Market opacity and fragility: Why liquidity evaporates when it is most needed*

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Abstract

Lack of market transparency can make liquidity demand upward sloping, inducing strategic complementarity and multiple equilibria, by impairing the liquidity provision of non-standard liquidity suppliers. Then an initial dearth of liquidity may degenerate into a liquidity rout (as in a "flash crash") and traders faced with the largest cost of trading are those trading more intensely at equilibrium. An increase in order flow transparency and/or in the mass of dealers who are in the market at all times has a positive impact on total welfare.

Keywords: Liquidity fragility, flash crash, strategic complementarity, order flow transparency.

JEL Classification Numbers: G10, G12, G14

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Introduction

Concern for the stability and resilience of financial markets has recently revived, in the wake of the sizeable number of "flash events" and other disruptions that have occurred in recent years.[1] Disrupted markets impair policy makers' ability to implement stabilizing macroeconomic policies, which compromises their capacity to pursue their mandate. The debate over the ultimate cause at the root of these episodes is still open. However, some consensus seems to have gathered around the hypothesis that they are related to the presumably overall liquidity and efficiency-enhancing process of "electronification" that has affected different types of securities' markets over the past two decades. The emergence of non-standard liquidity providers has accompanied this process, with new trading firms such as hedge funds and high frequency traders (HFTs), as major actors.[2] There's a suspicion that this process has occurred at the cost of increased fragility: small changes in market parameters may have large effects on liquidity.[3] At the same time, episodes of extreme market turbulence, where liquidity seems to inexplicably disappear and markets become somewhat inelastic have also occurred in the past. As the experience of the stock market crash of October 19, 1987 makes clear, (apparently) fundamentals-unrelated crashes have been a worrying, regular feature of financial markets.[4]

A unifying characteristic of these episodes seems to be the jamming of the "rationing" function of the cost of trading associated with market illiquidity. In "normal" market conditions, traders perceive a lack of liquidity as a cost, while arbitrageurs and liquidity suppliers regard it as an opportunity. Thus, an illiquidity hike leads the former to limit their demand for immediacy, and the latter to increase their supply of liquidity (i.e., the demand for and supply of liquidity, are respectively decreasing and increasing in the illiquidity of the market). In normal conditions, then, an illiquidity hike leads the net demand for a security to abate, producing a stabilizing effect on the market. However, on occasions, a bout of illiquidity, which often can hardly be construed as fundamentals-driven, has a destabilizing impact, and fosters a disorderly "run for the exit" that is conducive to a rout. In these cases, traders attempt to place orders *despite* the liquidity shortage, and arbitrageurs flee the market, foregoing profitable (but risky) opportunities. In such conditions, liquidity is fragile. What can account for such a dualistic feature of market illiquidity?

[1] A "flash event" is a situation in which market liquidity suddenly evaporates in conjunction with a rapid increase in liquidity demand and the occurrence of extreme price changes, typically in the absence of fundamentals news, over a short time interval. Flash events have hit different markets. Starting with the May 6, 2010 U.S. "flash-crash" (equity, centralized) where the Dow Jones Industrial Average dropped by 9% in the middle of the trading day, and partially recovered by the end of trading; moving to the October 15, 2014 Treasury Bond crash (bonds, mainly OTC), where the yield on the benchmark 10-year U.S. government bond, dipped 33 basis points to 1.86% and reversed to 2.13% by the end of the trading day; to end with the August 25, 2015 ETF market freeze (ETF and equity, centralized), during which more than a fifth of all U.S.-listed exchange traded funds and products were forced to stop trading. More evidence of flash events is provided by NANEX and Bank of International Settlements (2017).

[2] See "New titans of Wall Street: How trading firms stole a march on big banks", J. Franklin and C. Mourselas, *Financial Times, September 2024*.

[3] See Chapter 4 in Duffie et al. (2022).

[4] See https://en.wikipedia.org/wiki/List_of_stock_market_crashes_and_bear_markets and also Ian Domowitz's "Will the real market failure please stand up?" for an account of a 1962 flash-crash forerunner.

In this paper, we argue that lack of transparency about relevant market conditions is an important ingredient in the answer to this question. In current markets, trading automation arguably creates informational frictions by hampering some traders' access to reliable and timely market information (Ding et al. (2014)), thus impairing their ability to potentially enhance the risk-bearing capacity of the market. Furthermore, participation of some liquidity suppliers is variable (for technical or regulatory reasons).[5] Several accounts of the August 24, 2015 "flash-crash," point to the fact that uncertainty over the price of ETF constituents contributed to a huge investors' sellout, and sidelined the actions of arbitrageurs, exacerbating the liquidity dry-up in some ETFs.[6] In less automated markets, impaired access to market information arose because of different reasons.[7] The upshot is that accessibility to market information is vital to trade.

We use a stylized model of liquidity provision to show that, access to order flow information allows traders to supply liquidity via marketable orders, thereby improving the risk-bearing capacity of the market.[8] This is consistent with empirical evidence pointing to the importance of non-standard liquidity suppliers in modern markets (as collected in our literature review). Conversely, the absence of reliable order flow information limits the participation of non-standard liquidity providers, which can seriously dent the ability of a market to absorb risk, to the extent that, in extreme conditions, it can cause a market crash. This can happen due to an unexpected increase in uncertainty or dealer risk aversion, as during the COVID crisis. Therefore, our model has the potential to explain both liquidity "dry-ups" and "flash crashes." We do so without introducing any market irrationality in the form of noise trading or exogenous demand or supply. We also find that both an increase in market transparency and/or in the participation of liquidity providers who are continuously in the market, has a positive effect on total welfare. But, when market transparency is low, a small reduction in the mass of dealers continuously present in the market (say because of a cyberattack) may cause a liquidity crash.[9]

[5]Ding et al. (2014) argue that in the U.S. "[n]ot all market participants have equal access to trade and quote information. Both physical proximity to the exchange and the technology of the trading system contribute to the latency." In the EU the situation is possibly even worse, as testified by the lack of a consolidated tape in a market environment displaying an even higher degree of market fragmentation than in the US (see, e.g. European Commission progress update on action 14 of the capital markets union 2020 action plan. Action 14: Consolidated tape., see also EU faces a last-ditch challenge from exchanges over trading reforms, " *Financial Times, April 2023.*).

[6]In the morning of August 24, 2015, the Dow dropped roughly 1,100 points in the first five minutes of trading, and trading in several stocks was halted due to unusual market turbulence. The ensuing lack of reliable price information allowed profitable, but risky, arbitrage opportunities to go unexploited, leading to a widening of spreads and a thinning of market depth. For example, during the event, the spread between the SPDR S&P500 (SPY) and the Guggenheim S&P 500 Equal Weight ETF (RSP), two very similar ETFs whose prices are normally in sync, at one point reached $21 (see What The E-T-F Happened On August 24? Forbes, 28 August, 2015). In a similar vein, in their account of the May 10, 2010 "Flash Crash" Easley et al. (2011) state: "This generalized severe mismatch in liquidity was exacerbated by the withdrawal of liquidity by some electronic market makers and by uncertainty about, or delays in, market data affecting the actions of market participants.

[7]For example, in the 80s, access to the NYSE trading floor was crucial to have a good view of market conditions, but obviously constrained by physical limitations.

[8]A marketable (limit) order is a priced order with the limit price set at, or better than, the opposite side quote (bid price for sell orders and ask price for buy orders).

