affecting the actions of market participants." Amihud et al. (1990), in their analysis of the 1987 "Black Monday," argue that a number of operational issues affected the opening trade session on the day of the event "[O]rders could not be executed, and information on market conditions, and on order execution was delayed." This impaired the ability of traders outside of the market to provide liquidity, restricting total liquidity supply.

[7] A marketable (limit) order is a priced order with the limit price set at, or better than, the opposite side quote (bid price for sell orders and ask price for buy orders).

[8] Several authors find that liquidity is provided by (contrarian) marketable orders both at high trading frequencies (Brogaard et al. (2014) and Biais et al. (2017)) and at lower frequencies (Biais et al. (2017), Anand et al. (2021), Anand et al. (2013)).

risky security is traded by dealers and traders who hedge an endowment shock. Liquidity demand comes from two cohorts of risk-averse liquidity traders who submit market orders. The first cohort observes its endowment shock exposure to a non-tradable good (whose value is perfectly correlated with that of the risky security) prior to the first period and trades at both rounds. The second cohort enters the market at the second period, observes its endowment shock exposure, which is independent from the first period traders' one, possibly a signal about the first period order imbalance (which reflects that period endowment shock), and trades. Liquidity is supplied by a continuum of risk-averse dealers who post limit orders at both rounds and are thus able to efficiently rebalance their risk exposure.[9]

Being in the market at both rounds, first period traders split their hedging (liquidity demand) across periods: when they receive a positive (negative) endowment shock, they sell (buy) the risky security in both periods. With full transparency, second period traders perfectly observe the first period imbalance (endowment shock), and take a contrarian position against first period liquidity traders' second period order–in this way de-facto providing liquidity to them. In this case, we show that traders' demand for liquidity is a decreasing function of the cost of trading they face–that is, *higher illiquidity discourages liquidity demand* and illiquidity works as a *rationing* device. Additionally, a unique equilibrium obtains. Along this equilibrium, we show that at the first round, dealers supply liquidity and also speculate on the anticipated impact of first period traders' order at the second round. Indeed, due to their ability to be in the market in both periods, dealers also demand liquidity by trading in the same direction as first period liquidity traders, thus exploiting these traders' demand predictability. At the second round, dealers absorb the orders of both cohorts of liquidity traders. First period liquidity traders' split liquidity demand is also responsible for the positive return autocovariance that obtains at equilibrium. That is, in our model, returns are positively autocovariant in the *absence* of any fundamentals information.

A deterioration of second period traders' information (about the first period order imbalance) impairs these traders' ability to supply liquidity via contrarian orders. This reduces the risk-bearing capacity of the market and can increase market fragility. Specifically, we find that, for some plausible parameterizations, the model displays multiple equilibria with different levels of market depth. In this case, a larger cost of trading leads traders to demand more liquidity and *higher illiquidity incentivizes liquidity demand*. There is strategic complementarity in liquidity demands and the cost of trading. A drop in liquidity may increase the demand for liquidity, thus generating a further drop in liquidity. Specifically, when the market is opaque, an increase in the price impact of cohort 2 liquidity traders' orders hikes the execution risk faced by the traders belonging to cohort 1. This lowers (increases) the liquidity demand and consumption of the latter (former).[10] Thus, the initial second period illiquidity spike leads

[9] In a related paper (Cespa and Vives (2019)), which the present one supersedes, we study the case in which first period liquidity traders have a short-term trading horizon, obtaining qualitatively similar results.

[10] This is because a higher execution risk faced by first period liquidity traders limits these traders' liquidity demand, allowing dealers to offer "more" liquidity to second period traders who, because of such a liquidity

second period traders to demand more (rather than less) liquidity, increasing dealers' exposure to their endowment shock and thereby further boosting the price impact of their order.

We show that when dealers' risk bearing capacity is small, liquidity traders have an urge to trade (because the dispersion of their endowment shock is large and they have a low risk tolerance) and the security's payoff volatility is large, if the market is fully opaque (second period traders have no information on the first period imbalance), the above described loop generates three equilibria, which can be ranked in terms of market liquidity. Indeed, in these conditions dealers cannot count on the additional risk sharing provided by liquidity traders' contrarian orders. When traders' demand for liquidity spikes, this widens the gap between liquidity demand and supply, making the market fragile. We also show that only the extreme equilibria are stable and that trading costs for traders at the second round are heterogeneous.[11] At the two stable solutions of the model, first and second period cohorts' price impact (of endowment shocks) *and* their liquidity consumption are negatively correlated. Thus, a spike in liquidity consumption by second (first) period traders crowds out first (second) period traders' liquidity consumption.

Importantly, in this situation, illiquidity stops working as a rationing device of liquidity consumption. That is, at equilibrium the trader cohort facing the highest price for liquidity is also the one consuming more of it (hedging a larger proportion of the endowment shock). We show that, as long as the market is fully opaque, an increase in the risk-tolerance of liquidity traders or a reduction in the dispersion of their endowment shock, weakens strategic complementarity, leading to a unique equilibrium. However, in such equilibrium liquidity demand is still positively related to illiquidity. Thus, even when strategic complementarity is not strong enough to generate multiple equilibria, order flow opaqueness jams the rationing role of illiquidity.

In the last part of the paper, we consider the effect of three extensions to the baseline model. We first allow second period traders to observe a noisy signal about the first period endowment shock. In this context we show, by way of numerical simulations, that a low precision of such signal delivers equilibrium multiplicity, generalizing the results we obtain in the "fully opaque" case. We also show that an increase in the precision of such signal leads to a unique equilibrium in which second period traders' cost of trading is higher than the one of their first period peers. Thus, an increase in order flow transparency makes the market less fragile, but increases trading costs for some market participants.

In the second extension, we consider the case in which liquidity is also supplied by a mass of dealers who are in the market only at the first round, which we refer to as "restricted" dealers. The analysis of this case allows us to show that an increase in the mass of the dealers who are always in the market may, for low levels of order flow transparency, have a non-linear effect on market stability, moving the market from a unique equilibrium to a regime with multiplicity.

demand decline, are instead faced with a lower execution risk. This feedback loop, which works through the link between illiquidity and dealers' risk exposure, is reminiscent of the purported risk faced by dealers in US Treasury markets due to a potential increase in yield volatility (see Duffie (2020)).

[11]Stability is with respect to the best response stability criterion.

Finally, the third extension tackles welfare analysis. We compute the welfare functions of market participants and use them to numerically measure total welfare. Our results show that, when the equilibrium is unique, an increase in market transparency and in the mass of dealers who are always in the market are both welfare enhancing. Interestingly, an increase in the transparency of order flows seems to be in line with a recent proposal by the US Treasury department to make public the transactions for "on-the run" bonds from 2023 to improve market resilience.[12]

Related literature Our paper is related to–and has implications for–four streams of the finance literature. First, it is related to the literature on liquidity fragility (see, e.g., Brunnermeier and Pedersen (2009)). Most of the contributions in this framework focus on the possibility that liquidity may evaporate because of self-sustaining loops that limit the ability of *dealers* to meet customers' demand, be it because of funding problems (Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002)), lack of price information (Cespa and Foucault (2014)), or the effect of retrospective learning about the security's payoff (Cespa and Vives (2015)). In light of such effects, scholars have argued that regulation impairing access to capital for financial institutions may have a negative impact on the risk sharing capacity of the liquidity provision sector, precisely when this is needed the most (see, e.g. Bao et al. (2018)). However, accounts of market crashes often attribute the inception of these events to "aggressive" or "unusually large" liquidity demand realizations which are not met by a sufficiently responsive increase in liquidity supply.[13] In this paper we thus propose a theory in which liquidity fragility arises because of a self-sustaining loop affecting *liquidity demanders*, which exhausts liquidity suppliers' limited risk-bearing capacity. Indeed, in our model poor market information impairs second period traders' ability to speculate against the aggregate order imbalance, creating the loop which impairs risk sharing.[14] In view of the documented decline in quoted depth that has occurred over the past twenty years, this should reinforce regulatory concerns over the paucity of *public, affordable* order flow information in current markets.

Second, the paper is also related to the literature documenting liquidity provision via (contrarian) market orders. Several authors find this phenomenon at high trading frequencies (Brogaard et al. (2014) and Biais et al. (2017)). There is, however, evidence that it also occurs at lower frequencies (Biais et al. (2017)). Anand et al. (2021) provide evidence that far from contributing to market fragility, some corporate bond mutual funds actively supply liquidity during periods of market stress. A similar behavior is also found in equity mutual funds during the recent financial crisis (see Anand et al. (2013)). In this respect, our paper argues that informational impediments to liquidity provision via market orders can negatively affect risk

[12] See Financial Times, 16, November, 2022.

[13] For example, the CFTC-SEC report on the flash-crash attributes the inception of the crash to an aggressive E-mini S&P500 futures sell order initiated by a large mutual fund identified as Waddell & Reed (see CFTC and SEC (2010)), which appears to have persisted during the crash (see Aldrich et al. (2017)). See also Aquilina et al. (2018) for evidence of market participants' behavior during flash events in the UK.

[14] For evidence of demand driven "commonality" in liquidity, see e.g. Karolyi et al. (2012).

sharing and make liquidity fragile.[15]

Third, the paper is related to the early literature on price crashes. Gennotte and Leland (1990) provide a model tracing the 1987 stock market crash to traders not taking into account the possibility of portfolio insurance affecting the security demand. Jacklin et al. (1992) also analyse the crash-inducing effect of mis-estimating the actual magnitude of portfolio insurance in a dynamic model à la Glosten and Milgrom (1985). Madrigal and Scheinkman (1997) study a model in which traders have private fundamental information and together with noise traders post orders who are accommodated by market makers who act strategically to control the information flow implied by the security price. The authors show that, under some conditions, the need to control the information flow conveyed by prices leads to crashes. All of the above papers rely on some form of irrationality either due to the presence of noise trading, or to the fact that some rational traders are unaware of one component of the aggregate demand for the stock, to generate price discontinuities. In our model, as explained above, all traders are rational expected utility maximizers, and the crash occurs because of the self-sustaining loop triggered by traders' liquidity demand.

Finally, the paper is related to the literature highlighting the impact of multi-dimensional fundamentals for price discovery and the equilibrium properties of the market (see, e.g., Subrahmanyam and Titman (1999), Cespa and Foucault (2014), Goldstein and Yang (2015), and Goldstein et al. (2021)). Differently from this literature, in this paper we assume that prices are driven by multiple, independent, non-fundamentals-driven shocks (i.e., the hedging demands of different liquidity traders' cohorts) and show that, when liquidity demand reacts to prices, this can have important consequences for market stability.

The rest of the paper is organized as follows. In the next section we present the model. In Section 2 we study the fully transparent benchmark, in which we assume that second period traders perfectly observe the endowment shock affecting their first period peers. In Section 3 we assume that such information is not available (the fully opaque case) and show that this can generate multiple equilibria. In Section 4 we look at the relationship between liquidity trading by hedgers and noise trading. In Section 5, we analyze the model's extensions. The final section contains concluding remarks. Most of the proofs are relegated to the Appendix.

1 The model

A single risky asset with liquidation value $v \sim N(0, \tau_v^{-1})$, and a risk-less asset with unit return are exchanged in a market during two periods (we interchangeably also use the expression "trading rounds").[16] Two classes of traders are in the market. First, a continuum of compet-

[15] Li et al. (2021) modify Budish et al. (2015) to study competition for liquidity provision between HFTs and "execution algorithms," some of which can choose whether to trade via market or limit orders. They show that under continuous pricing, at equilibrium HFTs provide liquidity via market orders to execution algorithms who post aggressive limit orders.

[16] A model of the same family was used in Cespa and Vives (2022) to study competition among exchanges.

itive, risk-averse dealers of unit mass, active in both periods. Second, a unit mass of liquidity traders who enter the market at the first round and post their orders at round 1 and 2. In the second period, a new cohort of liquidity traders (of unit mass) who enter the market and trade. The asset is liquidated in period 3. We now illustrate the preferences and orders of the different players as well as the market clearing conditions.[17]

1.1 Dealers

A dealer has CARA preferences with risk-tolerance γ, and submits price-contingent orders x_t^D, $t = 1, 2$, to maximize the expected utility of his final wealth: $W^D = (v - p_2)x_2^D + (p_2 - p_1)x_1^D$. At each trading round dealers condition their positions on the sequence of equilibrium prices up to that period. Thus, at the first round, they condition on p_1 and at the second round on $\{p_1, p_2\}$.[18]

1.2 Liquidity traders (hedgers)

The liquidity demand side of the model is represented by a unit mass of risk-averse traders who, prior to entering the market at time t, learn about the value of an endowment shock u_t in a non-tradable security that they will receive at the liquidation date ($t = 3$). We assume that the non-tradable security's value is perfectly correlated with that of the risky security traded in the market. This assumption, which is common in the literature (see, e.g. Wang (1994), Vayanos and Wang (2012), and Llorente et al. (2002)), induces a hedging demand for the risky security.

More in detail, in the first period, a unit mass of CARA traders with risk-tolerance γ_H is in the market. Traders learn the value of the endowment shock u_1 and post a market order x_{t1}, at round $t \in \{1, 2\}$ to maximize the expected utility of their wealth $\pi_1 = u_1 v + (v - p_2)x_{21} + (p_2 - p_1)x_{11}$:

$$E\left[-\exp\{-\pi_1/\gamma_H\}|\Omega_1\right],$$

where $\Omega_1 \equiv \{u_1\}$ denotes their information set. In period 2, a new (unit) mass of CARA traders (with the same risk tolerance γ_H) enters the market, learns the realization of the non-tradable endowment shock u_2 that they will receive at $t = 3$, and observes a noisy signal of the previous period endowment shock $s_{u_1} = u_1 + \eta$. Second period traders submit a market order x_2 to maximize the expected utility of their wealth $\pi_2 = u_2 v + (v - p_2)x_2$:

$$E\left[-\exp\{-\pi_2/\gamma_H\}|\Omega_2\right],$$

where $\Omega_2 \equiv \{u_2, s_{u_1}\}$ denotes their information set. Note that they do not observe p_1 reflecting

[17]In Section 5, we show that our results are qualitatively robust to a generalization of the model which includes a class of "Restricted Dealers" who can only trade at the first round.

[18]We assume, without loss of generality with CARA preferences, that the non-random endowment of dealers is zero. Also, as equilibrium strategies will be symmetric, we drop the subindex i.

that they have only imperfect information on the order flow. We assume $u_t \sim N(0, \tau_u^{-1})$, $\eta \sim N(0, \tau_\eta^{-1})$ and $\text{Cov}[u_t, v] = \text{Cov}[u_t, \eta] = \text{Cov}[u_1, u_2] = 0$, $t = 1, 2$.

As an example of the "non-tradable" security, one can think of a portfolio of assets that traders are unwilling to liquidate (or that are intrinsically illiquid). In view of the assumed correlation structure, protection against changes in the non-tradable value is then obtained by taking an offsetting position in the risky security. For instance, traders could be long in a portfolio of stocks that tracks the market, say a fund, and hedge by shorting a market-tracking ETF; alternatively, they could be long on a S&P500 ETF, like the SPY, and setup a hedge by trading the Emini (while the former trades from 6am to 8pm, including extended trading hours, the latter trades 24/7, thus allowing overnight hedging).[19]

To simplify notation, in the following we denote by $E_t^D[Y]$, and $\text{Var}_t^D[Y]$, the conditional expectation and variance that a dealer forms about random variable Y, in period $t = 1, 2$. Note that since dealers submit limit orders, at a linear equilibrium they will infer the endowment shocks hitting hedgers' budget constraints. Similarly, $E_t[Y]$, $\text{Var}_t[Y]$, and $\text{Cov}_t[X, Y]$ denote the conditional expectation, variance, and conditional covariance that a period-t hedger forms about random variables Y and X.

1.3 Market clearing

We will restrict attention to equilibria in which prices are linear functions of the endowment shocks and the error term affecting second period traders' signal. With hindsight, these will have the following form:

$$p_1 = -\Lambda_1 u_1 \tag{1a}$$

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22} \eta, \tag{1b}$$

where Λ_1, Λ_2, Λ_{21}, Λ_{22} are coefficients which will be pinned down at equilibrium. At equilibrium, dealers absorb the orders of first period traders x_{11}:

$$x_1^D + x_{11} = 0. \tag{2}$$

Traders know u_1, while, at equilibrium, dealers infer it from the price, which justifies (1a).

Consider now the second period equilibrium condition. First period liquidity traders split their hedging needs by posting an order x_{21} together with second period traders. Additionally, dealers rebalance their position at the second round. Formally, from the second period market

[19] For an example involving SPY, see https://money.stackexchange.com/questions/54373/why-dont-spy-spx-and-the-e-mini-sp-500-track-perfectly-with-each-other, and http://tastytradenetwork.squarespace.com/tt/blog/equating-futures-to-etfs, and for other ETF related examples, see https://investorplace.com/2017/10/portfolio-hedge-fund-consider-etfs/.

clearing equation we have

$$(x_2^D - x_1^D) + (x_{21} - x_{11}) + x_2 = 0 \iff x_2^D + x_{21} + x_2 = 0, \tag{3}$$

where the expression on the right hand side in (3) follows from using the first period market clearing condition (2). At equilibrium, dealers' and traders' strategies are a function of their information sets–$\{p_1, p_2\}$ for dealers and Ω_2 for second period traders. As a consequence, the price will load on $\{u_1, u_2, \eta\}$, justifying (1b).

Note that since liquidity traders have the possibility to retrade at the second round, to hedge their endowment shock, both the first and second period price depend on u_1. This, in turn, suggests the following alternative way to write the second period equilibrium price:

$$p_2 = -\Lambda_2 \theta_2 - \Lambda_{22}\eta, \tag{4}$$

where $\theta_2 = u_2 + \beta u_1$ and $\beta = \Lambda_{21}/\Lambda_2$. The expression in (4) shows how our model can be made equivalent to models postulating noise trading as an AR(1) process, thus endogenizing the persistence coefficient β and relating it to the relative weight that endowment shocks receive in the second period price.[20]

Figure 1 displays the timeline of the model.

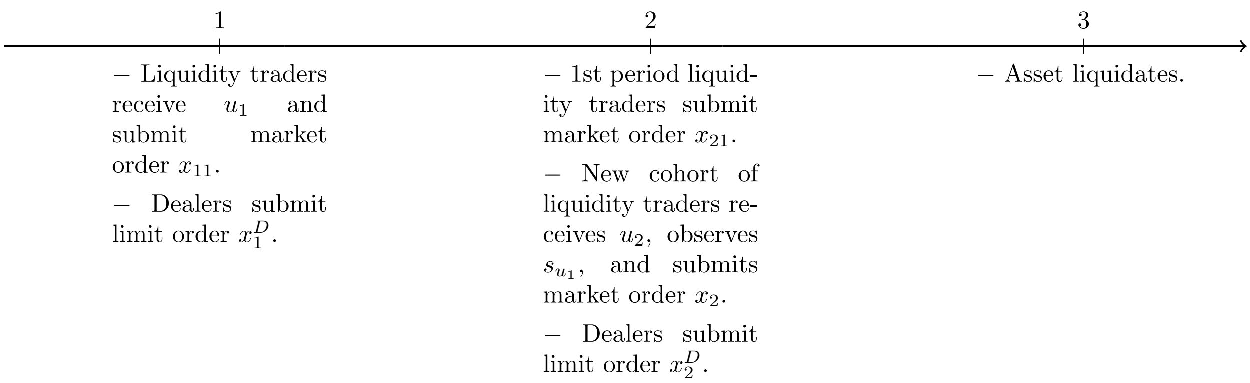


Figure 1: The timeline.

2 Fully transparent benchmark

We start the analysis by assuming that second period traders observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$). This assumption implies that the market is fully transparent and has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period traders is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second

[20]Several authors have assumed this form for noise trading. Among others: Campbell et al. (1993), He and Wang (1995), Cespa and Vives (2015), Cespa et al. (2021).

period price only reflects endowment shocks:

$$\begin{aligned} p_2 &= -\Lambda_2 u_2 - \Lambda_{21} u_1 \\ &= -\Lambda_2 \theta_2, \end{aligned} \tag{5}$$

while the first period price is as in (1a).

Due to the linearity assumption for prices, equilibrium strategies will also be linear. Specifically, we posit $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, $x_2 = a_2 u_2 + b u_1$, where the coefficients a_1, a_{21} and a_2 denote the hedging intensity of liquidity traders and the corresponding absolute values of such coefficients denote their hedging "aggressiveness." The coefficient b denotes second period traders' "speculative" aggressiveness (see below). In the Appendix, we show that in this case the equilibrium is identified by the unique solution to a system of simultaneous equations in $\Lambda_1, \Lambda_{21}, \Lambda_2$. We thus obtain the following:

Proposition 1. *When the market is fully transparent, there exists a unique equilibrium in linear strategies. The coefficients of equilibrium prices* $p_1 = -\Lambda_1 u_1$ *and* $p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1$, *are given by:*

$$\Lambda_2 = -\frac{1}{\gamma \tau_v} a_2 > 0 \tag{6a}$$

$$\Lambda_1 = -\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_H}{\gamma_H} a_1 > 0 \tag{6b}$$

$$\Lambda_{21} = -\frac{1}{\gamma \tau_v} \big(b + a_{21}\big) > 0. \tag{6c}$$

The coefficients of traders' strategies $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, $x_2 = a_2 u_2 + b u_1$ *are as follows:*

$$a_1 = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} \in (-1, 0),\ \ a_{21} = \frac{\gamma_H \tau_v \Lambda_{21} - 1}{\tau_v \mathrm{Var}_1[v - p_2]} \in (-1, 0), \tag{7a}$$

$$a_2 = \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \mathrm{Var}_2[v - p_2]} \in (-1, 0),\ \ b = \gamma_H \tau_v \Lambda_{21} > 0, \tag{7b}$$

where $\mathrm{Var}_1[p_2] = \Lambda_2^2 \tau_u^{-1}$, $\mathrm{Var}_1[v - p_2] = \Lambda_2^2 \tau_u^{-1} + \tau_v^{-1}$ *and* $\mathrm{Var}_2[v - p_2] = \tau_v^{-1}$. *Furthermore,* $-1 < a_{21} < a_1 < 0$, $0 < \Lambda_1 < \Lambda_{21} < \Lambda_2$ *(explicit expressions for the price coefficients are in* (A.34a), (A.36a) *and* (A.36b)*).*

According to (7a), first period liquidity traders demand liquidity by hedging part of their risk exposure at both trading rounds. Comparing their hedging intensities: $a_{21} - a_1 < 0$. Hence, if $u_1 > 0$, they hedge their exposure shorting at the first round, and increasing their short position at the second round (that is, their second period *trade* is a sell), when second period traders are in the market.

The liquidity that accommodates such demand is offered by dealers. In the Appendix (see (A.28)), we show that a dealer's strategy is given by:

$$\begin{aligned} X_1^D(p_1) &= \frac{\gamma}{\mathrm{Var}_1^D[p_2]} E_1^D[p_2] - \gamma \left(\frac{1}{\mathrm{Var}_1^D[p_2]} + \frac{1}{\mathrm{Var}[v]} \right) p_1 \\ &= -\gamma \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1^D[p_2]} u_1 - \gamma \tau_v p_1. \end{aligned} \tag{8}$$

According to the above expression, a dealer's strategy reflects two trading motives: liquidity supply (captured by the price dependent component in (8), $-\gamma\tau_v p_1$), and short-term return speculation (captured by the component $-u_1\gamma(\Lambda_{21} - \Lambda_1)/\mathrm{Var}_1[p_2]$). That is, due to their ability to infer traders' endowment shock and the fact that they know these traders repeatedly hedge such shock, dealers exploit the anticipated effect the shock has on expected returns. To see this, note that at the second round dealers in aggregate hold (see (A.10))

$$\begin{aligned} X_2^D(p_1, p_2) &= \gamma \frac{E_2^D[v - p_2]}{\mathrm{Var}_2^D[v - p_2]} = \\ &= -\gamma \tau_v p_2 \\ &= \gamma \tau_v \Lambda_{21} u_1 + \gamma \tau_v \Lambda_2 u_2, \end{aligned} \tag{9}$$

where the expression at the third line in (9) originates from substituting (1b) in dealers' second period aggregate position. Expression (9) implies that at the second round dealers hold $\gamma\tau_v\Lambda_{21}$ of the first period endowment shock. Based on (8), at the first round their position is given by

$$x_1^D = \gamma \left(\tau_u \frac{\Lambda_1 - \Lambda_{21}}{\Lambda_2^2} + \tau_v \Lambda_1 \right) u_1.$$

Hence, keeping the assumption $u_1 > 0$, at the first round dealers provide liquidity by absorbing part of first period traders' endowment shock ($\Lambda_1 > 0$). Additionally, they *consume* liquidity by taking a short position in the risky security ($\Lambda_1 - \Lambda_{21} < 0$).

At the second round, based on what said above, they provide liquidity to the additional sell order of first period traders: their trade with respect to the latter is given by

$$\gamma \tau_v \Lambda_{21} u_1 - x_1^D = \gamma \frac{\tau_u + \Lambda_2^2 \tau_v}{\Lambda_2^2} (\Lambda_{21} - \Lambda_1) u_1,$$

i.e., a buy order. Thus, because of their ability to anticipate returns, dealers gain from short term speculation at the first round (selling at a higher price at the first round and buying back at a lower price at the second round).[21] At the second round, their activity is instead limited to liquidity provision (see (9)).

[21]This is akin to "order anticipation" which, according to SEC (2010), occurs when "...a proprietary firm seeks to ascertain the existence of one or more large buyers (sellers) in the market and to buy (sell) ahead of the large orders with the goal of capturing a price movement in the direction of the large trading interest (a price rise for buyers and a price decline for sellers)."

At the second round, based on (7b), liquidity traders hedge their risk exposure ($a_2 \in (-1, 0)$). Additionally, because of their ability to perfectly infer the direction of the demand pressure due to first period traders' second round trade, they also post a contrarian market order ($b > 0$), which provides additional risk-sharing.[22]

Corollary 1. *When the market is transparent, second period liquidity traders supply liquidity by posting a contrarian market order with aggressiveness $b > 0$ (see (7b)).*

In our setup, trading occurs because liquidity traders are exposed to a non-tradable endowment shock which induces a hedging demand. Due to risk aversion, dealers have a limited capacity to bear risk. This implies the following

Corollary 2. *The price coefficients in (6a)–(6c) capture the risk-tolerance weighted risk compensation dealers require to absorb the aggregate liquidity demand.*

To see this note that at the first round a_1 reflects the marginal position of liquidity traders, that is their "liquidity demand":

$$a_1 = \frac{\partial x_{11}}{\partial u_1} = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]}. \tag{10}$$

As observed above, dealers also demand liquidity, since they speculate on the price impact of u_1 and their aggregate liquidity demand is given by

$$-\gamma \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]} = \gamma \frac{a_1}{\gamma_H}.$$

Aggregating across liquidity traders' and dealers' demands yields the aggregate liquidity demand at the first round:

$$a_1 + \gamma \frac{a_1}{\gamma_H} = \frac{\gamma + \gamma_H}{\gamma_H} a_1.$$

At equilibrium, replacing dealers and liquidity traders' equilibrium strategies (respectively, (8) and the first in (7a)) in the first period equilibrium condition (2), we have:

$$\begin{aligned} x_1^D + x_{11} = 0 &\iff \gamma \frac{a_1}{\gamma_H} u_1 - \gamma \tau_v p_1 + a_1 u_1 = 0 \\ &\iff \frac{\gamma + \gamma_H}{\gamma_H} a_1 u_1 = \gamma \tau_v p_1 \end{aligned} \tag{11}$$

At a linear equilibrium the price is proportional to the aggregate endowment shock u_1: $p_1 =$

[22] Because of the informativeness of the signal they observe about u_1, at equilibrium, second period traders are able to perfectly infer the first period endowment shock and thus p_2. This makes their order akin to a contrarian marketable order. Indeed, based on (7b), we have $x_2 = (\gamma_H \tau_v \Lambda_2 - 1) u_2 + \gamma_H \tau_v \Lambda_{21} u_1 = \gamma_H \tau_v (\Lambda_2 u_2 + \Lambda_{21} u_1) - u_2 = -\gamma_H \tau_v p_2 - u_2$.

$-\Lambda_1 u_1$. Identifying $-\Lambda_1$ in the latter expression yields:

$$\underbrace{\frac{1}{\gamma\tau_v}\frac{\gamma+\gamma_H}{\gamma_H}a_1}_{-\Lambda_1} u_1 = p_1. \tag{12}$$

Thus, $-\Lambda_1$ measures the price impact of a marginal increase in the endowment shock hitting the first period cohort and market illiquidity at the first round is given by:

$$\Lambda_1 = -\frac{1}{\gamma\tau_v}\frac{\gamma+\gamma_H}{\gamma_H}a_1. \tag{13}$$

According to (13), Λ_1 captures the risk-weighted compensation that liquidity suppliers demand to absorb the aggregate marginal position of liquidity traders and dealers (the aggregate "liquidity demand"). Since this covers a "cost" incurred to supply immediacy, we interpret (somewhat loosely) Λ_1 as the first period "liquidity supply" function.

At the second round, liquidity demand comes from first and second period traders coefficients a_{21} and a_2:

$$\begin{aligned} x_{21} &= \gamma_H \frac{E_1[v-p_2]}{\mathrm{Var}_1[v-p_2]} - \frac{\mathrm{Cov}_1[v, v-p_2]}{\mathrm{Var}_1[v-p_2]}u_1 \\ &= \underbrace{\frac{(\gamma_H\tau_v\Lambda_{21}-1)\tau_u}{\tau_u+\Lambda_2^2\tau_v}}_{=a_{21}} u_1. \end{aligned} \tag{14}$$

and

$$\begin{aligned} x_2 &= \gamma_H \frac{E_2[v-p_2]}{\mathrm{Var}_2[v-p_2]} - \frac{\mathrm{Cov}_2[v, v-p_2]}{\mathrm{Var}_2[v-p_2]}u_2 \\ &= \underbrace{(\gamma_H\tau_v\Lambda_2-1)}_{=a_2} u_2 + \underbrace{\gamma_H\tau_v\Lambda_{21}}_{=b} u_1. \end{aligned} \tag{15}$$

We can interpret the expressions for a_{21} and a_2 in the following way. A liquidity trader hedges a larger fraction of his shock (demands more liquidity), the lower is the impact the endowment shock has on p_2 (as a larger price impact reduces a trader's expected return from hedging), and the lower is the return uncertainty he faces (as a higher return variance dents his utility since he is risk averse). Consider now the second period market clearing condition:

$$\begin{aligned} (x_2^D - x_1^D) + x_{21} - x_{11} + x_2 = 0 &\iff x_2^D + x_{21} + x_2 = 0 \\ &\iff -\gamma\tau_v p_2 + a_2 u_2 + (a_{21}+b)u_1 = 0 \\ &\iff p_2 = \underbrace{\frac{a_2}{\gamma\tau_v}}_{=-\Lambda_2} u_2 + \underbrace{\frac{a_{21}+b}{\gamma\tau_v}}_{=-\Lambda_{21}} u_1. \end{aligned} \tag{16}$$

At the second line of the above expression we make use of the first period market clearing equation: $x_1^D + x_{11} = 0$. We then replace strategies with their equilibrium expressions and finally solve for p_2, identifying the price coefficients.

Similarly to Λ_1, the coefficients Λ_2 and Λ_{21} reflect the risk-weighted compensation that liquidity suppliers demand to absorb first and second period liquidity traders' aggregate demand. To understand the numerator of Λ_{21}, note that first period liquidity traders' demand at the second round (i.e., the marginal position a_{21}), is not absorbed by dealers in its entirety. Indeed, at the second round part of first period liquidity traders' endowment shock exposure is absorbed by second period traders' speculation (the coefficient b). Similarly to what we have done for Λ_1, we interpret Λ_{21} and Λ_2 as the second period liquidity supply functions to first and second period traders.

2.1 Liquidity demand and supply in a transparent market

In this section we focus on the behavior of liquidity demand and supply in the fully transparent benchmark. In Proposition 1, we show that the hedging intensities a_1, a_{21} and a_2 are negatively valued functions (ranging between -1 and 0) since they capture first and second period liquidity traders' reaction to the endowment shock they receive. To ease the exposition, we measure liquidity traders' demand for liquidity via their "hedging aggressiveness," that is the absolute values of a_1, a_{21}, and a_{22}. Because of the way they are defined, liquidity supply functions are instead positively valued. In sum, the liquidity demand and supply functions are given by the following expressions:

$$|a_2| = |\gamma_H \tau_v \Lambda_2 - 1|,\ \Lambda_2 = -\frac{a_2}{\gamma\tau_v} \tag{17a}$$

$$|a_{21}| = \left|\frac{(\gamma+\gamma_H)^2(\gamma_H\tau_v\Lambda_{21}-1)\tau_u\tau_v}{1+(\gamma+\gamma_H)^2\tau_u\tau_v}\right|,\ \Lambda_{21} = -\frac{a_{21}}{(\gamma+\gamma_H)\tau_v} \tag{17b}$$

$$|a_1| = |-\gamma_H\tau_u(\gamma+\gamma_H)^2\tau_u\tau_v^2(\Lambda_{21}-\Lambda_1)|,\ \Lambda_1 = -\frac{\gamma+\gamma_H}{\gamma\gamma_H\tau_v}a_1. \tag{17c}$$

Inspection of the above expressions shows that:

Corollary 3. *When the market is transparent, liquidity demand is decreasing in the aggregate price impact it induces and liquidity supply increases in traders' aggregate demand.*

Therefore, in a transparent market, the cost of trading works as a rationing device: the pricier liquidity becomes, the less traders choose to hedge. Conversely, an increase in traders' liquidity demand prompts dealers to make the market less liquid (i.e., make liquidity pricier). In Figure 2 we plot the liquidity supply and demand functions (respectively, in blue and green) for second period traders. The unique equilibrium corresponds to the crossing point between the two curves.

$\tau_u = \tau_v = 0.1,\ \tau_\eta \to \infty,\ \gamma = 1,\ \gamma_L = 0.1$

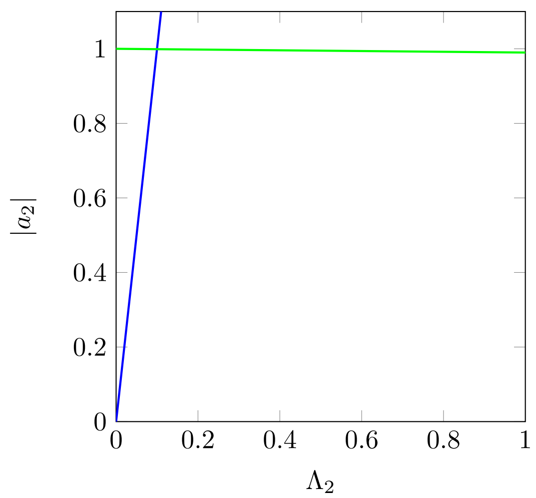


Figure 2: Second period traders' liquidity demand (in green) and supply (in blue) at the second round with a fully transparent market.

Summarizing, when the market is transparent, liquidity demand decreases in the price impact coefficients of the endowment shocks and price impact coefficients increase in liquidity demand. In these conditions, a unique equilibrium obtains. In this equilibrium dealers speculate on short-term returns and second period liquidity traders hedge their risk exposure and provide liquidity via contrarian market(able) orders, sharing with dealers the risk exposure of first period traders.