[9]See for example, Ransomware attack on ICBC disrupts trades in US Treasury market, *Financial Times, November 2023.*

Consider a market where overlapping cohorts of risk averse investors, such as portfolio managers or hedge funds, suffer endowment shocks and must rebalance their portfolios. In this context, each cohort has a different impact on the price, when trading the security. This happens in electronified modern markets with all-to-all trading where liquidity is provided not only by standard dealers but also by algorithmic traders and where all traders can supply liquidity, including typical liquidity demanders such as portfolio managers or hedge funds.[10] However, for the latter to be able to supply liquidity they need to be informed about the order imbalance in the market. If they are informed, then their orders offset the order imbalance (e.g., a buy order that balances out an observable selling pressure), which eliminates price risk and makes the price impact of the current investor cohort's endowment shock independent of the price impact of the endowment shock of the previous cohort of investors. If they are poorly informed about the order flow, then their orders may no longer balance out (e.g., a sell order in addition to an unobserved selling pressure), which increases price risk and makes the two price impacts related to the point that when one tends to increase, the other tends to decrease. Compounding these effects yields a source of strategic complementarity in the cost of transaction (illiquidity). Sufficient conditions for our results are overlapping cohorts of risk averse investors suffering endowment shocks, enough opacity about the order flow, and risk averse dealers. A necessary condition for multiple equilibria and fragility in general is that strategic complementarity is strong enough, and in our context, strategic complementarity is increasing in opacity. The effects may be reinforced by a lower mass of dealers continuously present on the market.

More specifically, when the current cohort of hedgers has good information on the past order flow (order imbalance), they can react to the hedging needs of previous cohorts and provide liquidity by speculating. This makes for a stable market. Furthermore, in this case, traders' demand for liquidity is a decreasing function of the cost of trading they face–that is, higher illiquidity discourages liquidity demand. However, when current hedgers have poor information, they can only speculate in a very limited way, and not at all under full opacity. In this case, the risk-bearing capacity of the market is reduced (only dealers can absorb the imbalance), and illiquidity may feed into itself and provoke market fragility. Suppose that the market impact of the current period hedgers' endowment shock increases. This reduces these traders' expected profit from hedging and increases the execution risk of previous hedgers' cohort, since they do not know the hedging needs of the current hedgers, leading them to scale down their liquidity demand. Other things equal, the scaling down reduces the price impact of the endowment shock of the previous cohort because dealers need to absorb a smaller share of their endowment shock. This, in turn, lowers the execution risk faced by the current hedgers' cohort, leading them to scale up their liquidity demand and further boosts the price impact of their endowment shock since dealers need to absorb a larger share of their endowment shock.[11] In this situation, a

[10] In Biais et al. (2017), at Euronext the hedgers are proprietary traders, in Hendershott et al. (2021) liquidity providers are non-standard, similarly to the equity markets analyzed by Hendershott et al. (2014) and Brogaard et al. (2014a) where they are algorithmic and high-frequency traders.

[11] This feedback loop between illiquidity and dealers' risk exposure, is reminiscent of the purported risk faced by dealers in US Treasury markets due to a potential increase in yield volatility (see Duffie (2020)).

larger cost of trading leads traders to demand more liquidity and *higher illiquidity incentivizes liquidity demand*.

The information friction creates strategic complementarity in price impacts (illiquidity). The source of strategic complementarity is the interaction between the price impact of the current and past endowment shocks. The composition of both impacts yields strategic complementarity because there is strategic substitutability between them when there is opacity. Under (full) transparency, the price impact of the current endowment shock is independent of the price impact of the past endowment shock since current hedgers do not face price risk. The complementarity is increasing in the degree of opacity. If it is high enough, and together with a low risk bearing capacity of dealers, high dispersion of the endowment shock of hedgers, and large security payoff volatility, then the market displays multiple equilibria, which can be fragile. This fragility is obtained even when strategic complementarity is not large enough to induce multiple equilibria. That is, a small change in a parameter may provoke large effects on liquidity with moderate but high enough strategic complementarity.

We obtain the above results in a two-period (trading rounds) model where competitive traders have CARA preferences over a risky security. The two cohorts of hedgers receive independent endowment shocks [12] and submit market orders to hedge their exposure. The first cohort has the opportunity to trade again during the second period.[13] Dealers are present in both periods posting limit orders. All random variables are normally distributed, and there are no noise traders (our model rationalizes an AR(1) process for noise trading where the persistence parameter is endogenous), and we solve for linear equilibria of the model. We also introduce a novel measure of total illiquidity which is appropriate when prices react to different endowment shocks (i.e., the endowment shocks' price impacts in period 2 differ).

Our model has the potential to explain liquidity "dry-ups" and "flash crashes." In a liquidity dry up, liquidity suddenly vanishes when demand for it surges rapidly, accompanied by extreme price fluctuations; it then reappears later on. A flash crash is similar, but everything, including recovery, happens over a short time interval. In our model, a liquidity dry-up happens when an unexpected shock substantially increases hedgers' endowment uncertainty (or the risk aversion of dealers) and introduces multiple equilibria making the market transit from a unique equilibrium with high liquidity to a low liquidity equilibrium. When the increased endowment uncertainty is removed or dealers recover their usual risk aversion (as in the unexpected shock caused by Covid and the unanticipated early arrival of vaccines) the original equilibrium is restored as in a flash-crash.[14]

In the last part of the paper, we extend the baseline model to consider the case in which a proportion of dealers is not continuously present in the market. In the model extention,

[12]Some of our results would be weakened if the endowment shocks were correlated, although the main effects would be there as long as there is no perfect correlation.

[13]In a related previous paper (Cespa and Vives (2019)), which the present one supersedes, we study the case in which first period liquidity traders have a short-term trading horizon, obtaining qualitatively similar results which are exacerbated by the forced liquidation of positions of first period hedgers in period two.

[14]Note that a shock to the dispersion of the random endowment impacts a deep parameter of the model and can therefore be considered akin to a shock to market fundamentals. It affects the demand for the security for a *given* liquidation value of the payoff.

liquidity is also supplied by a mass of dealers in the market only at the first round. The analysis of this case allows us to show that a small decrease in the mass of dealers who are always present in the market (say because of a cyberattack or a computer glitch) may decrease liquidity substantially when market transparency is low.

Finally, we tackle welfare analysis in the extended model. We compute the total welfare of market participants using a utilitarian criterion and numerically evaluate it. Our results show that, when the equilibrium is unique, an increase in market transparency and in the mass of dealers who are always in the market increase total liquidity in the second period and are both welfare enhancing. The improvement is driven by the increase in utility of the hedgers while all types of dealers may suffer.

Fragmented versus centralized markets We are modeling trading in a centralized market but our results should hold a fortiori in a fragmented OTC *competitive* market since an OTC market is more opaque than a centralized market. Fragmentation may augment welfare in the presence of strategic behavior but not with competitive traders (see the literature discussion below). In our model, opacity is represented by the imperfect signal traders in period 2 have about the endowment shock in period 1, and dealer market participation by the proportion of dealers continuously present in the market. Those two frictions can be related to fragmentation. We can identify trading rounds with different (geographically separated) trading venues, 1 and 2, where the same security can be traded. Opacity can then be interpreted as the extent to which information flows across different trading venues (instead of rounds as we do in the paper). With full opacity, traders in market 2 do not observe the hedgers' endowment shock in market 1, preventing them from supplying liquidity to these traders. We can interpret the proportion of dealers continuously present in the market as a reduced form proxy for the degree of fragmentation, capturing the idea that in a more fragmented market, the continuous presence of dealers is reduced (e.g., due to technological limitations). We find that, in an opaque market, fragmentation related to dealer participation has a nonmonotonic effect on fragility and liquidity.

Related literature We present a novel market fragility mechanism. In short, in contrast to the literature, in our paper: i) the disruptive effect of strategic complementarities is on the liquidity demand side instead of the supply side; ii) fragility does not rely on any irrationality on the part of traders, such as exogenous demand or supplies; and iii) there is no asymmetric information about payoffs but about the order flow.