3 The opaque market

Suppose now that second period traders observe a noisy signal of the first period order imbalance ($\tau_\eta \in (0, \infty)$). In this case, $\Omega_2 = \{u_2, s_{u_1}\}$ which implies that second period traders cannot perfectly anticipate p_2. As a consequence, their strategy is affected by their return uncertainty (see (A.6)):

$$x_2 = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\mathrm{Var}_2[v - p_2]}}_{a_2} u_2 + \underbrace{\gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_u + \tau_\eta)\mathrm{Var}_2[v - p_2]}}_{b} s_{u_1}, \tag{18}$$

with $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$, and the second period price is as in (1b). Intuitively, traders' inability to exactly infer u_1 impacts their return uncertainty, exposing their strategy to execution risk. This, in turn, affects both their hedging and speculative aggressiveness ($|a_2|$ and b) and the cost of trading of their order. Given the risk-sharing enhancing role of traders' speculation, this impacts market stability. To see this, it is useful to start from the extreme case in which $\tau_\eta \to 0$.

3.1 The fully opaque market

Suppose second period traders' signal becomes unboundedly noisy (i.e., $\tau_\eta \to 0$). In this case, we obtain the following result:

Proposition 2. *When the market is fully opaque, the expressions for the equilibrium price coefficients Λ_2 and Λ_1 are as in (6a) and (6b), while*

$$\Lambda_{21} = -\frac{a_{21}}{\gamma\tau_v}, \tag{19}$$

The coefficients of traders' strategies are as follows:

$$a_1 = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} < 0,\ a_{21} = \frac{\gamma_H \tau_v \Lambda_{21} - 1}{\mathrm{Var}_1[v - p_2]} \in (-1, 0) \tag{20a}$$

$$a_2 = \frac{\gamma_H \tau_v \Lambda_2 - 1}{\mathrm{Var}_2[v - p_2]} \in (-1, 0),\ b = 0, \tag{20b}$$

where $\mathrm{Var}_1[p_2] = \Lambda_2^2 \tau_u^{-1}$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_2^2 \tau_u^{-1}$ *and* $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}$. *Furthermore, at equilibrium $\Lambda_{21} > \Lambda_1 > 0$ and $\Lambda_2 > 0$.*

According to (19) and the second expression in (20b), when the market is fully opaque, second period traders do not speculate. This is because their signal on u_1 is infinitely noisy, which makes it impossible for them to predict the direction of the first period imbalance. As a consequence, $\Lambda_{22} = 0$ and we have:

Corollary 4. *When the market is fully opaque, second period liquidity traders do not supply liquidity via contrarian market orders and the second period price only reflects traders' endowment shocks.*

According to (20a) and (20b), liquidity traders' second period hedging aggressiveness, $|a_{21}|, |a_2|$ depends on two forces: the expected return from holding the endowment shock, and the variance of the second period return $v - p_2$ (respectively captured by the terms at the numerator–which is negative–and denominator of the expressions in (20a) and (20b)).[23] For given return variance, a higher price impact of the t-period traders' endowment shock, increases these traders' expected return from holding the endowment shock, decreasing their hedging aggressiveness. For given expected return from holding the endowment shock, a higher price impact of the t-period traders' endowment shock increases $s \neq t$-period traders' execution risk, lowering the latter hedging aggressiveness. Therefore, changes in the price impacts of the trades of different cohorts have opposite effects on the execution risk faced by each cohort, and this effect, when second period traders are not informed about u_1, can be responsible for self-sustaining demand loops.

[23]In the Appendix we show that $E_2[v - p_2] = \Lambda_2 u_2$, $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}$, $E_1[v - p_2] = \Lambda_{21} u_1$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}$, $\mathrm{Cov}_1[v, v - p_2] = \mathrm{Cov}_2[v, v - p_2] = \tau_v^{-1}$.

To see this, assume that the market impact of the second period traders' endowment shock (Λ_2) increases. This reduces these traders' expected profit from hedging the endowment and heightens the cohort 1 traders' execution risk, leading them to scale down their liquidity demand ($|a_{21}|$ decreases). All else equal, this reduces the price impact of cohort 1's endowment shock (Λ_{21} decreases), because liquidity providers need to absorb a smaller share of cohort 1's endowment shock. This in turn lowers the execution risk faced by cohort 2 traders, potentially leading them to scale up their liquidity demand ($|a_2|$ increases), and further boosting Λ_2, because dealers need to absorb a larger share of cohort 2's endowment shock, which reinforces the initial spike (see (19)).

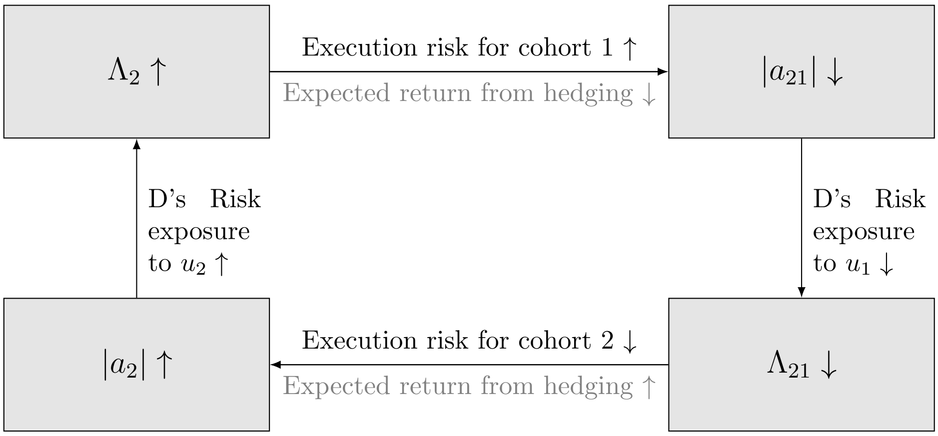


Figure 3: Strategic complementarity. A diagrammatical representation of the self-reinforcing loop between liquidity consumption and illiquidity arising with market opacity.

The loop described above is diagrammatically sketched in Figure 3 and formally captured by the "aggregate" best response function (21) which reflects the impact of an exogenous change in Λ_2 on traders' strategies, yielding a new value for Λ_2 (see (A.40) in the Appendix for its formal derivation):

$$\Phi(\Lambda_2) \equiv \frac{((\gamma+\gamma_H)\tau_u+\gamma\Lambda_2^2\tau_v)^2}{\gamma\tau_u+((\gamma+\gamma_H)\tau_u+\gamma\Lambda_2^2\tau_v)^2(\gamma+\gamma_H)\tau_v}. \tag{21}$$

Differentiating (21), it is possible to see that

$$\frac{\partial\Phi(\Lambda_2)}{\partial\Lambda_2} = \frac{4\gamma^2\Lambda_2((\gamma+\gamma_H)\tau_u+\gamma\Lambda_2^2\tau_v)\tau_u\tau_v}{(\gamma\tau_u+((\gamma+\gamma_H)\tau_u+\gamma\Lambda_2^2\tau_v)^2(\gamma+\gamma_H)\tau_v)^2} > 0, \tag{22}$$

which provides the formal counterpart to the heuristic argument developed above–that is the existence of strategic complementarity in illiquidity with market opacity.

Because of the way it is defined, a fixed point of (21), $\Lambda_2 = \Phi(\Lambda_2)$, corresponds to an equilibrium of the market and in Figure 4 we show that, depending on parameters' values, either a unique equilibrium or multiple equilibria can obtain. Specifically, with the hypothesized parameterization, when the dispersion of the endowment shock is sufficiently low (case $\tau_u = 2$, in Panel (a)), strategic complementarity is "weak" and a unique equilibrium arises (in which case $\Lambda_{21} = \Lambda_2 = 4.61$ and $\Lambda_1 = 0.01$). Conversely, when the dispersion of the endowment shock increases (case $\tau_u = 0.1$, in Panel (b)), strategic complementarity is "strong," and multiple equilibria arise, where $\Lambda_2 \in \{8.96, 1.98, 0.12\}$, and the corresponding values for the other price coefficients are $\Lambda_{21} \in \{0.12, 1.98, 8.96\}$, $\Lambda_1 \in \{0.1 \times 10^{-2}, 0.43, 8.84\}$. Our simulations suggest that equilibrium multiplicity is more likely to obtain when payoff and endowment shock dispersion are larger and liquidity traders are more risk averse (this is corroborated in Corollary 5 below).

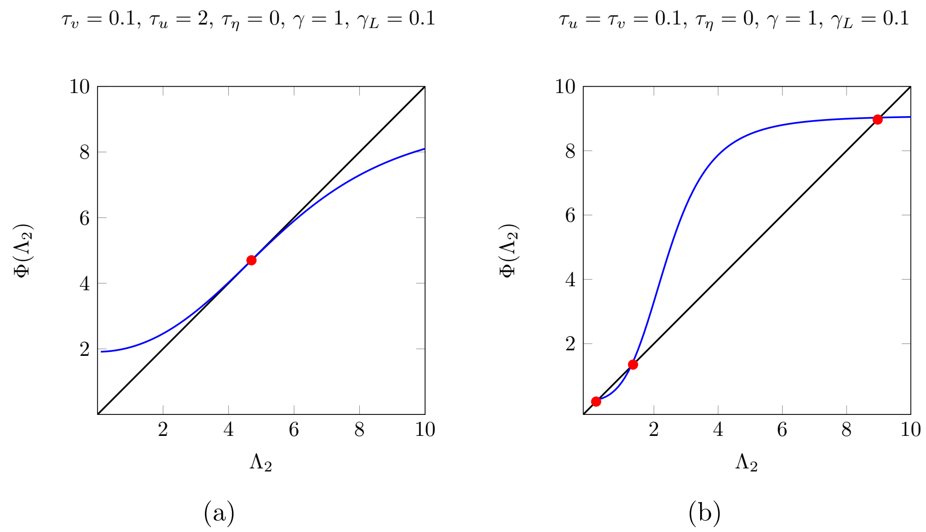


Figure 4: Market opacity: single equilibrium (Panel (a)), and multiple equilibria (Panel (b)).

For $\tau_\eta \to 0$, the system of equations which pins down the price impacts becomes:

$$\Lambda_2 = \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v} \tag{23a}$$

$$\Lambda_{21} = \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\tau_v} \tag{23b}$$

$$\Lambda_1 = \frac{(\gamma + \gamma_H)\tau_u\Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}. \tag{23c}$$

Manipulating (23a) and (23b) in the Appendix we show that in this case the equilibrium obtains as a solution to the following quadratic equation:

$$(\gamma + \gamma_H)\gamma\tau_v\Lambda_2^2 - \gamma\Lambda_2 + (\gamma + \gamma_H)^2\tau_u = 0, \tag{24}$$

which, thus, has a closed form solution. Note that in this case, the price impact of the first period endowment shock (Λ_1) does not affect the second period price coefficients (Λ_2, Λ_{21}) but is determined by their equilibrium values. Formally, we obtain the following corollary of the previous result:

Corollary 5. *When the market is fully opaque, at equilibrium*

$$\Lambda_1 = (\gamma + \gamma_H)\tau_v\Lambda_{21}^2. \tag{25}$$

If

$$0 < \tau_u\tau_v < \gamma/(4(\gamma + \gamma_H)^3), \tag{26}$$

three equilibria arise, where

$$\Lambda_2 = \frac{\gamma \pm \sqrt{(\gamma - 4(\gamma + \gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_H)\gamma\tau_v}, \Lambda_{21} = \frac{\gamma \mp \sqrt{(\gamma - 4(\gamma + \gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_H)\gamma\tau_v}, \tag{27a}$$

and $\Lambda_2 = \Lambda_{21}$ obtaining as the unique root of the following cubic

$$\varphi(\Lambda_2) \equiv ((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\Lambda_2\tau_v - \tau_u = 0. \tag{27b}$$

If $\tau_u\tau_v \geq \gamma/(4(\gamma + \gamma_H)^3)$, then there is a unique equilibrium where $\Lambda_2 = \Lambda_{21}$ is the unique root of the cubic (27b).

Condition (26) defines the parameter restriction for the region where equilibrium multiplicity occurs. According to such condition, multiplicity obtains when liquidity demand is likely to be stronger, the volatility of the security's payoff is larger and traders are more risk averse, i.e. when the gap between liquidity demand and liquidity provision is likely to be *wider*. Indeed, in these conditions traders need to hedge the most (due to the higher unpredictability of their endowment shock and their higher risk aversion), while dealers are less willing to supply liquidity (due to the higher volatility of the security's payoff). Interestingly, an increase in dealers' risk-bearing capacity has a non-monotonic impact on the magnitude of this region. This is because for given hedging aggressiveness ($|a_{21}|$ and $|a_{22}|$), an increase in γ lowers the cost of trading (see (6a) and (19)) which, for low levels of risk tolerance, induces more liquidity consumption on traders' side (see (20a) and (20b)). However, as γ grows large this effect becomes second order, and an increase in dealers' risk tolerance reduces the magnitude of the multiplicity region. Importantly, in the latter case, this implies that a decrease in dealers' risk bearing capacity can be responsible for an increase in market instability. Indeed for $\gamma > \gamma_H/2$ a lower γ enlarges the region of parameter values for which multiplicity obtains.

The expressions in (27a) show that with equilibrium multiplicity, the second period price sensitivities to the endowment shock correspond to the two roots of the quadratic (24). This implies that at the second round the trading costs faced by traders in different cohorts are heterogeneous: the price impact of first and second period liquidity traders' endowment shocks are *negatively correlated.*

The next result characterizes the stability properties of the equilibrium and the liquidity consumption patterns arising with multiple equilibria. For ease of exposition we denote by Λ_2^* and Λ_2^{***} the low and high root in the first of (27a), and with Λ_2^{**} the unique real root of the cubic (27b). Correspondingly, Λ_{21}^{***}, Λ_{21}^{*}, and Λ_{21}^{**}, denote the low and high root in the second of (27a), and the unique real root of the cubic (27b) (recall that in this case $\Lambda_2 = \Lambda_{21}$). Finally, Λ_1^{***}, Λ_1^{*} and Λ_1^{**} denote the first period price impact coefficient obtained via (25). Accordingly, we rank traders' hedging intensities in a similar way: a_2^* corresponds to the case where $\Lambda_2 = \Lambda_2^*$ (and $\Lambda_{21} = \Lambda_{21}^*$), and so on.

Corollary 6. *When the market is fully opaque, with uniqueness, the equilibrium is stable. When multiple equilibria arise,*

1. *The two extreme equilibria are stable, while the intermediate equilibrium is unstable.*

2. *Equilibria can be ranked in terms of the price sensitivity to first and second period endowment shocks:*

$$\Lambda_2^* < \Lambda_2^{**} < \Lambda_2^{***},\ \Lambda_{21}^{***} < \Lambda_{21}^{**} < \Lambda_{21}^{*},\ \Lambda_1^{***} < \Lambda_1^{**} < \Lambda_1^{*}. \tag{28}$$

   Thus, at a stable equilibrium we have either that p_2 reacts more to u_2 than to u_1, or the opposite. Correspondingly, in the former (latter) case the first period market is more (less) liquid. Comparing liquidity across trading rounds, we have

$$\Lambda_1 < \Lambda_{21}^{***} < \Lambda_2^{***},\ \text{or}\ \Lambda_1 < \Lambda_2^{*} < \Lambda_{21}^{*}.$$

3. *Traders' hedging intensity is increasing in the cost of trading it induces: $-1 < a_2^{***} < a_2^{**} < a_2^{*} < 0$, $-1 < a_{21}^{*} < a_{21}^{**} < a_{21}^{***} < 0$, and $-1 < a_1^{*} < a_1^{**} < a_1^{***} < 0$.*

Therefore, only the extreme equilibria are stable. Additionally, at equilibrium the traders belonging to the cohort that faces the *highest market impact demand more liquidity*. In other words, with multiple equilibria, illiquidity no longer operates as a rationing device. This is because the price impact induced by the endowment shock (affecting traders in cohort) t, has a proportionally stronger effect on the execution risk faced by cohort $s \neq t$ traders than on the expected return obtained by traders in cohort t.

An important implication of Corollary 6 is that when multiple equilibria arise, at the first round of trade, dealers tend to speculate more aggressively (consume more liquidity) when the market is more illiquid. Indeed, with opacity the first period strategy of a liquidity provider is still as in (8), which implies that the equilibrium coefficient of the speculative component in that strategy is given by:

$$-\frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} = \gamma \frac{a_1}{\gamma_H}.$$

Given part 3 of the above corollary, it then follows that dealers speculate more aggressively along the equilibrium with the highest illiquidity. This prediction is consistent with the findings in Brogaard et al. (2018) and Bellia et al. (2022). The former show that when extreme price movements occur across different securities, high frequency traders step up their liquidity demand. The latter argue that HFT consume liquidity during flash crashes, contributing to triggering or exacerbating these events.

Notably, the positive relationship between liquidity consumption and illiquidity is preserved even when (26) is not satisfied and a unique equilibrium arises (i.e, $|a_2|$ and Λ_2 move together). In that situation, since $\Lambda_{21} = \Lambda_2$, we have

$$a_{21} = a_2 = \frac{(\gamma_H \tau_v \Lambda_2 - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}. \tag{29}$$

Rearranging (27b) to isolate $\Lambda_2^2 \tau_v$ yields:

$$\Lambda_2^2 \tau_v = \frac{(1 - (\gamma + \gamma_H)\Lambda_2 \tau_v)\tau_u}{\gamma \Lambda_2 \tau_v},$$

which can be substituted at the denominator of (29) to obtain

$$a_2 = -\gamma \tau_v \Lambda_2.$$

This implies the following result.

Corollary 7. *When the market is fully opaque and a unique equilibrium obtains, at the second round both traders' cohorts hedge the same fraction of their endowment shock, facing the same illiquidity:*

$$a_2 = -\gamma \tau_v \Lambda_2, \tag{30}$$

where Λ_2 is the unique real solution to (27b).