Our paper is related to five streams of the literature. First, most of the contributions to liquidity fragility focus on the potential shortcomings of the supply side, be it because of funding problems (Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002)), lack of price information (Cespa and Foucault (2014)), or the effect of retrospective learning about the security's payoff (Cespa and Vives (2015)). For example, scholars have argued that regulation impairing access to capital for financial institutions may have a negative impact on the risk sharing capacity of the liquidity provision sector (see, e.g. Bao et al. (2018)). However, accounts of market crashes often attribute the inception of these events to "aggressive" or "unusually large" liquidity demand realizations which are not met by a sufficiently responsive increase in liquidity supply.[15] In our paper, we propose a theory, based on information access, in which liquidity fragility arises because of a self-sustaining loop affecting *liquidity demanders*, which exhausts liquidity suppliers' risk-bearing capacity.

Second, the paper is also related to the literature documenting liquidity provision via (contrarian) orders. Several authors provide evidence at high trading frequencies (Brogaard et al. (2014b)[16] and Biais et al. (2017)) and at lower frequencies (Biais et al. (2017)). Anand et al. (2021) provide evidence that some corporate bond mutual funds actively supply liquidity during periods of market stress and Anand et al. (2013) have a similar findings for equity mutual funds during the global financial crisis. In this respect, our paper argues that informational impediments to liquidity provision via market orders can negatively affect risk sharing and make liquidity fragile.[17]

Third, the paper is related to the early literature on price crashes. Gennotte and Leland (1990) provide a model tracing the 1987 stock market crash to traders not taking into account the possibility of exogenous portfolio insurance strategies affecting the security demand. Jacklin et al. (1992) also analyse the crash-inducing effect of mis-estimating the actual magnitude of portfolio insurance in a model à la Glosten and Milgrom (1985). Madrigal and Scheinkman (1997) show that the need to control the information flow conveyed by prices may lead to crashes. All of the above papers rely on some form of irrationality either due to the presence of noise trading, or to the fact that some rational traders are unaware of some component of the aggregate demand for the stock, to generate price discontinuities. In our model, all traders are rational expected utility maximizers, and the crash occurs because of the self-sustaining loop triggered by traders' liquidity demand.

Fourth, the paper is related to the literature highlighting the impact of multi-dimensional fundamentals for price discovery and the equilibrium properties of the market (see, e.g., Subrahmanyam and Titman (1999), Cespa and Foucault (2014), Goldstein and Yang (2015), and Goldstein et al. (2021)). Differently from this literature, in this paper, we assume that prices are driven by multiple, independent endowment shocks unrelated to the liquidation value of the asset and show that when liquidity demand reacts to prices, this can have important consequences for market stability.

Finally, the paper is related to the literature on market fragmentation. Some papers have argued that fragmentation may be good for welfare. Chen and Duffie (2021) state: "Although fragmentation reduces market depth on each exchange, it also isolates cross-exchange price

[15] For example, the CFTC-SEC report on the flash-crash attributes the inception of the crash to an aggressive E-mini S&P500 futures sell order initiated by a large mutual fund identified as Waddell & Reed (see CFTC and SEC (2010)), which appears to have persisted during the crash (see Aldrich et al. (2017)). See also Aquilina et al. (2018) for evidence of market participants' behavior during flash events in the UK.

[16] The authors analyze Nasdaq data and find that HFTs trade (buy or sell) in the direction of permanent price changes and the opposite direction of transitory pricing errors. This is done through their liquidity demanding (marketable) orders and is true on average and on the more volatile days.

[17] Li et al. (2021) modify Budish et al. (2015) to study competition for liquidity provision between HFTs and "execution algorithms," some of which can choose whether to trade via market or limit orders. They show that under continuous pricing, at equilibrium HFTs provide liquidity via market orders to execution algorithms who post aggressive limit orders.

impacts, leading to more aggressive overall order submission and better rebalancing of unwanted positions across traders". The result may be an improvement of overall liquidity where it is key that traders (individually) have price impact. Malamud and Rostek (2017) consider a multi-exchange demand function submission game with strategic traders in which each exchange operates a double auction similar to our model. They find that, in certain settings, when agents' risk preferences are sufficiently heterogeneous, fragmented markets can produce welfare outcomes superior to centralized markets. Manzano and Vives (2021) also find a similar result when traders have market power (market integration always increases welfare with competitive traders). All the potential advantages of fragmentation come when traders have a price impact, while in our setting, traders are competitive.[18]

The rest of the paper is organized as follows. In the next section we present the model. In Section 2 we study the fully transparent benchmark, in which we assume that second period traders perfectly observe the endowment shock affecting their first period peers. In Section 3 we assume that such information is not available (the fully opaque case) and show that this can generate multiple equilibria and consider also the intermediate case where second period traders have partial information. In Section 4, we extend the model to consider dealers not always present in the market together with dealers continuously present and perform a welfare analysis. Section 5 presents a calibrated contrast between opaque and transparent markets. The final section contains concluding remarks, including policy implications. Most of the proofs are relegated to Appendix A, extensions and additional material are provided in Internet Appendices.[19]

1 The model

This section presents the model, the market participants, market clearing conditions, and introduces a measure of total liquidity useful to compare different equilibrium regimes.

A single risky asset with liquidation value $v \sim N(0, \tau_v^{-1})$, and a risk-less asset with unit return are exchanged in a market during two periods (we interchangeably also use the expression "trading rounds").[20] Two classes of traders are in the market. First, a continuum of competitive, risk-averse dealers of unit mass, active in both periods. Second, a unit mass of liquidity traders who enter the market at the first round and post their orders at round 1 and 2. In the second period, a new cohort of liquidity traders (of unit mass) who enter the market

[18] Vairo and Dworczak (2023) also study transparency. In most OTC markets, traders do not observe each other's transactions and hence cannot rely on data about past prices to inform their trading strategies as in our model with opacity. In the paper, a search cost prevents investors from fully learning about the currently available prices. In our model, the equivalent friction would be the noise in the signal received by traders in the second period.

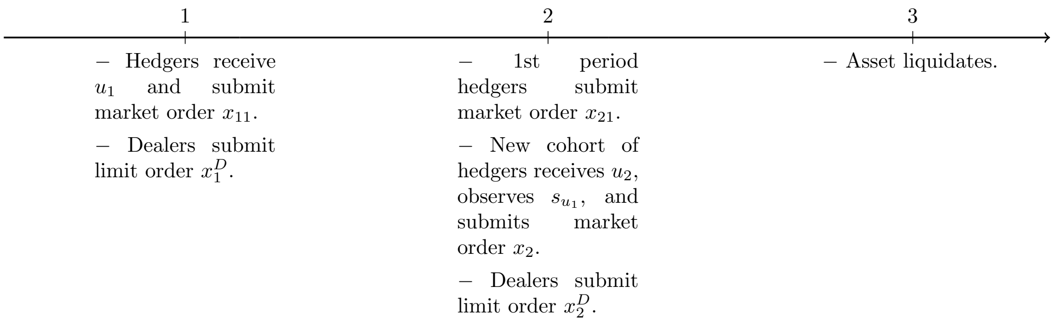
[19] Appendix B offers an extended explanation of market participants' strategies in the fully transparent benchmark; Appendix C simulates the model with a partially informative signal; Appendix D contains comparative statics exercises performed on the model of Section 4.1; Appendix E, relates the behavior of hedgers' demand to the persistence of a noise trading process; Appendix F considers the model in which first period hedgers observe the second period endowment shock and Appendix G compares hedging aggressiveness between opaque and transparent markets.

[20] A model of the same family was used in Cespa and Vives (2022) to study competition among exchanges.

and trade.[21] The asset is liquidated in period 3. We now illustrate the preferences and orders of the different players as well as the market clearing conditions.

As will become clear from the discussion in Section 3, in our model liquidity can become "fragile" (in the sense that a small shock to a deep parameter has a disproportionately large impact on liquidity). The sufficient conditions for such result are overlapping cohorts of risk averse investors suffering endowment shocks, enough "opacity" about the first period order flow, and risk averse dealers. A necessary condition for multiple equilibria and fragility in general is that strategic complementarity is strong enough, and in our context, strategic complementarity is increasing in opacity.

The timeline of the model is as follows:



1.1 Dealers

A dealer has CARA preferences with risk-tolerance γ, and submits price-contingent orders x_t^D, $t = 1, 2$, to maximize the expected utility of his final wealth: $W^D = (v - p_2)x_2^D + (p_2 - p_1)x_1^D$. At each trading round dealers condition their positions on the sequence of equilibrium prices up to that period. Thus, at the first round, they condition on p_1 and at the second round on $\{p_1, p_2\}$.[22]

1.2 Hedgers

The liquidity demand side of the model is represented by a unit mass of risk-averse traders who, prior to entering the market at time t, learn about the value of an endowment shock u_t in a non-tradable security that they will receive at the liquidation date ($t = 3$). We assume that the non-tradable security's value is perfectly correlated with that of the risky security traded in the market. This assumption, which is common in the literature (see, e.g. Wang (1994), Vayanos and Wang (2012), and Llorente et al. (2002)), induces a hedging demand for the risky security. We refer to these traders as "hedgers" (or, equivalently, "liquidity traders") and indicate them with the letter H.

[21]For example, the dealers could be HFTs, the first cohort of traders could be portfolio managers and the second slow traders who are not always in the market.

[22]We assume, without loss of generality with CARA preferences, that the non-random endowment of dealers is zero. Also, as equilibrium strategies will be symmetric, we drop a trader subindex.

More in detail, in the first period, a unit mass of CARA traders with risk-tolerance γ_H is in the market. Traders learn the value of the endowment shock u_1 and post a market order x_{t1}, at round $t \in \{1, 2\}$ to maximize the expected utility of their wealth $\pi_1 = u_1 v + (v - p_2)x_{21} + (p_2 - p_1)x_{11}$:

$$E\left[-\exp\{-\pi_1/\gamma_H\}|\Omega_1\right],$$

where $\Omega_1 \equiv \{u_1\}$ denotes their information set. In period 2, a new (unit) mass of CARA traders (with the same risk tolerance γ_H) enters the market, learns the realization of the non-tradable endowment shock u_2 that they will receive at $t = 3$, and observes a noisy signal of the previous period endowment shock $s_{u_1} = u_1 + \eta$. Second period traders submit a market order x_2 to maximize the expected utility of their wealth $\pi_2 = u_2 v + (v - p_2)x_2$:

$$E\left[-\exp\{-\pi_2/\gamma_H\}|\Omega_2\right],$$

where $\Omega_2 \equiv \{u_2, s_{u_1}\}$ denotes their information set. Note that they do not observe p_1 reflecting that they have only imperfect information on the order flow. We assume $u_t \sim N(0, \tau_u^{-1})$, $\eta \sim N(0, \tau_\eta^{-1})$ and $\text{Cov}[u_t, v] = \text{Cov}[u_t, \eta] = \text{Cov}[u_1, u_2] = 0$, $t = 1, 2$.

As an example of the "non-tradable" security, one can think of a portfolio of assets that traders are unwilling to liquidate (or that are intrinsically illiquid). In view of the assumed correlation structure, protection against changes in the non-tradable value is then obtained by taking an offsetting position in the risky security. For instance, traders could be long in a portfolio of stocks that tracks the market, say a fund, and hedge by shorting a market-tracking ETF; alternatively, they could be long on a S&P500 ETF, like the SPY, and setup a hedge by trading the Emini (while the former trades from 6am to 8pm, including extended trading hours, the latter trades 24/7, thus allowing overnight hedging).[23]

To simplify notation, in the following we denote by $E_t^D[Y]$, and $\text{Var}_t^D[Y]$, the conditional expectation and variance that a dealer forms about random variable Y, in period $t = 1, 2$. Note that since dealers submit limit orders, at a linear equilibrium they will infer the endowment shocks hitting hedgers' budget constraints. Similarly, $E_t[Y]$, $\text{Var}_t[Y]$, and $\text{Cov}_t[X, Y]$ denote the conditional expectation, variance, and conditional covariance that a period-t hedger forms about random variables Y and X.

1.3 Market clearing

We will restrict attention to equilibria in which prices are linear functions of the endowment shocks and the error term affecting second period traders' signal. With hindsight, these will

[23] For an example involving SPY, see https://money.stackexchange.com/questions/54373/why-dont-spy-spx-and-the-e-mini-sp-500-track-perfectly-with-each-other, and http://tastytradenetwork.squarespace.com/tt/blog/equating-futures-to-etfs, and for other ETF related examples, see https://investorplace.com/2017/10/portfolio-hedge-fund-consider-etfs/.

have the following form:

$$p_1 = -\Lambda_1 u_1 \tag{1a}$$

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22}\eta, \tag{1b}$$

where Λ_1, Λ_2, Λ_{21}, Λ_{22} are coefficients which will be pinned down at equilibrium. At equilibrium, dealers absorb the orders of first period hedgers x_{11}:

$$x_1^D + x_{11} = 0. \tag{2}$$

Hedgers know u_1, while, at equilibrium, dealers infer it from the price, which justifies (1a).

Consider now the second period equilibrium condition. First period hedgers split their hedging needs by posting an order x_{21} together with their second period peers. Additionally, dealers rebalance their position at the second round. Formally, from the second period market clearing equation we have

$$(x_2^D - x_1^D) + (x_{21} - x_{11}) + x_2 = 0 \iff x_2^D + x_{21} + x_2 = 0, \tag{3}$$

where the expression on the right hand side in (3) follows from using the first period market clearing condition (2). At equilibrium, dealers' and hedgers' strategies are a function of their information sets–$\{p_1, p_2\}$ for dealers and Ω_2 for second period hedgers. As a consequence, the price will load on $\{u_1, u_2, \eta\}$, justifying (1b).

Note that since traders have the possibility to retrade at the second round, to hedge their endowment shock, both the first and second period price depend on u_1. This, in turn, suggests the following alternative way to write the second period equilibrium price:

$$p_2 = -\Lambda_2\theta_2 - \Lambda_{22}\eta, \tag{4}$$

where $\theta_2 = u_2 + \beta u_1$ and $\beta = \Lambda_{21}/\Lambda_2$. The expression in (4) shows how our model can be made equivalent to models postulating noise trading as an AR(1) process, thus endogenizing the persistence coefficient β and relating it to the relative weight that endowment shocks receive in the second period price.[24]

1.4 A measure of total illiquidity

At equilibrium p_2 depends on Λ_2, Λ_{21}, and Λ_{22}, which are the price impacts of respectively u_2, u_1, and η (see (1b)). This implies that none of the equilibrium price coefficients is by itself able to fully capture the illiquidity of the second period market. We thus introduce an illiquidity measure which accommodates this problem, by computing a weighted average of the price coefficients (Λ_{21}, Λ_2, and Λ_{22}) of the different factors influencing the price (that is, u_1, u_2,

[24]Several authors have assumed this process for noise trading. Among others: Campbell et al. (1993), He and Wang (1995), Cespa and Vives (2015). Peress and Schmidt (2021) provide rigorous empirical validation to this assumption.

and η). We weigh the price impacts by the volume associated with the responses to endowment shocks and possible noisy information.