3.2 Liquidity demand and supply in a fully opaque market

The discussion following the last result, suggests that when the market is opaque, liquidity demand should be an increasing function of the cost of trading it induces, that is, its *slope* should change compared to the case where the market is fully transparent. This is exactly what we display in Figure 5, where we substitute (21) and (23a) into the second of (20a) and take the absolute value of the resulting expression to obtain the hedging aggressiveness of first period traders when they re-trade at the second round: $|a_2|$. In the figure, we plot $|a_2|$ (in green) as a function of the cost of trading it generates and the liquidity supply function (in blue) as a function of the hedging intensity it induces, using the same parameter values of Figure 4. The crossing points between the two curves occur at equilibrium. In Panel (a) and (b) we use the same parameterizations of the corresponding panels in Figure 4, and, respectively, a unique equilibrium and three equilibria obtain. As shown by the figure, and differently from what shown in Figure 2 with a fully transparent market, a higher Λ_2 leads second period traders to demand more liquidity ($|a_2|$ increases), which leads to the positive association between liquidity consumption and illiquidity at equilibrium when $\tau_\eta \to 0$.

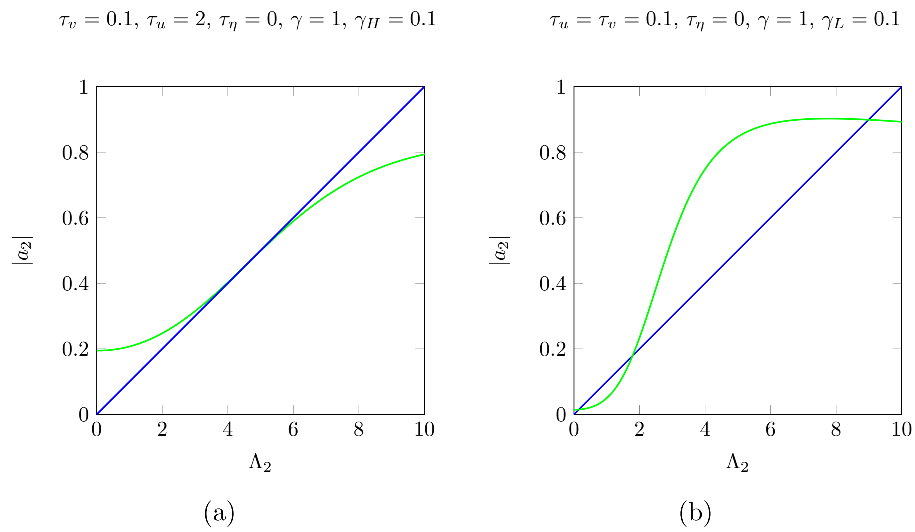


Figure 5: Liquidity demand and supply at the second round with a fully opaque market.

A liquidity "dry-up". Figure 5 also illustrates an important prediction of our model. Suppose the market is at a unique equilibrium and an unexpected shock boosts hedgers' endowment uncertainty. Then, the initial effect is that of reducing hedgers' liquidity demand. To see this, note that since $\varphi(\Lambda_2)$ is increasing in Λ_2, from (27b) we obtain $\partial\varphi(\Lambda_2)/\partial\tau_u = (\gamma+\gamma_H)\Lambda_2\tau_v - 1$, which can be shown to be negative, implying that at the intermediate equilibrium, a decline in τ_u reduces Λ_2. Intuitively, a lower τ_u increases execution risk for 2nd period traders (the denominator in (20b)), lowering $|a_2|$, which reduces dealers' exposure to u_2 and thus Λ_2.[24]

Moreover, when τ_u declines, the new aggregate best response becomes steeper at the intermediate equilibrium:

Corollary 8. *An increase in the volatility of the endowment shock affecting liquidity traders heightens strategic complementarity at the intermediate equilibrium:*

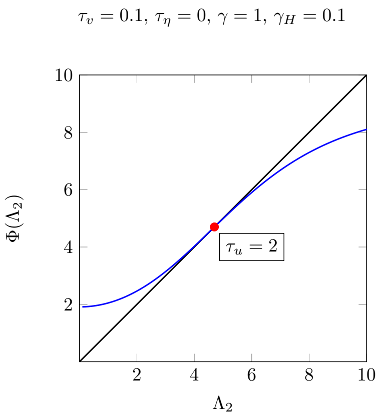
$$\frac{\partial}{\partial\tau_u}\left(\frac{\partial\Phi(\Lambda_2)}{\partial\Lambda_2}\right)\bigg|_{\Lambda_2=\Lambda_2^{**}} < 0. \tag{31}$$

Therefore, for a large enough shock (that fulfills condition (26)), the strengthening of strategic complementarity makes the effect on execution risk overpower that on expected returns, yielding multiple equilibria. As a consequence, when such a shock occurs, all else equal, the old equilibrium value of illiquidity Λ_2 falls between Λ_2^{**} and Λ_2^{***}, and because of best response adaptive dynamics, is attracted by the high illiquidity equilibrium. This yields the following result:

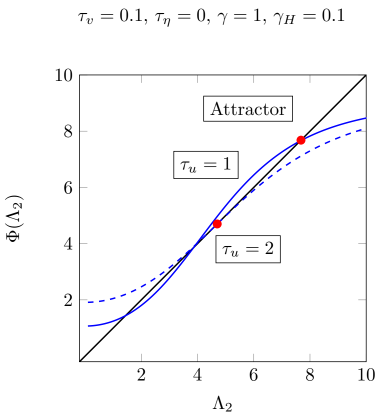
Corollary 9. *When the market is fully opaque and a unique equilibrium obtains, a shock increasing liquidity traders' endowment volatility which is large enough to induce multiple equilibria, leads the market to gravitate towards the high illiquidity equilibrium at the second round.*

[24]Formally, by chain rule, at the unique equilibrium $\partial\Lambda_2/\partial\tau_u = -(\partial\varphi(\Lambda_2)/\partial\tau_u)/(\partial\varphi(\Lambda_2)/\partial\Lambda_2) > 0$, since the numerator in the expression is negative at the unique equilibrium.

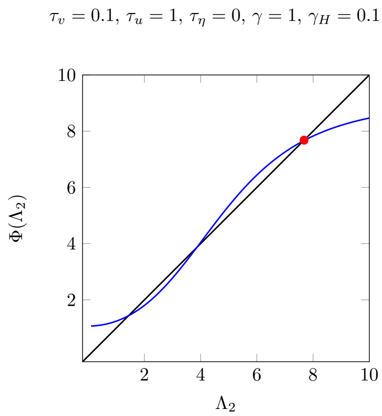
A "flash-crash". The above result implies that when the market is opaque, an unanticipated increase to traders' endowment shocks' dispersion is conducive to a liquidity crash. One example would be the case in which hedgers are investment banks with a position in the asset. If uncertainty over their endowments increases unexpectedly and is perceived permanent (e.g., because of an unanticipated macro event such as the Covid pandemic or the war in Ukraine), an opaque market triggers the loop we described above leading to a crash, characterized by a much higher illiquidity. When the additional uncertainty dissipates, and traders think again that the change is permanent, the market recovers, returning to the status quo ante, as in a "flash crash" (see, respectively, Figures 6 and 7).



(a) Unique equilibrium

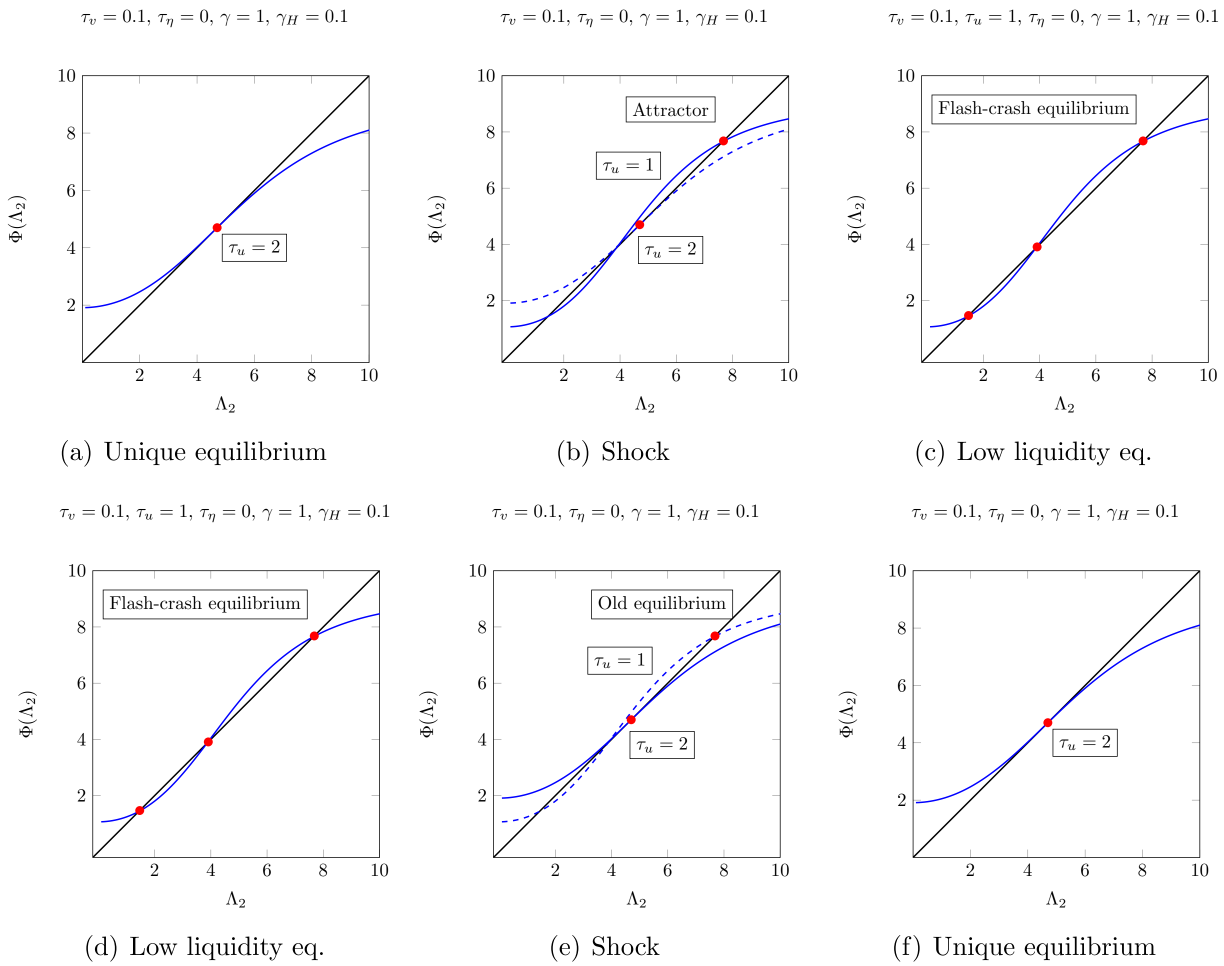


(b) Large shock



(c) Low liquidity equilibrium

Figure 6: An unanticipated, permanent increase in endowment shock dispersion leading to a liquidity dry up. Starting from the unique stable equilibrium when $\tau_u = 2$ (panel (a)), an unanticipated increase in hedgers' endowment shock dispersion (with $\tau_u \downarrow 1$) shifts the best response (21) to the left yielding three equilibrium points (panel (b)). Finally, best response dynamics leads the market to gravitate towards the high illiquidity equilibrium (panel (c)).



(a) Unique equilibrium (b) Shock (c) Low liquidity eq.

(d) Low liquidity eq. (e) Shock (f) Unique equilibrium

Figure 7: An unanticipated, thought permanent (but in fact temporary) increase in endowment shock dispersion leading to a "flash crash." Starting from the unique stable equilibrium when $\tau_u = 2$ (panel (a)), an unanticipated increase in hedgers' endowment shock dispersion (with $\tau_u \downarrow 1$) increases the steepness of the best response (21) yielding three equilibrium points (panel (b)). Best response dynamics leads the market to temporarily gravitate towards the high illiquidity equilibrium (panel (c)). Once the endowment shock dispersion returns to its initial value ($\tau_u \uparrow 2$), the best response mapping moves to the right, and the market returns to its original equilibrium value (panels (d)–(f)).

4 Liquidity trading and noise trading

In this section, we consider the implications of our analysis for the time series properties of noise trading and returns.

First, note that, based on Proposition 1 and the interpretation of the price impact coefficients in Corollary 2, we can say that with transparency, at the second round dealers absorb a smaller portion of the first period endowment shock (compared to the second period one), and the noise process is stable: $\beta \equiv \Lambda_{21}/\Lambda_2 < 1$. That is, the second period endowment shock impacts p_2 more than u_1.

Second, the first and second period returns are *positively* serially correlated. That is, the model displays momentum, in the absence of any fundamentals information:

$$\begin{aligned}\mathrm{Cov}[p_2 - p_1, p_1] &= \mathrm{Cov}[-(\Lambda_2 u_2 + (\Lambda_{21} - \Lambda_1)u_1), -\Lambda_1 u_1] \\ &= (\Lambda_{21} - \Lambda_1)\Lambda_1 \tau_u^{-1} > 0,\end{aligned} \tag{32}$$

due to Proposition 1.At the first round hedgers can count on the additional liquidity supplied by second period traders, which implies that they will increase their first period hedging position, inducing $\Lambda_{21} > \Lambda_1$.

We collect these results in the following corollary:

Corollary 10. *When the market is transparent: (1) liquidity trading behaves as a stable AR(1) process; (2) first and second period returns are positively serially correlated.*

The following result states the implications for the time series properties of noise trades and returns autocovariance when the market is fully opaque:

Corollary 11. *With multiple equilibria, (1) $\beta < 1$ ($\beta > 1$) when $\Lambda_2 = \Lambda_2^{***}$ ($\Lambda_2 = \Lambda_2^*$); (2) the autocovariance of first and second period returns increases in Λ_{21} and also increases compared to the case with full transparency at both equilibria.*

Low noise trading persistence is therefore compatible both with strong opacity (at the equilibrium with a high Λ_2) or with full transparency.

5 Extensions

In this section, we consider three extensions to the model we developed so far. In the first one, we allow the market to be "partially" opaque (i.e., $\tau_\eta \in (0, \infty)$). Next, we assume that liquidity is also supplied by a class of dealers (of mass $1 - \mu$) who can only trade at the first round and which we term "Restricted Dealers"–we denote them by RD and use D (of mass $0 < \mu < 1$) to denote the dealers we introduced in Section 1.1. Finally, we analyze the welfare properties of our model.

We start by considering the effect of an informative signal, keeping $\mu = 1$.

5.1 An informative signal

When $\tau_\eta \in (0, \infty)$, prices are as in (1a) and (1b), and we have the following result:

Proposition 3. *With partial opacity, the equilibrium obtains as a solution to the system of non-linear, simultaneous equations (A.17a)–(A.53) and (A.29). The expressions for the equilibrium prices' coefficients Λ_2, Λ_1, Λ_{21} and Λ_{22} are as in (A.29), and (A.30a)-(A.30c). The coefficients of traders' strategies are as in Proposition 2, with $\mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$, $\mathrm{Var}_1[v - p_2] =$*

$\tau_v^{-1} + \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$ *and* $\text{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$, *except for* b, *which is given by:*

$$b = \gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\text{Var}_2[v - p_2]}. \tag{33}$$

At equilibrium, $\Lambda_2 > 0, \Lambda_{21} > \Lambda_1 > 0$, *and* $\Lambda_{22} < 0$.

Corollary 12. *As* $\tau_\eta \to \infty$, *we have the unique equilibrium of Proposition 1.*

For $\tau_\eta < \infty$, we are not able to analytically study the equilibrium and we resort to numerical simulations to investigate the properties of the model. As in the fully opaque case we can have one or three equilibria.

According to the above result, an informative signal about u_1 ($\tau_\eta \in (0, \infty)$) leads second period traders to speculate against the price pressure created by first period traders' liquidity demand, taking a contrarian position (in our simulations, $b > 0$), thus enhancing the risk-bearing capacity of the market. This dampens the strategic complementarity responsible for multiple equilibria and for τ_η large enough, leads to a unique equilibrium (see Figure 8).[25]

In Figure 8, we plot the price and strategy coefficients for one of our simulations. As shown in the figure, for τ_η small, three equilibria arise. We plot them using the color green, blue and red to indicate the equilibrium that corresponds to the two "extreme," stable price impacts (respectively in green and red) and the unstable one (in blue) when $\tau_\eta = 0$. Importantly, when multiple equilibria obtain, order flow partial transparency does not modify an important conclusion we reached in Section 3.2: liquidity demand and illiquidity are positively related at equilibrium (see panels (c), (d), (e) and (f) in Figure 8).

[25]This result is reminiscent of Bernardo and Welch (2004), who argue that a way to stabilize the market in the face of a run on liquidity, is to increase the risk bearing capacity of the market making sector. This is precisely what a better signal about u_1 achieves in our setup.