More in detail, at a linear equilibrium strategies will be of the following form: $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, and $x_2 = a_2 u_2 + b s_{u_1}$, with a_1, a_{21}, and a_2 the hedging intensities and b the speculative intensity (see below). We term such measure "Weighted Average Price Impact" ($WAPI$):[25]

$$WAPI = \frac{\Lambda_{21}\sqrt{\text{Var}[(a_{21}+b)u_1]} + \Lambda_2\sqrt{\text{Var}[a_2 u_2]} + \Lambda_{22}\sqrt{\text{Var}[b\eta]}}{\sqrt{\text{Var}[(a_{21}+b)u_1]} + \sqrt{\text{Var}[a_2 u_2]} + \sqrt{\text{Var}[b\eta]}} = \frac{(|a_{21}+b|\Lambda_{21} + |a_2|\Lambda_2)\sigma_u + \Lambda_{22} b\sigma_\eta}{(|a_{21}+b| + |a_2|)\sigma_u + b\sigma_\eta}. \tag{5}$$

2 Fully transparent benchmark

In this section, we consider a fully transparent market, characterize the strategies of hedgers and dealers, propose a heuristic way to think about liquidity demand, and check that this schedule has the "normal" downward slope, indicating that hedgers trade less intensely when it is more costly to do so. With full transparency, second period hedgers can provide liquidity since they are well informed about period 1. This is not the case with opacity and the hedgers' liquidity demand can slope upwards.

We deal with the case where the second period hedgers observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$). This assumption implies that the market is fully transparent and has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period hedgers is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second period price only reflects endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 \tag{6}$$

while the first period price is as in (1a), $p_1 = -\Lambda_1 u_1$.

Due to the linearity assumption for prices, equilibrium strategies will also be linear. Specifically, we posit $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, $x_2 = a_2 u_2 + b u_1$, where the coefficients a_1, a_{21} and a_2 denote the hedging intensity of liquidity traders and the corresponding absolute values of such coefficients denote their hedging "aggressiveness". The coefficient b denotes second period traders' "speculative" aggressiveness as we explain below. In the Appendix, we show that in this case the equilibrium is identified by the unique solution to a system of simultaneous equations in $\Lambda_1, \Lambda_{21}, \Lambda_2$. We obtain the following:

Proposition 1. *When the market is fully transparent, there exists a unique equilibrium in linear strategies. The coefficients of equilibrium prices* $p_1 = -\Lambda_1 u_1$ *and* $p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1$*,*

[25] This measure of illiquidity can be related to the volume-weighted spread measures often used to gauge liquidity across different venues (see, e.g., ESMA).

are given by:

$$\Lambda_2 = -\frac{1}{\gamma\tau_v}a_2 \tag{7a}$$

$$\Lambda_1 = -\frac{1}{\gamma\tau_v}\frac{\gamma+\gamma_H}{\gamma_H}a_1 \tag{7b}$$

$$\Lambda_{21} = -\frac{1}{\gamma\tau_v}\big(b+a_{21}\big). \tag{7c}$$

The coefficients of hedgers' strategies $x_{11} = a_1u_1$, $x_{21} = a_{21}u_1$, $x_2 = a_2u_2 + bu_1$ *are as follows:*

$$a_1 = -\gamma_H\frac{\Lambda_{21}-\Lambda_1}{\mathrm{Var}_1[p_2]} \in (-1,0),\ a_{21} = \frac{\gamma_H\tau_v\Lambda_{21}-1}{\tau_v\mathrm{Var}_1[v-p_2]} \in (-1,0), \tag{8a}$$

$$a_2 = \gamma_H\tau_v\Lambda_2 - 1 \in (-1,0),\ b = \gamma_H\tau_v\Lambda_{21} > 0, \tag{8b}$$

where $\mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1}$, *and* $\mathrm{Var}_1[v-p_2] = \Lambda_2^2\tau_u^{-1} + \tau_v^{-1}$. *Furthermore,* $-1 < a_{21} < a_1 < 0$, $0 < \Lambda_1 < \Lambda_{21} < \Lambda_2$ *(explicit expressions for the price coefficients are in Appendix A). Dealers strategies are given by*

$$X_1^D(p_1) = -\frac{\gamma}{\gamma_H}a_1u_1 - \gamma\tau_vp_1 \tag{9a}$$

$$X_2^D(p_1,p_2) = -\gamma\tau_vp_2. \tag{9b}$$

Corollary 1. *When the market is fully transparent total illiquidity at period 2 is given by:*

$$WAPI|_{transparency} = \frac{|a_{21}+b|\Lambda_{21} + |a_2|\Lambda_2}{|a_{21}+b| + |a_2|}.$$

In the case of transparency s_{u_1} is perfect ($\sigma_\eta \to 0$), implying that $\Lambda_{22} = 0$, and yielding a simplified expression for $WAPI$. It is worth noting also that the price impact of the second period endowment shock Λ_2 is independent of the price impact of the first period endowment shock Λ_{21}.

In the following, we explain hedgers' and liquidity providers' strategies in the fully transparent case (a more detailed explanation of this material is contained in Appendix B).

2.1 Hedgers' strategies

First period hedgers demand liquidity by hedging part of their risk exposure at both trading rounds–that is, they *split* their hedging order with $-1 < a_{21} < a_1 < 0$. Hence, if $u_1 > 0$, they hedge their exposure shorting at the first round, and increasing their short position at the second round (i.e., their second period *trade* is a also a sell).

Second period liquidity traders also hedge their risk exposure. We can interpret the expressions for a_{21} and a_2 in the following way. A liquidity trader hedges a larger fraction of his shock (demands more liquidity), the lower is the impact the endowment shock has on p_2 (as a larger price impact reduces a trader's expected return from hedging), and the lower is the return uncertainty he faces (as a higher return variance dents his utility since he is risk averse). Crucially, second period hedgers face no price risk (even using market orders) since they know both u_1 and u_2 and $\text{Var}_2[v - p_2] = \tau_v^{-1}$ (note that there is no variance correction in a_2).

Additionally, because of their ability to perfectly infer the direction of the demand pressure due to first period traders' second round trade, second period hedgers post a *contrarian market order* ($b > 0$), which provides additional risk-sharing and rationalizes first period traders' decision to split their hedging order.[26] *Therefore, when the market is transparent, second period liquidity traders provide additional risk sharing by posting a contrarian market order with aggressiveness $b > 0$.*

2.2 Dealers' strategies

Dealers submit price contingent orders (generalized limit orders) at both rounds and can infer the endowment shocks at a linear equilibrium. $X_1^D(p_1)$ reflects two trading motives: short-term return speculation (captured by $-(\gamma(\Lambda_{21} - \Lambda_1)/\text{Var}_1[p_2])u_1 = -(\gamma a_1/\gamma_H)u_1$) and liquidity supply (captured by $-\gamma\tau_v p_1 = \gamma\tau_v\Lambda_1 u_1$). Speculation originates from dealers' ability to infer hedgers' endowment shock and its impact on p_2. To see this, note that at the second round dealers in aggregate hold

$$
\begin{aligned}
X_2^D(p_1, p_2) &= -\gamma\tau_v p_2 \\
&= \gamma\tau_v\Lambda_{21}u_1 + \gamma\tau_v\Lambda_2 u_2,
\end{aligned} \tag{10}
$$

that is, they hold $\gamma\tau_v\Lambda_{21}$ of the first period endowment shock. At the first round, dealers take the counterpart of hedgers and their position is given by

$$
X_1^D(p_1) = \gamma\left(\frac{\Lambda_1 - \Lambda_{21}}{\text{Var}_1[p_2]} + \frac{\Lambda_1}{\text{Var}[v]}\right)u_1. \tag{11}
$$

If $u_1 > 0$, $x_{11} < 0$ and from market clearing we have that $x_1^D > 0$. Dealers anticipate providing further trading opportunities to first period hedgers at the second round, which increases their risk exposure, so the price impact of u_1 will be larger, yielding $\Lambda_{21} > \Lambda_1$. We can think that they defer part of their buy order to the second round (since $\Lambda_1 - \Lambda_{21} < 0$), speculating on the opportunity to buy more of u_1 at a lower price. Such speculation is risky (depending on the realization of u_2, it may turn out that $p_2 > p_1$), and dealers' second period trades decrease in $\text{Var}_1[p_2]$.[27] In expectation, they are to gain: $E_1^D[p_2 - p_1] = (\Lambda_1 - \Lambda_{21})u_1 < 0$ if $u_1 > 0$ because all else equal, p_2 has to fall in expectation compared to p_1 for the market to absorb the extra

[26] Because of the informativeness of the signal they observe about u_1, at equilibrium, second period traders can perfectly infer the first period endowment shock and thus p_2. This makes their order akin to a contrarian "marketable" order–an order is marketable when it is submitted at a price that matches (or is better than) the best quotes, implying that it obtains immediate execution. Indeed, based on (8b), we have $x_2 = (\gamma_H\tau_v\Lambda_2 - 1)u_2 + \gamma_H\tau_v\Lambda_{21}u_1 = \gamma_H\tau_v(\Lambda_2 u_2 + \Lambda_{21}u_1) - u_2 = -\gamma_H\tau_v p_2 - u_2$.

[27] Note that then $p_1 < 0$ since $\Lambda_1 > 0$ and $p_1 = -\Lambda_1 u_1$. The price reflects the expected value of the security net of the compensation demanded by the market to absorb hedgers' endowment shock. Hedgers receiving a positive endowment shock share their risk by selling the security. Since we assume that the security value's expectation is null, this implies that in this case the price at the first round is depressed and becomes negative.

liquidity demand from the first period hedgers.[28] At the second round, based on (10), dealers provide additional liquidity to first period hedgers by trading

$$\gamma\tau_v\Lambda_{21}u_1 - X_1^D(p_1) = \gamma(\Lambda_{21} - \Lambda_1)\left(\frac{1}{\mathrm{Var}[v]} + \frac{1}{\mathrm{Var}_1[p_2]}\right)u_1,$$

a buy order if $u_1 > 0$. Note that with risk neutral dealers we would have that $E_1^D[p_2 - p_1] = 0$, since in this case, dealers do not need compensation for holding risk (indeed then the market is infinitely deep, $\Lambda_1 = \Lambda_{21} = \Lambda_2 = 0$).[29]

Due to risk aversion, dealers have a limited capacity to bear risk, and *the price coefficients in* (7a)–(7c) *capture the risk-tolerance weighted risk compensation dealers require to absorb the aggregate liquidity demand.*

The coefficient Λ_1 captures the risk-weighted compensation that liquidity suppliers demand to absorb the aggregate marginal position of liquidity traders and dealers (the aggregate "liquidity demand"). Since this covers a "cost" incurred to supply immediacy, we interpret (somewhat loosely) Λ_1 as a function of a_1 as the first period "liquidity supply" function.

Similarly to Λ_1, the coefficients Λ_2 and Λ_{21} reflect the risk-weighted compensation that liquidity suppliers demand to absorb first and second period liquidity traders' aggregate demand. To understand the numerator of Λ_{21}, note that first period liquidity traders' demand at the second round (i.e., the marginal position a_{21}), is not absorbed by dealers in its entirety. Indeed, at the second round part of first period liquidity traders' endowment shock exposure is absorbed by second period traders' speculation (the coefficient b). Similarly to what we have done for Λ_1, we interpret Λ_{21} and Λ_2 as the second period liquidity supply functions (of a_{21} and a_2, respectively) to first and second period traders.

2.3 Liquidity demand and supply in a transparent market

We are now ready to explain the behavior of liquidity demand and supply in the fully transparent benchmark. In Proposition 1, we show that the hedging intensities a_1, a_{21} and a_2 are negatively valued functions (ranging between -1 and 0) since they capture first and second period liquidity traders' reaction to the endowment shock they receive. We measure liquidity traders' demand for liquidity via their "hedging aggressiveness," that is the absolute values of a_1, a_{21}, and a_{22}. Because of the way they are defined, liquidity supply functions are positively valued. We display $|a_1|$ as function of Λ_{21} and Λ_1 and $|a_{21}|$ as a function of Λ_{21}, by solving for

[28] This is akin to "order anticipation" which, according to SEC (2010), occurs when "…a proprietary firm seeks to ascertain the existence of one or more large buyers (sellers) in the market and to buy (sell) ahead of the large orders with the goal of capturing a price movement in the direction of the large trading interest (a price rise for buyers and a price decline for sellers)."

[29] Prices are also semi-strong efficient when second period hedgers do not observe u_1 while first period traders do observe u_2 since in this case first period hedgers have no benefit in splitting their orders and $\Lambda_1 = \Lambda_{21}$ (see Appendix F).

Λ_2. In sum, the liquidity demand and supply functions are given by the following expressions:

$$|a_2| = |\gamma_H \tau_v \Lambda_2 - 1|, \ \Lambda_2 = -\frac{a_2}{\gamma \tau_v} \tag{12a}$$

$$|a_{21}| = \left| \frac{(\gamma + \gamma_H)^2(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u \tau_v}{1 + (\gamma + \gamma_H)^2 \tau_u \tau_v} \right|, \ \Lambda_{21} = -\frac{a_{21}}{(\gamma + \gamma_H)\tau_v} \tag{12b}$$

$$|a_1| = | - \gamma_H \tau_u (\gamma + \gamma_H)^2 \tau_u \tau_v^2 (\Lambda_{21} - \Lambda_1)|, \ \Lambda_1 = -\frac{\gamma + \gamma_H}{\gamma \gamma_H \tau_v} a_1. \tag{12c}$$

Inspection of the above expressions shows that:

Corollary 2. *When the market is transparent, liquidity demand is decreasing in the aggregate price impact it induces and liquidity supply increases in traders' aggregate demand.*

Therefore, in a transparent market, the cost of trading works as a rationing device: the pricier liquidity becomes, the less traders choose to hedge. Conversely, an increase in traders' liquidity demand prompts dealers to make the market less liquid (i.e., make liquidity pricier). In Figure 1 we plot the liquidity supply and demand functions (respectively, in blue and green) for second period traders. The unique equilibrium corresponds to the crossing point between the two curves.

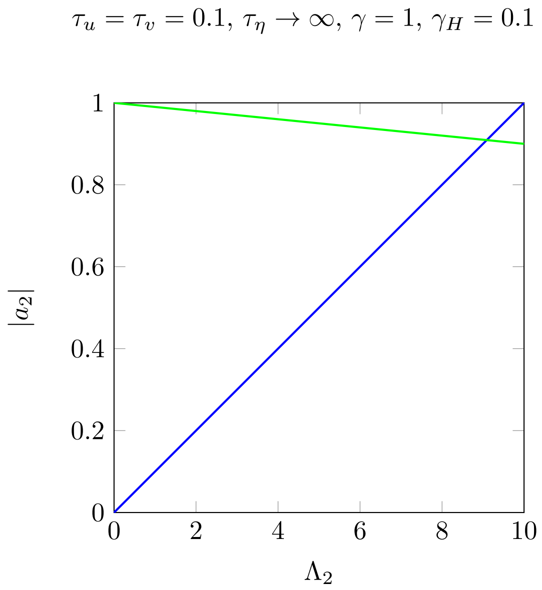


Figure 1: Second period traders' liquidity demand (in green) and supply (in blue) at the second round with a fully transparent market.

Summarizing, when the market is transparent, liquidity demand decreases in the price impact coefficients of the endowment shocks and price impact coefficients increase in liquidity demand. Then, a unique equilibrium obtains. In this equilibrium dealers speculate on short-term returns and second period liquidity traders hedge their risk exposure and provide liquidity via contrarian market(able) orders, sharing with dealers the risk exposure of first period traders.

3 The opaque market

In this section, we look at an opaque market, starting with the case where second period traders have no information on the first period endowment shock. This is the extreme version of the case where, in the current period, hedgers have imperfect information about the previous trading period order imbalance. We characterize the equilibrium, examine strategic complementarity in illiquidity, the parameter regions where unique or multiple equilibria are obtained, and the conditions for flash events to occur. A necessary condition for this type of fragility is upward sloping demand for liquidity. Under opacity, the interaction between the price impact of the first and second-period endowment shocks in period 2 is source of strategic complementarity.