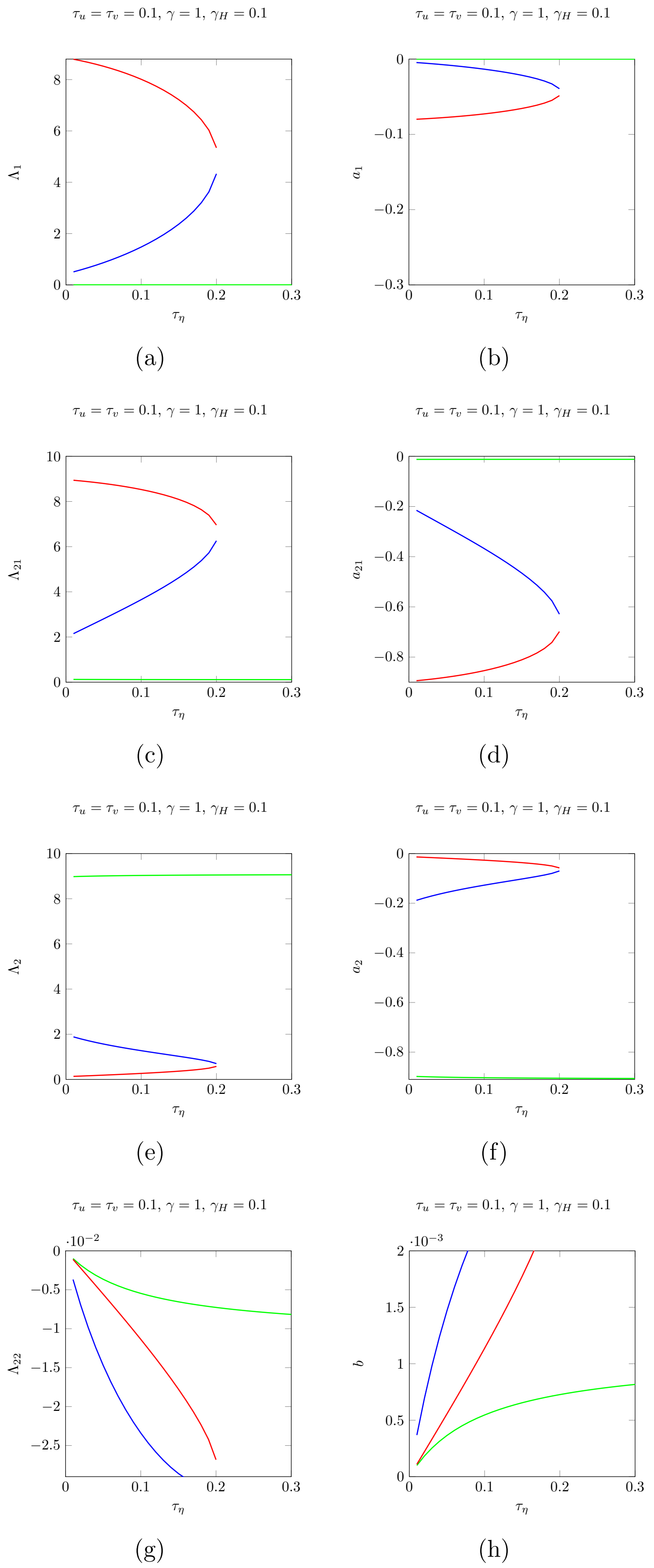


Figure 8: Price impact coefficients (panel (a), (c), (e), (g)) and strategy coefficients with a partially informative signal ($\tau_\eta < \infty$). Parameter values are as in Figure 5 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$.

5.2 Restricted dealers

In this section, we assume that in the first round, liquidity is provided by a mass $\mu \in (0,1]$ of dealers D and a complementary mass $1-\mu$ of RD (Restricted Dealers). An RD has CARA preferences with the same risk-tolerance γ as a D. However, as he is in the market only in the first period, he submits a price-contingent order x^{RD} to maximize the expected utility of his wealth $(v-p_1)x^{RD}$ which, as we show in the Appendix (see (A.14)), has the following expression: $x^{RD} = -\gamma\tau_v p_1$. The inability of RD to trade in the second period captures some liquidity suppliers' limited market participation. This friction could be due to technological reasons as in the case of dealers with impaired access to a technology that allows trading at high frequencies. Alternatively, it could arise from limited access to the trading venue, as in the case of those liquidity suppliers who in the 80s could not access the NYSE trading floor.

Due to the heterogeneity of liquidity suppliers' types, market clearing conditions change compared to (2)-(3):

$$\mu x_1^D + (1-\mu)x^{RD} + x_{11} = 0 \tag{34a}$$

$$(x_2^D - x_1^D)\mu + (x_{21} - x_{11}) + x_2 = 0 \iff \mu x_2^D + (1-\mu)x^{RD} + x_{21} + x_2 = 0, \tag{34b}$$

where in the latter we make use of the first period market clearing condition to obtain the expression at the right hand side of (34b). Figure 9 displays the timeline of the model.

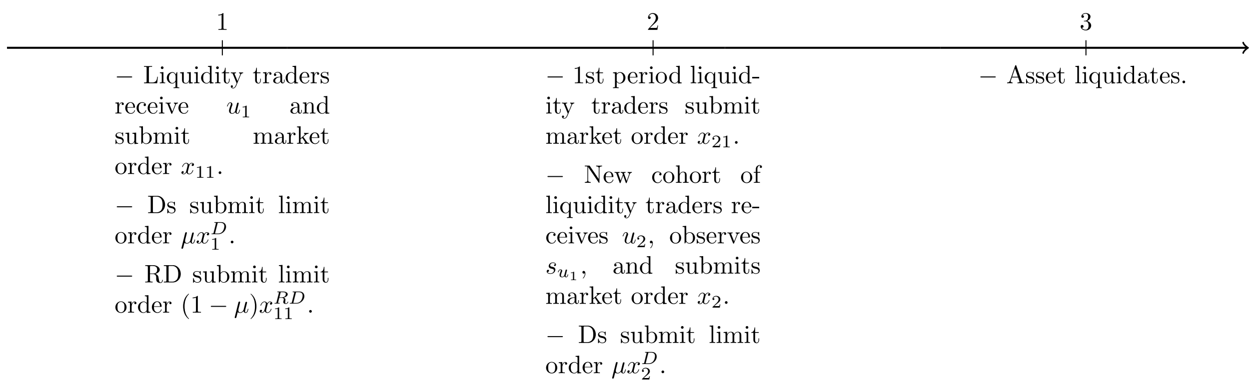


Figure 9: The timeline with heterogeneous liquidity supply.

This version of the model is also analytically challenging, and we resort to numerical simulations to investigate its properties.[26]

Unique and multiple equilibria. We first show that multiple equilibria also arise when second period traders observe an informative signal about u_1 and $\mu \in (0,1]$. In Figure 10 we partition the space $\mu \in (0,1], \tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium

[26] The analytical characterization of the equilibrium is very similar to the one illustrated in Proposition 3.

(three equilibria). According to the figure, uniqueness obtains when second period traders' signal is of sufficiently good quality, in line with the results of Sections 3 and 5.1. The effect of an increase in μ is less obvious. As the figure illustrates, we find that when τ_η is low, an increase in μ leads the market to switch from multiple equilibria to a unique equilibrium, and, eventually, back to multiple equilibria. The intuition is as follows.

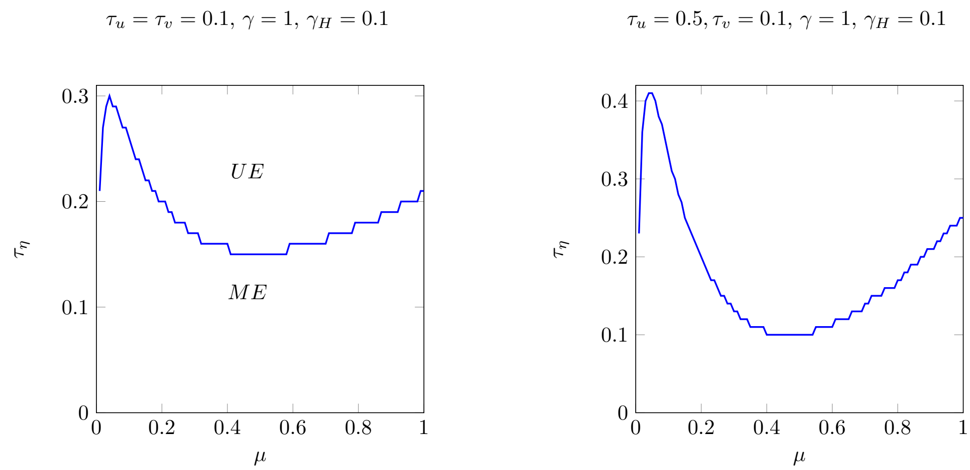
Liquidity fragility is a byproduct of imperfect risk sharing and market opacity. When second period traders' information is noisy (τ_η low), as μ increases from zero, with multiple equilibria, initially risk sharing improves (as there are more dealers absorbing traders' liquidity demand). This stabilizes the market and improves liquidity for second period traders (i.e., lowering Λ_2 at the HIE). However, as argued in Section 3.1, the decline in Λ_2 means that first period traders face lower execution risk which boosts their liquidity demand, eventually heightening strategic complementarity and leading back to the region with multiple equilibria and liquidity fragility.

The bottom line is that with low order flow transparency, an increase in the mass of dealers that are continuously in the market has a non-monotonic effect on strategic complementarity. Thus, to eliminate fragility, enhancing transparency is key.

Effects on liquidity fragility of risk aversion, volatility, and dispersion of the endowment shock. Comparing the areas below the blue curve in panel (a) and (b) in Figure 10 shows that that for $\mu \in (0, 1)$, consistently with what we have found in Corollary 5, an increase in τ_u reduces the chances of liquidity fragility. For extreme values of μ (that is, for μ close to 0 or 1) the figure indicates that when τ_η is low an increase in τ_u increases the chances of liquidity fragility. The intuition is as follows.[27]

Recall that when the signal is not perfect second period traders (1) may speculate in the "wrong" direction and (2) use p_2 and the signal to predict u_1. With a higher τ_u there is less noise in the price, which reduces second period traders speculative intensity. When μ is close to 0, almost only second period traders provide liquidity at the second round, and the reduction in speculation by these traders has a large impact on overall risk sharing. Conversely, when μ is close to 1, almost only D provide liquidity at 2 and the reduction in speculation by 2nd period traders means that dealers have less liquidity traders to share risk with. In either case this increases liquidity fragility. For intermediate values of μ, D have a smaller exposure to the risky security, and the additional risk sharing provided by 2nd period traders is less important. In this case, the reduction in these traders' speculation rids the market of the "wrong" trades with a positive impact on liquidity fragility.

[27] The comparative statics result for τ_v and γ_H align with the intuition gained in Corollary 5 and are deferred to Appendix B.



(a) (b)

Figure 10: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.

The effect of changes in the mass of D (μ) and order flow transparency (τ_η) on liquidity fragility is highly non linear. An increase in μ may be destabilizing for μ and τ_η low (as shown in panel (a)), and a small decrease in μ may induce a liquidity crash for μ large and τ_η low as we see below.

A small shock to the mass of dealers. In Figure 11, we show that when the market is at the low illiquidity equilibrium (with $\Lambda_2^* = 1.47$), a small reduction in the mass of D (from $\mu = 0.9$ to $\mu = 0.8$), plunges the market to the high illiquidity equilibrium ($\Lambda_2 = 9.6$, corresponding to a 653% increase in illiquidity). This may explain how a disconnection of a small percentage of dealers from the market (say because of a cyberattack) may cause a liquidity crash.[28]

[28] See for example, Ransomware attack on ICBC disrupts trades in US Treasury market, *Financial Times, November 2023.*

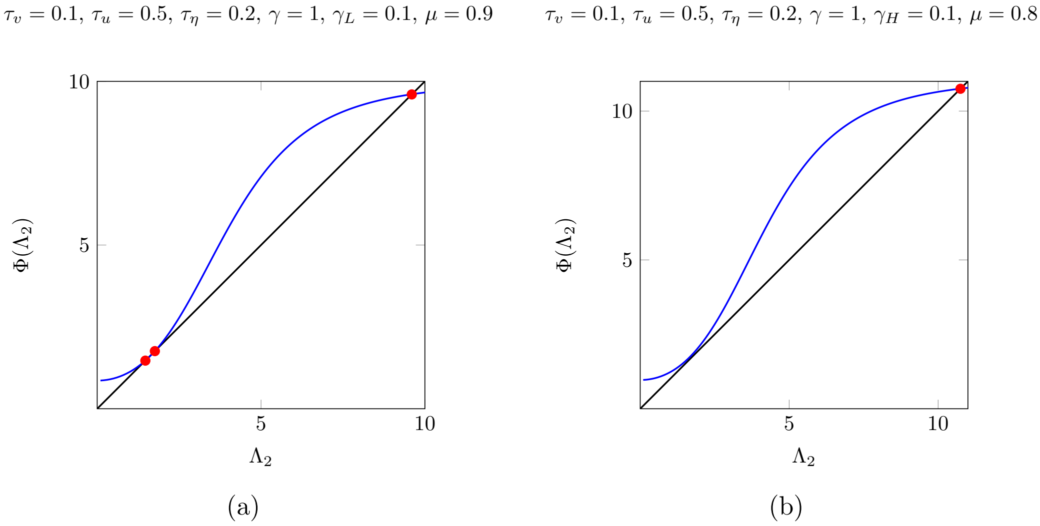


Figure 11: The effect of a small reduction in μ when τ_η is low.

5.3 Welfare analysis

In this section, we study the welfare implications of the general version of the model we presented in Section 5.2. Denoting by EU^D, EU^{RD}, and EU_t^H, respectively the unconditional expected utilities of D, RD and round $t \in \{1,2\}$ hedgers, we measure traders' payoffs by computing their certainty equivalents:

$$CE^D = -\gamma \ln(-EU^D),\ CE^{RD} = -\gamma \ln(-EU^{RD}),\ CE_t^H = -\gamma_H \ln(-EU_t^H).$$

The next result provides a condition for traders' payoffs to be well-defined.

Proposition 4. *Assuming that*

$$\gamma_H^2 \tau_u \tau_v > 1, \tag{35}$$

traders' payoffs are well defined and their expressions are given in the appendix (see (A.55)*,* (A.58)*,* (A.61)*, and* (A.64)*).*

Using (A.55), (A.58), (A.61), and (A.64), we define the total (utilitarian) welfare of market participants as follows:

$$TW(\mu;\tau_\eta) = \mu CE^D + (1-\mu)CE^{RD} + CE_1^H + CE_2^H. \tag{36}$$

We then numerically evaluate (36) to assess:

1. The welfare ranking of the equilibria that arise with multiplicity, taking as a reference the parameter values of Figure (12), panel (a).
2. The welfare properties of the unique equilibrium as either the market becomes less opaque (τ_η increases), or the mass of D increases (μ increases). In this case, we assume $\gamma = 0.5$,

$\gamma^L = 0.25$, a 10% annual volatility for the endowment shock, and consider a "high" and a "low" payoff volatility scenario (respectively, $\tau_v = 3$, which corresponds to a 60% annual volatility for the liquidation value, and $\tau_v = 25$ which corresponds to a 20% annual volatility). With this set of parameters, we solve for the equilibrium of the market and compute traders' payoffs and $TW(\mu; \tau_\eta)$, for $\mu \in \{0.1, 0.2, \ldots, 1\}$ and $\tau_\eta \in \{0.1, 25, 50, 75, 100\}$.

Regarding the welfare ranking with multiplicity, we find that whenever multiple equilibria obtain, traders' payoffs (that is (A.61), and (A.64)) are complex valued functions, which prevents obtaining a general welfare ranking result across equilibria. Turning now to the welfare properties of the unique equilibrium, our numerical simulations yield the following result:

Numerical Result 1. *When a unique equilibrium obtains, $TW(\mu; \tau_\eta)$ is increasing in μ and τ_η. The total welfare improvement is driven by the increase of CE_t^H, $t = 1, 2$, with μ and τ_η. CE^{RD} decreases with τ_η as well as CE^D when τ_η is not too small and CE^D decreases in μ.*

Therefore, policies aimed at increasing market transparency and/or increase the mass of dealers who are always in the market to supply liquidity, achieve a higher total welfare by effecting a welfare transfer from liquidity providers to liquidity consumers.

6 Concluding remarks

We analyse a two-period market in which a risky security is traded by dealers that are continuously in the market and traders hedging an endowment shock. We show that the properties of the market equilibrium crucially depend on the information environment. With full transparency, second period traders perfectly observe the order flow. This allows them to take a contrarian position against first period liquidity traders' second period order–in this way defacto providing liquidity to them. In this case we show that traders' demand for liquidity is a decreasing function of the cost of trading it induces–that is, illiquidity works as a *rationing* device. Additionally, a unique equilibrium obtains. A deterioration of second period traders' information impairs these traders' ability to supply liquidity via contrarian orders. This reduces the risk-bearing capacity of the market and can increase market fragility. With market opacity, the model displays strategic complementarity and multiple equilibria with different levels of market depth. Additionally, a larger cost of trading leads traders to demand and consume more liquidity. Thus, our model predicts that market opacity can make markets fragile and jam the rationing function of illiquidity.[29] We also find that for intermediate levels of market transparency, increasing the mass of dealers that are continuously in the market has

[29]The importance of equal access to market information for market stability is also underlined in a recent opinion paper by PIMCO on the ways to improve the resiliency of the US Treasury market. "[I]n our view, an effective all-to-all platform for Treasuries would function similarly to a utility and would 1) include all legitimate, professional market participants; 2) require that participants trade under the same rules with the same access to price, information, etc.. . . "

a non-monotonic effect on strategic complementarity and can be destabilizing. However, for high enough transparency we have always equilibrium uniqueness and increasing both μ and τ_η increases total welfare. This offers an economic justification to policies aimed at enhancing access to order flow information such as the ones recently pursued by the SEC for the US Treasury market.[30]

Our model provides a plausible explanation for several recent events in which market liquidity "crashes" in the absence of any observable change in the value of the risky asset. In these events, it looks as if traders chased liquidity while liquidity suppliers withdrew from the market. We argue that opacity of the trading process can be the responsible for this type of effect, as it can severely impair the market participation of "non-standard" liquidity suppliers.[31]

Our model is also consistent with the narrative of the impact of the COVID pandemic on the illiquidity of the US Treasury market in March 2020. On March 12, 2020, the World Health Organization declared COVID-19 to be a global pandemic and liquidity deteriorated in the US Treasury market, with spreads increasing ten folds compared to their normal level and depth virtually disappearing at times (Duffie (2023)). In our setup, a similar effect is explained because market opacity impairs the ability of second period traders to speculate against the first period endowment shock, which reduces the risk bearing capacity of the market.[32]

Finally, our model predicts that when the market is fragile, trading costs are heterogeneous across different cohorts of investors. Specifically, the investors paying most for liquidity are those that consume more of it.

Our model is in line with the calls to increase post-trade price transparency in the US Treasury market[33] and offers an additional argument in support of the introduction of a "consolidated tape" in the EU stock markets. Indeed, the level of stock market fragmentation in the EU is higher than in the US. However, differently from their US peers, traders in the EU cannot rely on a common signal displaying the best quotes available across trading venues. To obtain such a "consolidated" market view, they need to piece together the more expensive feeds offered by each exchange, which creates a suboptimal two-tiered market (Cespa and Foucault (2013); Brogaard et al. (2021)). In an attempt to level the playing field, the European Commission is seeking to introduce the supply of a consolidated tape, at a reasonable price. However, this effort is facing fierce resistance from exchanges.[34] Such resistance is likely to

[30] See, e.g. SEC moves to unmask high-speed traders in Treasury bond market, Financial Times, March 2022.