We end the section considering the partially opaque market case where second period traders observe a *noisy* signal of the first period order imbalance ($\tau_\eta \in (0, \infty)$). In this case, $\Omega_2 = \{u_2, s_{u_1}\}$ which implies that second period traders cannot perfectly anticipate p_2. As a consequence, their strategy is affected by their return uncertainty:

$$x_2 = \gamma_H \frac{E_2[v - p_2]}{\text{Var}_2[v - p_2]} - \frac{\text{Cov}_2[v - p_2, v]}{\text{Var}_2[v - p_2]} u_2 = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\text{Var}_2[v - p_2]}}_{a_2} u_2 + \gamma_H \underbrace{\frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_u + \tau_\eta)\text{Var}_2[v - p_2]}}_{b} s_{u_1}, \quad (13)$$

where $\text{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$, and the second period price is as in (1b). Traders' inability to exactly infer u_1 impacts their return uncertainty, exposing their strategy to execution risk. This, in turn, affects both their hedging and speculative aggressiveness ($|a_2|$ and b) and the cost of trading of their order. Given the risk-sharing enhancing role of traders' speculation, this impacts market stability. To see this, it is useful to start from the extreme case in which $\tau_\eta \to 0$.

3.1 The fully opaque market

Suppose second period traders' signal becomes unboundedly noisy (i.e., $\tau_\eta \to 0$). In this case, we obtain the following result:

Proposition 2. *When the market is fully opaque, the expressions for dealers strategies are as in* (9a) *and* (9b)*; the equilibrium price coefficients* Λ_2 *and* Λ_1 *are as in* (7a) *and* (7b)*, while*

$$\Lambda_{21} = -\frac{a_{21}}{\gamma \tau_v}, \quad (14)$$

The coefficients of traders' strategies $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, $x_2 = a_2 u_2 + b u_1$ *are as follows:*

$$a_1 = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]} < 0, \ a_{21} = \frac{\gamma_H \tau_v \Lambda_{21} - 1}{\tau_v \text{Var}_1[v - p_2]} \in (-1, 0) \quad (15a)$$

$$a_2 = \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \text{Var}_2[v - p_2]} \in (-1, 0), \ b = 0, \quad (15b)$$

where $\text{Var}_1[p_2] = \Lambda_2^2 \tau_u^{-1}$, $\text{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_2^2 \tau_u^{-1}$ *and* $\text{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}$. *Furthermore, at equilibrium* $\Lambda_{21} > \Lambda_1 > 0$ *and* $\Lambda_2 > 0$*;* Λ_{21} *can be larger, equal or smaller than*

Λ_2.

Corollary 3. *When the market is fully opaque, total illiquidity at period 2 is given by:*

$$WAPI|_{opacity} = \frac{|a_{21}|\Lambda_{21} + |a_2|\Lambda_2}{|a_{21}| + |a_2|}.$$

With opacity s_{u_1} is infinitely noisy ($\tau_\eta^{-1} \to 0$), implying that $b = 0$, and yielding a simplified expression for $WAPI$. It is worth noting also that the price impact of the second period endowment shock Λ_2 is *not* independent of the price impact of the first period endowment shock Λ_{21}.When the market is fully opaque, second period traders do not speculate ($b = 0$). This is because their signal on u_1 is infinitely noisy, which makes it impossible for them to predict the direction of the first period imbalance. As a consequence, $\Lambda_{22} = 0$ and we have:

Corollary 4. *When the market is fully opaque, second period liquidity traders do not supply liquidity via contrarian market orders and the second period price only reflects traders' endowment shocks.*

Liquidity traders' second period hedging aggressiveness, $|a_{21}|, |a_2|$ depends on two forces: the expected return from holding the endowment shock, and the variance of the second period return $v - p_2$ (respectively captured by the terms at the numerator–which is negative–and denominator of the expressions in (15a) and (15b)).[30] For given return variance, a higher price impact of the t-period traders' endowment shock, increases these traders' expected return from holding the endowment shock, decreasing their hedging aggressiveness (e.g., for second period hedgers, a higher Λ_2, lowers the absolute value of the numerator $\gamma_H \tau_v \Lambda_2 - 1$ in (15b)). For given expected return from holding the endowment shock, a higher price impact of the t-period traders' endowment shock increases $s \neq t$-period traders' execution risk, lowering the latter hedging aggressiveness (again, for second period hedgers, a higher Λ_{21}, increases the denominator in (15b): $\text{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}$).

Therefore, changes in the price impacts of the different hedgers' cohorts trades have opposite effects on the execution risk faced by each cohort. In period 2, the liquidity demands of different hedgers' cohorts are substitutes: the more liquidity cohort 2 demands, the less cohort 1 demands. This is because when risk averse dealers are more exposed to the second period endowment shock, they require a larger risk compensation to absorb the endowment shock, which increases the sensitivity of the price to the second period endowment shock (Λ_2). This reduces cohort 2 traders' returns from hedging and increases execution risk for cohort 1 hedgers (these two effects are stated in Figure 2). So, when traders in cohort 2 demand lots of liquidity, they make the market less liquid for hedgers in cohort 1. These effects become self-sustaining when the lower liquidity demand displayed by hedgers in cohort 1 reduces dealers' exposure to cohort 1's endowment shock, lowering the risk compensation these traders demand to absorb the shock. This, in turn, reduces the price sensitivity to the first period endowment shock (Λ_{21}), reducing the execution risk faced by second period hedgers and leading the latter to increase their liquidity demand.

[30] With full opacity we have that $E_2[v - p_2] = \Lambda_2 u_2$, $E_1[v - p_2] = \Lambda_{21} u_1$, and $\text{Cov}_t[v, v - p_2] = \tau_v^{-1}$ for $t = 1, 2$.

When second period traders are not informed about u_1, Λ_2 and Λ_{21} are strategic substitutes and their composition is the source of strategic complementarity. To see this, assume that the market impact of the second period traders' endowment shock (Λ_2) increases. This reduces these traders' expected profit from hedging the endowment and heightens the cohort 1 traders' execution risk, leading them to scale down their liquidity demand ($|a_{21}|$ decreases). All else equal, this reduces the price impact of cohort 1's endowment shock (Λ_{21} decreases), because liquidity providers need to absorb a smaller share of cohort 1's endowment shock. This in turn lowers the execution risk faced by traders in cohort 2, potentially leading them to scale up their liquidity demand ($|a_2|$ increases), and further boosting Λ_2, because dealers need to absorb a larger share of cohort 2's endowment shock, which reinforces the initial spike (since $\Lambda_{21} = -a_{21}/\gamma\tau_v$ (14)).

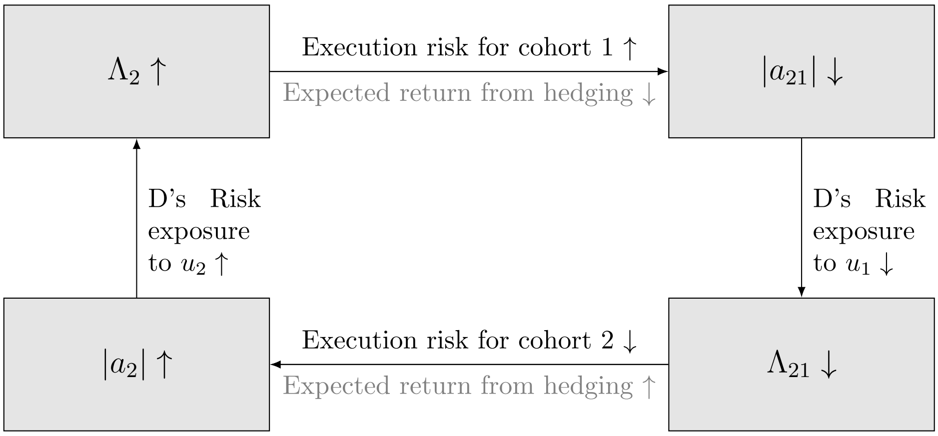


Figure 2: Strategic complementarity. A diagrammatical representation of the self-reinforcing loop between liquidity demand and illiquidity arising with market opacity.