[31] In a somewhat related manner Menkveld and Yueshen (2019) attribute the flash-crash of May 6, 2010 to the fleeing of cross-market arbitrageurs from the E-mini market, which considerably curtailed the liquidity supplied to that market during the event.

[32] Duffie (2023) argues that on a typical day, the illiquidity of the US Treasury market is well explained by treasuries' yield volatility but that dealers' balance sheet capacity, when constrained, acquires explanatory power. Indeed, facing a constrained balance sheet, dealers become less willing to trade with investors and among themselves, which impacts the risk bearing capacity of the market.

[33] For example Duffie (2023) states that "Improving post-trade price transparency with the real-time publication of Treasuries transactions would also improve market intermediation capacity through a more efficient matching of specific types of trades to specific dealer balance sheets".

[34] We do not see the consolidated tape as a sure remedy against flash events. Indeed, the US market has had a tape since the introduction of RegNMS (even though, according to the CFTC and SEC (2010) report on

lower the transparency of the trading process which, through the lenses of our model, can have undesirable side effects on market stability.

the flash-crash during the crash traders questioned the reliability of market information and took a pause from trading). We view the availability of reliable and prompt market information as an important ingredient that can help reducing the likelihood of market disruption.

References

Aldrich, E., J. Grundfest, and G. Laughlin (2017). The flash crash: a new deconstruction. *Working Paper*.

Amihud, Y., H. Mendelson, and R. A. Wood (1990). Liquidity and the 1987 stock market crash. *Journal of Portfolio Management 16*(3), 65–69.

Anand, A., P. Irvine, A. Puckett, and K. Venkataraman (2013). Institutional trading and stock resiliency: Evidence from the 2007-2009 financial crisis. *Journal of Financial Economics 108*(3), 773–797.

Anand, A., C. Jotikasthira, and K. Venkataraman (2021). Mutual fund trading style and bond market fragility. *Review of Financial Studies 34*(6), 2993–3044.

Aquilina, M., B. Eyeles, J. Shao, and C. Ysusi (2018). How do participants behave during flash events? evidence from the UK equity market. *FCA, Reserach note*.

Bank of International Settlements (2017). The Sterling 'flash event' of 7 October 2016. *BIS Working Paper*.

Bao, J., M. O'Hara, and X. (Alex) Zhou (2018). The volcker rule and corporate bond market making in times of stress. *Journal of Financial Economics 130*(1), 95–113.

Bellia, M., K. Christensen, A. Kolokov, L. Pelizzon, and R. Renò (2022). High-frequency trading during flash crashes: walk of fame or hall of shame? *Working paper*.

Bernardo, A. E. and I. Welch (2004, 02). Liquidity and financial market runs. *The Quarterly Journal of Economics 119*(1), 135–158.

Biais, B., F. Declerck, and S. Moinas (2017). Who supplies liquidity, how and when? *Working Paper, Toulouse school of Economics*.

Brogaard, J., J. Brugler, and D. Rösch (2021). Competition and exchange data fees. *Working Paper*.

Brogaard, J., A. Carrion, T. Moyaert, R. Riordan, A. Shkilko, and K. Sokolov (2018). High frequency trading and extreme price movements. *Journal of Financial Economics 128*, 253–265.

Brogaard, J., T. Hendershott, and R. Riordan (2014). High-frequency trading and price discovery. *Review of Financial Studies 27*(8), 2267–2306.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*(6), 2201–2238.

Budish, E., P. Cramton, and J. Shim (2015). The high-frequency trading arms race: frequent batch auctions as a market design response. *Quarterly Journal of Economics 130*, 1547–1621.

Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading volume and serial correlation in stock returns. *The Quarterly Journal of Economics 108*(4), 905–939.

Cespa, G. and T. Foucault (2013). Sale of price information by exchanges: Does it promote price discovery? *Management Science 60*(1), 148–165.

Cespa, G. and T. Foucault (2014). Illiquidity contagion and liquidity crashes. *Review of Financial Studies 27*(6), 1615–1660.

Cespa, G., A. Gargano, S. J. Riddiough, and L. Sarno (2021). Foreign Exchange Volume. *The Review of Financial Studies 35*(5), 2386–2427.

Cespa, G. and X. Vives (2015). The Beauty Contest and short-term trading. *Journal of Finance 70*(5), 2099–2154.

Cespa, G. and X. Vives (2019). Market transparency and fragility. *Working Paper*.

Cespa, G. and X. Vives (2022). Exchange competition, entry, and welfare. *Review of Financial Studies 35*(5), 2570–2624.

CFTC and SEC (2010). Findings regarding the market events of may 6, 2010. *Staff Report*.

Ding, S., J. Hanna, and T. Hendershott (2014). How slow is the NBBO? a comparison with direct exchange feeds. *Financial Review* (49), 313–332.

Duffie, D. (2020). Still the world's safe heaven? Redesigning the U.S. treasury market after the COVID-19 crisis. *Utchins Center, Working paper #62, Brookings institution*.

Duffie, D. (2023). Resilience redux in the US Treasury market. *Working paper*.

Duffie, D., T. Foucault, L. Veldkamp, and X. Vives (2022). *Technology and finance: the future of banking report.* CEPR London.

Easley, D., M. O'Hara, and M. López de Prado (2011). The microstructure of the Flash-Crash: flow toxicity, liquidity crashes, and the probability of informed trading. *Journal of Portfolio Management 37*, 118–128.

Gennotte, G. and H. Leland (1990). Market liquidity, hedging, and crashes. *American Economic Review 80*(5), 999–1021.

Glosten, L. and P. Milgrom (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics 14*, 71–100.

Goldstein, I., A. Kopytov, L. Shen, and H. Xiang (2021). On ESG investing: Heterogeneous preferences, information, and asset prices. *Working Paper*.

Goldstein, I. and L. Yang (2015). Information diversity and complementarities in trading and information acquisition. *Journal of Finance 70*, 1723–1765.

Gromb, D. and D. Vayanos (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics 66*(2-3), 361–407.

He, H. and J. Wang (1995). Differential information and dynamic behavior of stock trading volume. *8*(4), 919–972.

Jacklin, C. J., A. W. Kleidon, and P. Pfleiderer (1992). Underestimation of portfolio insurance and the crash of october 1987. *Review of Financial Studies 5*(1), 35–63.

Karolyi, G., K.-H. Lee, and M. van Dijk (2012). Understanding commonality in liquidity around the world. *Journal of Financial Economics 105*(1), 82–112.

Li, S., X. Wang, and M. Ye (2021). Who provides liquidity, and when? *Journal of Financial Economics 141*, 968–980.

Llorente, G., R. Michaely, G. Saar, and J. Wang (2002). Dynamic volume-return relation of individual stocks. *Review of Financial Studies 15*(4), 1005–1047.

Madrigal, V. and J. A. Scheinkman (1997). Price crashes, information aggregation, and market-making. *Journal of Economic Theory 75*(1), 16–63.

Menkveld, A. J. and B. Z. Yueshen (2019). The flash-crash: a cautionary tale about highly fragmented markets. *Management Science 10*, 4470–4488.

SEC (2010). Concept release on equity market structure. *Federal Register*.

Subrahmanyam, A. and S. Titman (1999). The going-public decision and the development of financial markets. *The Journal of Finance 54*(3), 1045–1082.

Vayanos, D. and J. Wang (2012). Liquidity and asset returns under asymmetric information and imperfect competition. *Review of Financial Studies 25*(5), 1339–1365.

Vives, X. (2008). *Information and learning in markets: the impact of market microstructure.* Princeton University Press.

Wang, J. (1994). A model of competitive stock trading volume. *Journal of Political Economy 102*(1), 2099–2154.

A Appendix A

The following is a standard results (see, e.g. Vives (2008), Technical Appendix, pp. 382–383) that allows us to compute the unconditional expected utility of market participants.

Lemma 1. *Let the n-dimensional random vector $z \sim N(0, \Sigma)$, and $w = c + b'z + z'Az$, where $c \in \mathbb{R}$, $b \in \mathbb{R}^n$, and A is a $n \times n$ matrix. If the matrix $\Sigma^{-1} + 2\rho A$ is positive definite, and $\rho > 0$, then*

$$E[-\exp\{-\rho w\}] = -|I + 2\rho\Sigma A|^{-1/2} \exp\{-\rho(c - \rho b'(\Sigma + 2\rho A)^{-1} b)\}.$$

We now derive the equilibrium for the general case in which $\tau_\eta \in (0, \infty)$ and $\mu \in (0, 1]$ that we discuss in Section 5.2. The two benchmarks with full transparency ($\mu = 1$ and $\tau_\eta \to \infty$) and full opacity ($\mu = 1$ and $\tau_\eta \to 0$) obtain as special cases of this result.

Proposition A.1. *When $\mu \in (0, 1]$ and $\tau_\eta \in (0, \infty)$, at a linear equilibrium:*

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22}\eta \tag{A.1a}$$

$$p_1 = -\Lambda_1 u_1 \tag{A.1b}$$

where the coefficients in the above expressions obtain as a solution to the following system of non-linear, simultaneous equations:

$$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v} \tag{A.2a}$$

$$\Lambda_{21} = -\frac{b + a_{21} + (1-\mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v} \tag{A.2b}$$

$$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v} \tag{A.2c}$$

$$\Lambda_1 = -\frac{\mu\gamma + \gamma_H}{\gamma\gamma_H\tau_v} a_1, \tag{A.2d}$$

and expressions for a_2, b, a_{21}, and a_1 are respectively given in (A.6), (A.15), and (A.25). At equilibrium, $\Lambda_2 > 0, \Lambda_{21} > \Lambda_1 > 0$, and $\Lambda_{22} < 0$.

Proof. Based on the market clearing condition (3), to pin down p_2 we need the strategies of first and second period traders, and dealers. We work by backward induction. In the second period, CARA and normality assumptions imply that the objective function of a liquidity trader is given by

$$E_2[-\exp\{-\pi_2/\gamma_H\}] = -\exp\left\{-\frac{1}{\gamma_H}\left(E_2[\pi_2] - \frac{1}{2\gamma_H}\mathrm{Var}_2[\pi_2]\right)\right\}, \tag{A.3}$$

where $\pi_2 \equiv (v-p_2)x_2+u_2v$. Maximizing (A.56) with respect to x_2, and solving for the optimal strategy yields:

$$x_2 = \gamma_H \frac{E_2[v-p_2]}{\mathrm{Var}_2[v-p_2]} - \frac{\mathrm{Cov}_2[v-p_2, v]}{\mathrm{Var}_2[v-p_2]} u_2, \tag{A.4}$$

where,

$$E_2[v-p_2] = \Lambda_2 u_2 + \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{\tau_\eta+\tau_u} s_{u_1} \tag{A.5a}$$

$$\mathrm{Var}_2[v-p_2] = \frac{1}{\tau_v} + \frac{(\Lambda_{21}-\Lambda_{22})^2}{\tau_\eta+\tau_u} \tag{A.5b}$$

$$\mathrm{Cov}_2[v-p_2, v] = \frac{1}{\tau_v}. \tag{A.5c}$$

Substituting (A.5a) and (A.5c) in (A.4), and rearranging yields:

$$X_2(u_2, s_{u_1}) = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \mathrm{Var}_2[v-p_2]}}_{a_2} u_2 + \underbrace{\gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta+\tau_u)\mathrm{Var}_2[v-p_2]}}_{b} s_{u_1}. \tag{A.6}$$

A dealer maximizes the expected utility of his second period wealth:

$$\begin{aligned} &E_2^D\left[-\exp\left\{-\frac{1}{\gamma}\Big((p_2-p_1)x_1^D + (v-p_2)x_2^D\Big)\right\}\right] = \\ &= \exp\left\{-\frac{1}{\gamma}(p_2-p_1)x_1^D\right\}\left(-\exp\left\{-\frac{1}{\gamma}\left(E_2^D[v-p_2]x_2^D - \frac{(x_2^D)^2}{2\gamma}\mathrm{Var}_2^D[v-p_2]\right)\right\}\right). \end{aligned} \tag{A.7}$$

For given x_1^D the above is a concave function of the second period strategy x_2^D. Solving the first order condition, yields that a second period D's strategy is given by:

$$X_2^D(p_1, p_2) = \gamma \frac{E_2^D[v-p_2]}{\mathrm{Var}_2^D[v-p_2]}. \tag{A.8}$$

Computing expectation and variance in the above expression:

$$E_2^D[v-p_2] = -p_2 \tag{A.9a}$$

$$\mathrm{Var}_2^D[v-p_2] = \frac{1}{\tau_v}, \tag{A.9b}$$

and substituting these in x_2^D yields:

$$X_2^D(p_1, p_2) = -\gamma\tau_v p_2. \tag{A.10}$$

Similarly, due to CARA and normality, in the first period a RD maximizes

$$E_1^{RD}\left[-\exp\left\{-\frac{1}{\gamma}(v-p_1)x_{11}^{RD}\right\}\right] = \\ -\exp\left\{-\frac{1}{\gamma}\left(E_1^{RD}[v-p_1]x_{11}^{RD} - \frac{(x_{11}^{RD})^2}{2\gamma}\text{Var}_1^{RD}[v-p_1]\right)\right\}. \tag{A.11}$$

Maximizing the above and solving for x_{11}^{RD} yields:

$$x_{11}^{RD}(p_1) = \gamma\frac{E_1^{RD}[v-p_1]}{\text{Var}_1^{RD}[v-p_1]}. \tag{A.12}$$

Computing the conditional expectation and variance:

$$E_1^{RD}[v-p_1] = -p_1 \tag{A.13a}$$

$$\text{Var}_1^{RD}[v-p_1] = \frac{1}{\tau_v}, \tag{A.13b}$$

so that

$$X_1^{RD}(p_1) = -\gamma\tau_v p_1. \tag{A.14}$$

At the second round, first and second period traders face the same utility maximization problem. This is because they both need to hedge the endowment shock, and have only one round to go. As a consequence, a first period trader's strategy reads as follows:

$$\begin{aligned} X_{21}(u_1) &= \gamma_H\frac{E_1[v-p_2]}{\text{Var}_1[v-p_2]} - \frac{\text{Cov}_1[v, v-p_2]}{\text{Var}_1[v-p_2]}u_1 \\ &= \underbrace{\frac{\gamma_H\Lambda_{21}\tau_v - 1}{\tau_v\text{Var}_1[v-p_2]}}_{a_{21}}u_1, \end{aligned} \tag{A.15}$$

where

$$\text{Var}_1[v-p_2] = \frac{1}{\tau_v} + \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta}. \tag{A.16}$$

Substituting (A.6), (A.10), (A.14), and (A.15) in (3), solving for p_2 and identifying the equilibrium price coefficients yields:

$$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v} \tag{A.17a}$$

$$\Lambda_{21} = -\frac{b + a_{21} + (1-\mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v} \tag{A.17b}$$

$$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v} \tag{A.17c}$$

According to (A.17a), at an equilibrium

$$\Lambda_2 = \frac{1}{(\gamma_H + \mu\gamma\tau_v \mathrm{Var}_2[v-p_2])\tau_v}, \tag{A.18}$$

so that at equilibrium $\Lambda_2 > 0$, and $\gamma_H \Lambda_2 \tau_v < 1$. Based on the expression for a_2 in (A.6), this implies that

$$a_2 \in (-1, 0). \tag{A.19}$$

To obtain the first period equilibrium price, we need to pin down the expressions for dealers' and liquidity traders' first period strategies. Starting from the latter, we obtain the second period value function of a first period trader by substituting (A.15) into the trader's objective function:

$$E_1[-\exp\{-((v-p_2)x_{21}+vu_1)/\gamma_H\}] = -\exp\{-(\mathrm{Var}_1[v-p_2]x_{21}^2 - \mathrm{Var}[v]u_1^2)/2\gamma_H^2\}. \tag{A.20}$$

As a consequence, at the first round, the trader's objective function reads as follows:

$$\begin{aligned} & E_1[-\exp\{-\pi_1/\gamma_H\}] \\ &\quad = E_1[-\exp\{-((p_2-p_1)x_{11} + (\mathrm{Var}_1[v-p_2]a_{21}^2u_1^2 - \mathrm{Var}[v]u_1^2)/2\gamma_H)/\gamma_H\}] \\ &\quad = E_1[-\exp\{-((p_2-p_1)x_{11} + \underbrace{((\mathrm{Var}_1[v-p_2]a_{21}^2 - \mathrm{Var}[v])/2\gamma_H)}_{C} u_1^2)/\gamma_H\}], \end{aligned} \tag{A.21}$$

where $\pi_1 = vu_1 + (v-p_2)x_{21} + (p_2-p_1)x_{11}$. Using the expression for p_2 in (A.54), the argument of the exponential in the latter expression of (A.21) can be written as follows:

$$(p_2-p_1)x_{11} + Cu_1^2 = -(\Lambda_{21}-\Lambda_1)u_1x_{11} + Cu_1^2 - (\Lambda_2u_2 + \Lambda_{22}\eta)x_{11}, \tag{A.22}$$

which is a quadratic form of the normal random variable $Z \equiv -(\Lambda_2u_2+\Lambda_{22}\eta)|u_1 \sim N(0, \mathrm{Var}_1[p_2-p_1])$ (the constant multiplying the squared term of Z in the quadratic form is in this case null), where

$$\mathrm{Var}_1[p_2-p_1] = \mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}. \tag{A.23}$$

We can then write the objective function of a trader at the first round as follows:

$$E[-\exp\{-\pi_1/\gamma_1\}|u_1] = -\exp\{-(-(\Lambda_{21}-\Lambda_1)u_1x_{11}+Cu_1^2-x_{11}^2\mathrm{Var}_1[p_2-p_1]/2\gamma_H)/\gamma_H\}. \tag{A.24}$$

Maximizing the above function with respect to x_{11} yields

$$X_{11}(u_1) = \underbrace{-\gamma_H\frac{\Lambda_{21}-\Lambda_1}{\mathrm{Var}_1[p_2]}}_{a_1} u_1. \tag{A.25}$$

We now obtain the strategy of a liquidity provider. Substituting a D's second period strategy (A.57) in (A.7), rearranging and applying Lemma 1 yields the following expression for the first period objective function of a D:

$$E_1^D[U((p_2-p_1)x_1^D+(v-p_2)x_2^D)] = -\left(1+\frac{\mathrm{Var}_1^D[p_2]}{\mathrm{Var}[v]}\right)^{-1/2}\times \tag{A.26}$$
$$\exp\left\{-\frac{1}{\gamma}\left(\frac{\gamma\tau_v}{2}(E_1^D[p_2])^2+(E_1^D[p_2]-p_1)x_1^D-\frac{(x_1^D+\gamma\tau_v E_1^D[p_2])^2}{2\gamma}\left(\frac{1}{\mathrm{Var}_1^D[p_2]}+\frac{1}{\mathrm{Var}[v]}\right)^{-1}\right)\right\},$$

where

$$E_1^D[p_2] = -\Lambda_{21}u_1 \tag{A.27a}$$
$$\mathrm{Var}_1^D[p_2] = \frac{\Lambda_{21}^2}{\tau_u}+\frac{\Lambda_2^2}{\tau_\eta}. \tag{A.27b}$$

Maximizing (A.26) with respect to x_1^D and solving for the first period strategy yields

$$\begin{aligned} x_1^D(p_1) &= \frac{\gamma}{\mathrm{Var}_1^D[p_2]}E_1^D[p_2]-\gamma\left(\frac{1}{\mathrm{Var}_1^D[p_2]}+\frac{1}{\mathrm{Var}[v]}\right)p_1 \\ &= -\gamma\frac{\Lambda_{21}-\Lambda_1}{\mathrm{Var}_1^D[p_2]}u_1-\gamma\tau_v p_1. \end{aligned} \tag{A.28}$$

Comparing (A.28) with (A.25) shows that in this model at the first round D and liquidity traders submit the same type of market order. That is, we can think of the strategy of a liquidity trader as being similar to the "directional bet" part of the D strategy (more on this in section 2).

Substituting (A.14), (A.25) and (A.28) into the first period market clearing condition (2) and identifying the equilibrium price coefficient yields:

$$\Lambda_1 = -\frac{\mu\gamma+\gamma_H}{\gamma\gamma_H\tau_v}a_1. \tag{A.29}$$

We have already signed Λ_2. To sign the remaining price coefficients, we substitute the expressions for the strategy coefficients into (A.17b), (A.53), and (A.29), obtaining:

$$\Lambda_{21} \tag{A.30a}$$
$$= \frac{(\tau_u+\tau_\eta)\mathrm{Var}_2[v-p_2]-(\gamma_H\Lambda_{22}\tau_u+(\tau_u+\tau_\eta)(1-\mu)\gamma\tau_v\Lambda_1\mathrm{Var}_2[v-p_2])\tau_v\mathrm{Var}_1[v-p_2]}{\gamma_H\tau_v\tau_\eta\mathrm{Var}_1[v-p_2]+(\tau_u+\tau_\eta)(\gamma_H+\mu\gamma\tau_v\mathrm{Var}_1[v-p_2])\tau_v\mathrm{Var}_2[v-p_2]}$$

$$\Lambda_{22} = -\frac{\gamma_H\Lambda_{21}\tau_\eta}{\mu\gamma\tau_v(\tau_u+\tau_\eta)\mathrm{Var}_2[v-p_2]+\gamma_H\tau_u} \tag{A.30b}$$
$$\Lambda_1 = \frac{(\mu\gamma+\gamma_H)\Lambda_{21}\tau_u\tau_\eta}{(\mu\gamma+\gamma_H)\tau_u\tau_\eta+(\Lambda_{22}^2\tau_u+\Lambda_2^2\tau_\eta)\gamma\tau_v}. \tag{A.30c}$$

Note that from (A.30c), the sign of Λ_{21} coincides with that of Λ_1. Now, suppose that $\Lambda_{21} \leq 0$, then this implies that $\Lambda_1 \leq 0$. However, because of (A.30a), we then have that $\Lambda_{21} > 0$, which is a contradiction. Once we have signed Λ_{21}, because of (A.30b), we know that $\Lambda_{22} < 0$, and by computing $\Lambda_{21} - \Lambda_1$ with (A.30c), we obtain $\Lambda_{21} - \Lambda_1 > 0$. □

Proof of Proposition 1

We prove here that that when second period traders observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$), the equilibrium obtained in Proposition 3, is unique. Note that this assumption has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period traders' is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second period price only reflects endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1,$$

and, using (A.4), second period traders' position reads as follows:

$$\begin{aligned} x_2 &= \gamma_H \frac{E_2[v - p_2]}{\mathrm{Var}_2[v - p_2]} - \frac{\mathrm{Cov}_2[v, v - p_2]}{\mathrm{Var}_2[v - p_2]} u_2 \\ &= \underbrace{(\gamma_H \tau_v \Lambda_2 - 1)}_{= a_2} u_2 + \underbrace{\gamma_H \tau_v \Lambda_{21}}_{= b} u_1, \end{aligned} \tag{A.31}$$

where we note that since second period traders perfectly observe u_1, $\mathrm{Var}_2[v - p_2] = \tau_v^{-1}$. First period traders, trading at the second round, can only anticipate the impact of u_1 on p_2. Thus, using (A.15), we obtain:

$$\begin{aligned} x_{21} &= \gamma_H \frac{E_1[v - p_2]}{\mathrm{Var}_1[v - p_2]} - \frac{\mathrm{Cov}_1[v, v - p_2]}{\mathrm{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}}_{= a_{21}} u_1. \end{aligned} \tag{A.32}$$

The strategy for D is as in (A.10), so that plugging it in the second period market clearing condition yields:

$$x_2^D + x_2 + x_{21} = 0 \iff p_2 = \underbrace{\frac{a_2}{\gamma \tau_v}}_{= -\Lambda_2} u_2 + \underbrace{\frac{b + a_{21}}{\gamma \tau_v}}_{= -\Lambda_{21}} u_1. \tag{A.33}$$

Based on the above, we can immediately identify the second period price impact coefficients:

$$\Lambda_2 = \frac{1}{(\gamma + \gamma_H)\tau_v} \tag{A.34a}$$

$$\Lambda_{21} = \frac{\tau_u}{((\gamma + \gamma_H)(\tau_u + \Lambda_2^2\tau_v) + \gamma_H\tau_u)\tau_v}. \tag{A.34b}$$

Finally, turning to the first period market, we have the following expression for the market clearing equation:

$$x_1^D + x_{11} = 0.$$

Replacing the expressions for traders and dealers' strategies (see, respectively (A.25), (A.28), and (A.14)), taking the limit for $\tau_\eta \to \infty$ and identifying the endowment shock price coefficient yields

$$p_1 = \underbrace{\frac{(\gamma + \gamma_H)\tau_u\Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}}_{= -\Lambda_1} u_1. \tag{A.35}$$

The equilibrium is uniquely pinned down by the solution to the linear system given by the expressions for the price coefficients of u_1 at the two trading rounds:

$$\Lambda_1 = \frac{\tau_u^2\tau_v(\gamma + \gamma_H)^4}{\tau_u\tau_v\left(2\gamma^2 + 4\gamma\gamma_H + \gamma_H^2\right)(\gamma + \gamma_H) + \tau_u^2\tau_v^2(\gamma + 2\gamma_H)(\gamma + \gamma_H)^4 + \gamma} \tag{A.36a}$$

$$\Lambda_{21} = \frac{\tau_u(\gamma + \gamma_H)\left(\tau_u\tau_v(\gamma + \gamma_H)^3 + \gamma\right)}{\tau_u\tau_v\left(2\gamma^2 + 4\gamma\gamma_H + \gamma_H^2\right)(\gamma + \gamma_H) + \tau_u^2\tau_v^2(\gamma + 2\gamma_H)(\gamma + \gamma_H)^4 + \gamma}, \tag{A.36b}$$

which possesses the unique solution illustrated in the text of the proposition. The ranking across the price impact coefficients follows immediately from their comparison.

□

Proof of Proposition 2

We obtain the equilibrium in the case with full opacity by setting $\mu = 1$ and taking the limit for $\tau_\eta \to 0$ of the equilibrium price coefficients obtained in the proof of Proposition 3.

Starting from Λ_{22}:

$$\Lambda_{22} = \lim_{\tau_\eta \to 0} -\frac{\gamma_H\Lambda_2\Lambda_{21}\tau_v\tau_\eta}{\tau_u + (1 - \gamma_2\tau_v\Lambda_2)} = 0. \tag{A.37a}$$

Based on (A.37a) we then have

$$\begin{aligned}\Lambda_2 = \lim_{\tau_\eta \to 0} \frac{1}{(\gamma_H + \gamma\tau_v\mathrm{Var}_2[v - p_2])\tau_v} &= \frac{\tau_u}{((\mu\gamma + \gamma_H)\tau_u + \gamma\tau_v(\Lambda_{21} - \Lambda_{22})^2)\tau_v} \\ &= \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v}\end{aligned} \tag{A.37b}$$

and

$$
\begin{aligned}
&\Lambda_{21} \\
&= \lim_{\tau_\eta \to 0} -\frac{(\tau_v \mathrm{Var}_1[v-p_2])^{-1}(\gamma_H \tau_v \Lambda_{21} - 1) + \gamma_H((\tau_u+\tau_\eta)\mathrm{Var}_2[v-p_2])^{-1}(\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u)}{\mu\gamma\tau_v} \\
&= -\frac{(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u}{(\tau_u + \Lambda_2^2 \tau_v)\gamma\tau_v}. \qquad \text{(A.37c)}
\end{aligned}
$$

Also,

$$
\begin{aligned}
\lim_{\tau_\eta \to 0} \frac{\Lambda_{22}^2}{\tau_\eta} &= \lim_{\tau_\eta \to 0} \left(\frac{\gamma_H \Lambda_2 \Lambda_{21} \tau_v}{(\tau_u/\tau_\eta^{1/2}) + (1 - \gamma_H \tau_v \Lambda_2)\tau_\eta^{1/2}} \right)^2 \\
&= 0,
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\Lambda_1 &= \lim_{\tau_\eta \to 0} \frac{(\gamma_H + \gamma)\tau_u \Lambda_{21}}{\gamma_H \tau_u + \gamma(((\Lambda_{22}^2/\tau_\eta)\tau_u + \Lambda_2^2)\tau_v + \tau_u)} \qquad \text{(A.37d)} \\
&= \frac{(\gamma_H + \gamma)\tau_u \Lambda_{21}}{\gamma_H \tau_u + \gamma(\Lambda_2^2 \tau_v + \tau_u)}.
\end{aligned}
$$

Based on the limits (A.37a)-(A.37d), the coefficients of traders' strategies are given by

$$
\begin{aligned}
a_1 &= -\gamma_H \tau_u \frac{\Lambda_{21} - \Lambda_1}{\Lambda_2^2} < 0 && \text{(A.38a)} \\
a_{21} &= \tau_u \frac{\gamma_H \tau_v \Lambda_{21} - 1}{\tau_u + \Lambda_2^2 \tau_v} \in (-1, 0) && \text{(A.38b)} \\
a_2 &= \tau_u \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_u + \Lambda_{21}^2 \tau_v} \in (-1, 0) && \text{(A.38c)} \\
b &= 0. && \text{(A.38d)}
\end{aligned}
$$

Additionally, an equilibrium is pinned down by solving the following system of simultaneous equations:

$$
\begin{aligned}
\Lambda_2 = \Phi_1(\Lambda_{21}) &\equiv \frac{\tau_u}{((\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v} && \text{(A.39a)} \\
\Lambda_{21} = \Phi_2(\Lambda_2) &\equiv \frac{\tau_u}{((\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\tau_v} && \text{(A.39b)} \\
\Lambda_1 &= \frac{(\gamma+\gamma_H)\tau_u\Lambda_{21}}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}. && \text{(A.39c)}
\end{aligned}
$$

An equilibrium obtains via the solution of the system (A.39a)–(A.39b). Replacing (A.39a) into (A.39b) and rearranging yields:

$$\Lambda_{21} = \Phi_2(\Lambda_{21}) \equiv \frac{(\gamma\tau_u + (\gamma+\gamma_H)B^2\tau_v)B^2}{(\gamma+\gamma_H)(\gamma+\gamma_H)B^4\tau_v^2 + 2(\gamma+\gamma_H)\gamma B^2\tau_u\tau_v + \gamma^2\tau_u^2}, \tag{A.40}$$

where $B \equiv (\gamma+\gamma_H)\tau_u + \gamma\Lambda_{21}^2\tau_v$. Inspection of (A.40) reveals (i) that $\Phi_2(\Lambda_{21}) > 0$, (ii) that $\Phi_2(0) > 0$, and (iii) that $\Lambda_{21} - \Phi_2(\Lambda_{21})$ is proportional to a 9-the degree polynomial in Λ_{21}, which thus always possesses at least one positive root Λ_{21}^*. Recursive substitution of such root first in (A.39a) and then in (A.39c) allows to pin down the set of equilibrium coefficients for p_1 and p_2.

Comparison of (A.39c) and (A.39b) shows that $\Lambda_{21}, \Lambda_1 > 0$ and $\Lambda_1 < \Lambda_{21}$. To see this, suppose $\Lambda_{21} \leq 0$. Then, because of (A.39c), $\Lambda_1 \leq 0$. However, because of (A.39b) this implies that $\Lambda_{21} > 0$, contradicting the initial assumption. Next, using (A.39c)

$$\begin{aligned}\Lambda_{21} - \Lambda_1 &= \Lambda_{21} - \frac{(\gamma+\gamma_H)\tau_u\Lambda_{21}}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2} \\ &= \frac{\gamma\tau_v\Lambda_2^2\Lambda_{21}}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2},\end{aligned}$$

which is positive. □

Proof of Corollary 5

Divide (23a) by (23b) to obtain

$$\frac{\Lambda_2}{\Lambda_{21}} = \frac{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2}.$$

Rearranging the above, yields the following equation

$$(\Lambda_2 - \Lambda_{21})((\gamma+\gamma_H)\tau_u - \gamma\tau_v\Lambda_{21}\Lambda_2) = 0. \tag{A.41}$$

One solution to the above equation is $\Lambda_2 = \Lambda_{21}$ which, substituted into (23a) after rearranging yields the following cubic in Λ_2:

$$\varphi(\Lambda_2) \equiv ((\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\Lambda_2\tau_v - \tau_u, \tag{A.42}$$

which, since $\varphi(0) < 0$ and $\varphi'(\Lambda_2) > 0$, is easily seen to posses a unique, positive root. Suppose instead that $\Lambda_{21} \neq \Lambda_2$. In this case, for (A.41) to be satisfied, we need

$$\Lambda_{21}\Lambda_2 = \frac{(\gamma+\gamma_H)\tau_u}{\gamma\tau_v}. \tag{A.43}$$

Solving the above for Λ_{21} and substituting the result into (23a), yields the following quadratic in Λ_2:

$$(\gamma + \gamma_H)\gamma\tau_v\Lambda_2^2 - \gamma\Lambda_2 + (\gamma + \gamma_H)^2\tau_u = 0. \tag{A.44}$$

The roots of the equation are given by

$$\Lambda_2^{*,**} = \frac{\gamma \pm \sqrt{(\gamma - 4(\gamma + \gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_H)\gamma\tau_v}.$$

Both roots are positive, which implies that, provided

$$0 < \tau_u\tau_v < \frac{\gamma}{4(\gamma + \gamma_H)^3},$$

there are two additional equilibria of the model and the corresponding value of Λ_{21} obtains by substituting either root into (A.43). Finally, note that when

$$\frac{\gamma}{4(\gamma + \gamma_H)^3} \leq \tau_u\tau_v,$$

the quadratic (A.44) has either two identical solutions $\Lambda_2^* = \Lambda_2^{**} = \Lambda_2 = 1/(2(\gamma + \gamma_H)\tau_v)$, or does not have a real solution, and only the equilibrium with $\Lambda_{21} = \Lambda_2$ obtains. □

Proof of Corollary 6

To analyze the stability properties of the equilibrium in this case, we use the aggregate best response function (A.40) which for $\mu = 1$ has the following expression:

$$\Phi_2(\Lambda_{21}) = \frac{((\gamma + \gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2}{\gamma\tau_u + ((\gamma + \gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma + \gamma_H)\tau_v}. \tag{A.45}$$

1. First, based on the above expression, $\Phi_2(0) > 0$ and differentiating (A.45) with respect to Λ_2 yields:

$$\Phi_2'(\Lambda_{21}) = \frac{4((\gamma + \gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)\gamma^2\Lambda_{21}\tau_u\tau_v}{(\gamma\tau_u + ((\gamma + \gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma + \gamma_H)\tau_v)^2} > 0, \tag{A.46}$$

   implying that the best response is always upward sloping. Thus, with uniqueness $\Phi_2(\Lambda_{21})$ cuts the 45-degree line from "above" implying that the equilibrium is stable. When multiple equilibria arise, it instead crosses the 45-degree line at three points, with a slope smaller (larger) than one at the two extreme (intermediate) crossings, which correspond to the three equilibria of the market. Hence, with multiplicity, the two extreme equilibria are stable, while the intermediate one is unstable.

2. Second, evaluating the cubic (A.42) at the low and high roots of the quadratic (A.44)

yields

$$\varphi\left(\frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}\right)=\frac{\gamma-4\tau_u\tau_v(\gamma+\gamma_H)^3-\sqrt{\gamma\left(\gamma-4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}}{2\tau_v(\gamma+\gamma_H)^3}<0 \tag{A.47a}$$

$$\varphi\left(\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}\right)=\frac{\gamma-4\tau_u\tau_v(\gamma+\gamma_H)^3+\sqrt{\gamma\left(\gamma-4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}}{2\tau_v(\gamma+\gamma_H)^3}>0, \tag{A.47b}$$

for $0<\tau_u\tau_v<\gamma/(4(\gamma+\gamma_H)^3)$. Hence, when multiple equilibria arise, the roots of the quadratic equation (A.44) "straddle" the root of the cubic (A.42).

3. Third, taking the product of the two extreme equilibrium values for Λ_2 yields:

$$\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}\times\frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}=\frac{(\gamma+\gamma_H)\tau_u}{\gamma\tau_v}.$$

Thus, in view of the second expression in (A.9b), at a stable equilibrium we have either that the price reacts more to u_2 than to u_1, or the opposite. Additionally, because of (25), when p_2 reacts more to u_1 than to u_2, the market is also less liquid at the first round.

4. Fourth, evaluating a_2 at the two extreme equilibria, we obtain:

$$a_2|_{\Lambda_2=\Lambda_2^{***}}=-\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)}>a_2|_{\Lambda_2=\Lambda_2^{*}}=\frac{-\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)},$$

which always holds within the parameter restriction needed for multiple equilibria to obtain. Given the symmetry of the equilibrium solution, this result also implies that when second period traders consume more liquidity, first period traders consume less of it. Finally, replacing (25) and (A.43) in the expression for a_1 yields:

$$\begin{aligned}a_1&=-\gamma_H\frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2\tau_u^{-1}}\\&=-\gamma_H\frac{(1-(\gamma+\gamma_H)\tau_v\Lambda_{21})\gamma^2\tau_v^2\Lambda_{21}^3}{(\gamma+\gamma_H)^2\tau_u},\end{aligned} \tag{A.48}$$

implying that also at the first round, liquidity consumption increases in the cost of trading generated.

□

Proof of Corollary 8

To prove our claim, we need to show that when evaluated at the intermediate equilibrium (A.49) is negative:

$$\frac{\partial}{\partial \tau_u}\left(\frac{\partial \Phi(\Lambda_2)}{\partial \Lambda_2}\right) \tag{A.49}$$
$$= \frac{4\gamma^2\Lambda_2\tau_v^2\left(\gamma^3\Lambda_2^6\tau_v^3(\gamma+\gamma_H) - \gamma^2\Lambda_2^2\tau_u - 3\gamma\Lambda_2^2\tau_u^2\tau_v(\gamma+\gamma_H)^3 - 2\tau_u^3(\gamma+\gamma_H)^4\right)}{\left(\gamma^2\Lambda_2^4\tau_v^3(\gamma+\gamma_H) + 2\gamma\Lambda_2^2\tau_u\tau_v^2(\gamma+\gamma_H)^2 + \tau_u^2\tau_v(\gamma+\gamma_H)^3 + \gamma\tau_u\right)^3},$$

To do this, it suffices to check that the cubic that pins down the intermediate equilibrium (see (27b)), satisfies the following condition:

$$\varphi(\hat{\Lambda}_2) > 0, \text{for } \hat{\Lambda}_2 = \frac{1}{\gamma_H\tau_v},$$

which implies that $\Lambda_2^{**} < \hat{\Lambda}_2$. Next, verify that the numerator in (A.49)

$$\gamma^3\Lambda_2^6\tau_v^3(\gamma+\gamma_H) - \gamma^2\Lambda_2^2\tau_u - 3\gamma\Lambda_2^2\tau_u^2\tau_v(\gamma+\gamma_H)^3 - 2\tau_u^3(\gamma+\gamma_H)^4 < 0,$$

for $\Lambda_2 < \hat{\Lambda}_2$.

□

Proof of Corollary 11

1. We can interpret the model as one that endogenously yields persistence in noise trading shocks. To see this, note that the equilibrium prices can be written as follows:

   $$p_2 = -\Lambda_2\theta_2,\ p_1 = -\Lambda_1\theta_1,$$

   where $\theta_1 = u_1$, and

   $$\theta_2 = u_2 + \underbrace{\frac{\Lambda_{21}}{\Lambda_2}}_{\beta} u_1.$$

   The properties of the noise process are related to the equilibrium that obtains. That is, if

   $$\tau_u\tau_v \in (0, \gamma/(4(\gamma+\gamma_H)^3),$$

   then $\beta \lesseqgtr 1$ depending on which equilibrium obtains. If $\tau_u\tau_v \geq \gamma/(4(\gamma+\gamma_H)^3$, $\beta = 1$.

2. We now evaluate the expression for returns autocovariance at the equilibrium with full transparency (and $\mu = 1$):

   $$\text{Cov}[p_2 - p_1, p_1] = \frac{\gamma\tau_u^2\tau_v(\gamma+\gamma_H)^5}{\left(\tau_u\tau_v\left(2\gamma^2 + 4\gamma\gamma_H + \gamma_H^2\right)(\gamma+\gamma_H) + \tau_u^2\tau_v^2(\gamma+2\gamma_H)(\gamma+\gamma_H)^4 + \gamma\right)^2} \tag{A.50}$$

and at both the equilibria that obtain under the parameter restriction ensuring multiplicity, when the market is fully opaque, when $\Lambda_{21} = \Lambda_{21}^{*}$ we have

$$\text{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}\right)^3\left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}+\gamma\right)}{16\gamma^4\tau_u\tau_v^2(\gamma+\gamma_H)^2}, \tag{A.51}$$

and when $\Lambda_{21} = \Lambda_{21}^{***}$ we have instead

$$\text{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}\right)\left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}+\gamma\right)^3}{16\gamma^4\tau_u\tau_v^2(\gamma+\gamma_H)^2}. \tag{A.52}$$

Comparing the two latter expressions shows that return autocovariance is higher when $\Lambda_{21} = \Lambda_{21}^{*}$. While comparing the latter expression above with (A.50) shows that it increases with respect to the case with full transparency.

□

Proof of proposition 3

The result follows from Proposition A.1 setting $\mu = 1$. If we take the limit for $\tau_\eta \to \infty$ in (A.18), (A.30a), and (A.30c), we obtain the unique linear equilibrium of Proposition 1.

□

Proof of proposition 4

We start by obtaining an expression for the unconditional expected utility of RD and D. Because of CARA and normality, an RD conditional expected utility evaluated at the optimal strategy is given by

$$\begin{aligned} E[U((v-p_1)x_1^{RD})|p_1] &= -\exp\left\{-\frac{(E[v|p_1]-p_1)^2}{2\text{Var}[v]}\right\} \\ &= -\exp\left\{-\frac{\tau_v\Lambda_1^2}{2}u_1^2\right\}. \end{aligned} \tag{A.53}$$

Thus, RD derive utility from the expected, long-term capital gain obtained supplying liquidity to first-period hedgers.

$$\begin{aligned} EU^{RD} \equiv E\left[U\left((v-p_1)x_1^{RD}\right)\right] &= -\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}\right)^{-1/2} \\ &= -\left(\frac{\tau_{u_1}}{\tau_{u_1}+\tau_v\Lambda_1^2}\right)^{1/2}, \end{aligned} \tag{A.54}$$

and

$$CE^{RD} = \frac{\gamma}{2} \ln \left(1 + \frac{\text{Var}[p_1]}{\text{Var}[v]}\right). \tag{A.55}$$

Turning to D, replacing the optimal x_1^D in (A.26) and rearranging yields

$$E[U((p_2 - p_1)x_1^D + (v - p_2)x_2^D)|u_1] = -\left(1 + \frac{\text{Var}[p_2|u_1]}{\text{Var}[v]}\right)^{-1/2} \times \exp\left\{-\frac{g(u_1)}{\gamma}\right\}, \tag{A.56}$$

where

$$g(u_1) = \frac{\gamma}{2}\left(\frac{(E[p_2|p_1] - p_1)^2}{\text{Var}[p_2|p_1]} + \frac{(E[v|p_1] - p_1)^2}{\text{Var}[v]}\right).$$

The argument of the exponential in (A.56) is a quadratic form of the first-period endowment shock. We can therefore apply Lemma 1 and obtain

$$\begin{aligned} EU^D \equiv E[U((p_2 - p_1)x_1^D + (v - p_2)x_2^D)] = \\ = -\left(1 + \frac{\text{Var}[p_2|p_1]}{\text{Var}[v]}\right)^{-1/2}\left(1 + \frac{\text{Var}[p_1]}{\text{Var}[v]} + \frac{\text{Var}[E[p_2|p_1] - p_1]}{\text{Var}[p_2|p_1]}\right)^{-1/2}. \end{aligned} \tag{A.57}$$

Computing the certainty equivalent yields:

$$\begin{aligned} CE^D = \frac{\gamma}{2}\Bigg(\ln\left(1 + \frac{\text{Var}[E[v - p_1|p_1]]}{\text{Var}[v - p_1|p_1]} + \frac{\text{Var}[E[p_2 - p_1|p_1]]}{\text{Var}[p_2 - p_1|p_1]}\right) \\ + \ln\left(1 + \frac{\text{Var}[E[v - p_2|p_1, p_2]]}{\text{Var}[v - p_2|p_1, p_2]}\right)\Bigg). \end{aligned} \tag{A.58}$$

To obtain the expression for first period hedgers' payoff we replace the strategy (A.25) into the objective function (A.24) and rearrange the result, to obtain

$$E[-\exp\{-\pi_1/\gamma_H\}|u_1] = -E\left[\exp\left\{-\frac{u_1^2}{\gamma_H}\left(\frac{\gamma_H(\Lambda_1 - \Lambda_{21})^2}{2\text{Var}_1[p_2]} + \frac{a_{21}^2\tau_v\text{Var}_1[v - p_2] - 1}{2\gamma_H\tau_v}\right)\right\}\right]. \tag{A.59}$$

The argument in the above expression is a quadratic form of the normal random variable $u_1 \sim N(0, \tau_u^{-1})$. Thus, to compute the unconditional expectation of (A.59), we apply Lemma 1 to obtain

$$E[-\exp\{-\pi_1/\gamma_H\}] = -\left(\frac{\gamma_H^2\tau_u\tau_v}{\gamma_H^2\tau_u\tau_v - 1 + (a_1^2\text{Var}_1[p_2] + a_{21}^2\text{Var}_1[v - p_2])\tau_v}\right)^{1/2}. \tag{A.60}$$

To obtain the certainty equivalent, we compute

$$\begin{aligned} CE_1^H &= -\gamma_H \ln\left(-E[-\exp\{-\pi_1/\gamma_1\}]\right) \\ &= \frac{\gamma_H}{2}\ln\left(1 + \frac{(a_1^2\text{Var}_1[p_2] + a_{21}^2\text{Var}_1[v - p_2])\tau_v - 1}{\gamma_H^2\tau_u\tau_v}\right). \end{aligned} \tag{A.61}$$

To obtain the payoff of second period liquidity traders we proceed similarly by replacing their equilibrium strategy into their objective function:

$$E_2[-\exp\{-(1/\gamma_H)((v-p_2)x_2+vu_2)\}] = -\exp\left\{-\frac{1}{\gamma_H}\left(\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_H}\right)\right\}. \quad \text{(A.62)}$$

The argument of the exponential at the right hand side of the above expression is a quadratic form of the normally distributed random vector

$$\begin{pmatrix} x_2 \\ u_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underbrace{\begin{pmatrix} \mathrm{Var}[x_2] & a_2/\tau_u \\ a_2/\tau_u & \tau_u^{-1} \end{pmatrix}}_{\Sigma}\right).$$

Indeed, we have

$$\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_H} = \frac{1}{2\gamma_H}\begin{pmatrix} x_2 & u_2 \end{pmatrix}\underbrace{\begin{pmatrix} \mathrm{Var}_2[v-p_2] & 0 \\ 0 & -\mathrm{Var}[v] \end{pmatrix}}_{A}\begin{pmatrix} x_2 \\ u_2 \end{pmatrix}.$$

Applying again Lemma 1, we then have

$$\begin{aligned} E\left[-\exp\left\{-\frac{1}{\gamma_H}\left(\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_H}\right)\right\}\right] &= -\left|I+(2/\gamma_H)\Sigma A\right|^{-1/2} \qquad \text{(A.63)} \\ &= -\left(\frac{\gamma_H^4\tau_u^2\tau_v}{a_2^2\mathrm{Var}_2[v-p_2]+(\mathrm{Var}_2[v-p_2]\mathrm{Var}[x_2]+\gamma_H^2)(\gamma_H^2\tau_u\tau_v-1)\tau_u}\right)^{1/2}. \end{aligned}$$

Finally, the certainty equivalent obtains by computing

$$\begin{aligned} CE_2^H &= -\gamma_H \ln\left(-E[-\exp\{-\pi_2/\gamma_H\}]\right) \qquad \text{(A.64)} \\ &= \frac{\gamma_H}{2}\ln\left(1+\frac{a_2^2\mathrm{Var}_2[v-p_2]\tau_v-1}{\gamma_H^2\tau_u\tau_v}+\frac{b^2\mathrm{Var}[s_{u_1}](\gamma_H^2\tau_u\tau_v-1)}{\gamma_H^4\tau_u\tau_v}\right). \end{aligned}$$

□

B Comparative statics in the model with a positive mass of "Restricted dealers"

In this appendix we present the full set of comparative statics results about the effects on liquidity fragility of traders' risk aversion (γ_H), payoff volatility (τ_v^{-1}) and endowment shock dispersion (τ_u^{-1}) in the model with "Restricted dealers." In Figure 12, we partition the space $\mu \in (0, 1]$, $\tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium (three equilibria).

Consistently with what we have found in Corollary 5, an increase in γ_H or τ_v tends to reduce the chances of liquidity fragility (compare the areas below the blue curve in panel (a) and panels (c) and (d)). The effect of an increase in τ_u is more complicated. Recall that when the signal is not perfect second period traders (1) may speculate in the "wrong" direction and (2) use p_2 and the signal to predict u_1. With a higher τ_u there is less noise in the price, which reduces second period traders speculative intensity. When μ is close to 0, almost only second period traders provide liquidity at the second round, and the reduction in speculation by these traders has a large impact on overall risk sharing. Conversely, when μ is close to 1, almost only D provide liquidity at 2 and the reduction in speculation by 2nd period traders means that dealers have less liquidity traders to share risk with. In either case this increases liquidity fragility. For intermediate values of μ, D have a smaller exposure to the risky security, and the additional risk sharing provided by 2nd period traders is less important. In this case, the reduction in these traders' speculation rids the market of the "wrong" trades with a positive impact on liquidity fragility.[35]

[35]The comparative statics result for τ_v and γ_H align with intuition and are deferred to Appendix B.

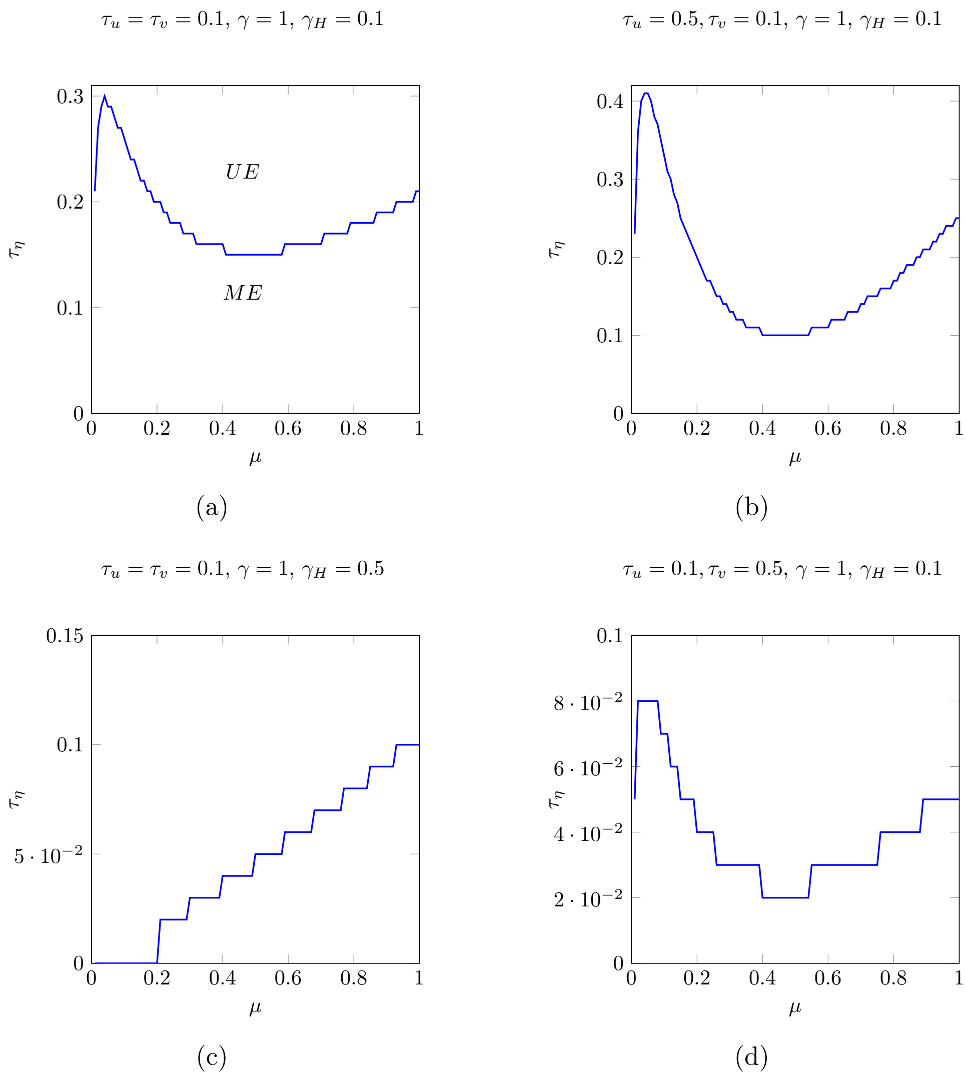


Figure 12: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.