We see thus that Λ_{21} and Λ_2 are strategic substitutes and this is at the root of the strategic complementarity in Λ_2. The loop described above is diagrammatically sketched in Figure 2 and formally captured by the "aggregate" best response function below which is derived in the Appendix and reflects the impact of an exogenous change in Λ_2 on traders' strategies, yielding a new value for Λ_2:

$$\Phi(\Lambda_2) \equiv \frac{((\gamma + \gamma_H)\tau_u + \gamma\Lambda_2^2\tau_v)^2}{\gamma\tau_u + ((\gamma + \gamma_H)\tau_u + \gamma\Lambda_2^2\tau_v)^2(\gamma + \gamma_H)\tau_v}. \tag{16}$$

It is possible to check that Φ is strictly increasing in Λ_2. which provides the formal counterpart to the heuristic argument developed above–that is the existence of strategic complementarity in illiquidity with market opacity.

A fixed point of Φ, $\Lambda_2 = \Phi(\Lambda_2)$, corresponds to an equilibrium of the market and in Figure 3 we show that, depending on parameters' values, either a unique equilibrium or multiple equilibria can obtain. Specifically, with the hypothesized parameterization, when the dispersion of the endowment shock is sufficiently low (case $\tau_u = 2$, in Panel (a)), strategic complementarity is "weak" and a unique equilibrium arises (in which case $\Lambda_{21} = \Lambda_2 = 4.61$ and $\Lambda_1 = 2.34$). Conversely, when the dispersion of the endowment shock increases (case

$\tau_u = 0.1$, in Panel (b)), strategic complementarity is "strong," and multiple equilibria arise, where $\Lambda_2 \in \{8.96, 1.98, 0.12\}$, and the corresponding values for the other price coefficients are $\Lambda_{21} \in \{0.12, 1.98, 8.96\}$, $\Lambda_1 \in \{0.1 \times 10^{-2}, 0.43, 8.84\}$.

In the figure, we also display with a green dot the equilibrium values for Λ_2 in the fully transparent market case for the two parameterizations. This allows us to compare equilibrium illiquidity across the two polar cases of full transparency and full opacity, showing that in our setup, a more transparent market leads to a *higher price impact for* u_2 at the second round–a higher Λ_2. This is consistent with the intuition that transparency (that is, observability of the first period endowment shock for second period hedgers) lowers second period hedgers' execution risk (compare $\text{Var}_2[v - p_2]$ in equation (15b) and in equation (8b)) and boosts their liquidity demand (a higher hedging aggressiveness $|a_2|$).[31] This increases the market exposure to u_2, leading to a higher Λ_2.[32]

Proposition 3 establishes the conditions for equilibrium multiplicity.

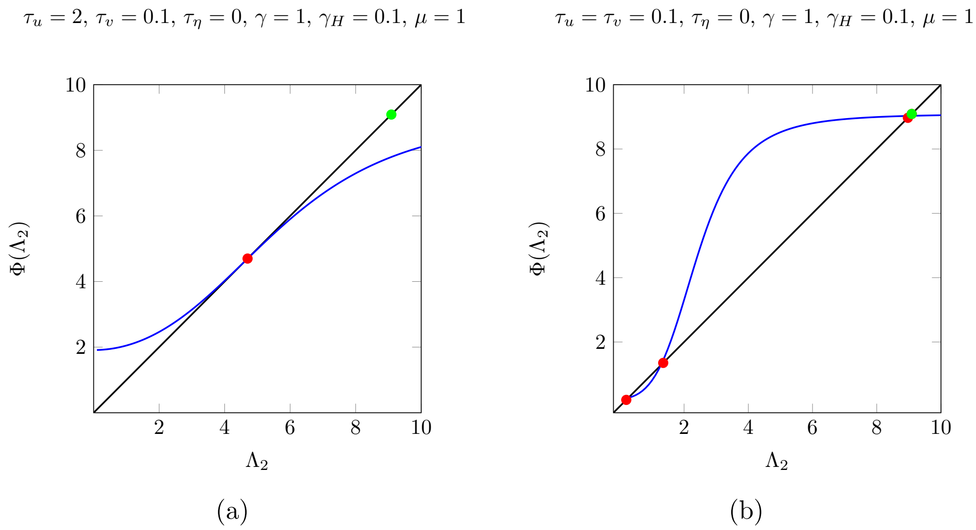


Figure 3: Market opacity: single equilibrium (Panel (a)), and multiple equilibria (Panel (b)). The green dot in both panels corresponds to the unique equilibrium in the fully transparent benchmark.

For $\tau_\eta \to 0$, the system of equations which pins down the price impacts becomes:

$$\Lambda_2 = \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v} \tag{17a}$$

$$\Lambda_{21} = \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\tau_v} \tag{17b}$$

$$\Lambda_1 = \frac{(\gamma + \gamma_H)\tau_u\Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}. \tag{17c}$$

From (17a) and (17b), it is clear that one solution is that $\Lambda_2 = \Lambda_{21}$ and this yields a cubic

[31] For the case of Figure 3 (a) we have that with transparency $|a_2| = 0.909$, while with opacity $|a_2| = 0.198$.

[32] This is a general result that holds independently of whether there is a unique or multiple equilibria with opacity. See Corollary 11 in Appendix G, where we also provide a ranking of second period hedging aggressiveness across regimes. Meli et al. (2024) find that when adverse selection is low and inventory costs are high (a low γ in our model), an increase in transparency widens the bid ask spread in the EU corporate bond market.

equation (which has a unique solution). If this is not the case, we show in the Appendix that two other equilibria obtain as a solution to a quadratic equation:

Note that the price impact of the first period endowment shock (Λ_1) does not affect the second period price coefficients (Λ_2, Λ_{21}) but is determined by their equilibrium values. The following corollary provides the conditions to obtain a unique equilibrium or three equilibria.

Proposition 3. *When the market is fully opaque, at equilibrium*

$$\Lambda_1 = (\gamma + \gamma_H)\tau_v \Lambda_{21}^2. \tag{18}$$

If

$$0 < \tau_u \tau_v < \gamma/(4(\gamma + \gamma_H)^3), \tag{19}$$

three equilibria arise, where in one extremal equilibrium Λ_2 is the smallest root of the following quadratic equation:

$$(\gamma + \gamma_H)\gamma\tau_v \Lambda_2^2 - \gamma\Lambda_2 + (\gamma + \gamma_H)^2 \tau_u = 0, \tag{20}$$

and Λ_{21} is the largest one (see (A.49) in the Appendix). In the other extremal equilibrium the opposite occurs, and in the the third equilibrium $\Lambda_2 = \Lambda_{21}$ obtains as the unique root of the following cubic

$$\varphi(\Lambda_2) \equiv ((\gamma + \gamma_H)\tau_u + \gamma\tau_v \Lambda_2^2)\Lambda_2 \tau_v - \tau_u = 0. \tag{21}$$

If $\tau_u \tau_v \geq \gamma/(4(\gamma + \gamma_H)^3)$, then there is a unique equilibrium where $\Lambda_2 = \Lambda_{21}$ is the unique root of the cubic equation, and $a_2 = a_{21}$.

Condition (19) defines the parameter restriction for the region where equilibrium multiplicity occurs. According to such condition, multiplicity obtains when liquidity demand is likely to be stronger, the volatility of the security's payoff is larger and traders are more risk averse, i.e. when the gap between liquidity demand and liquidity provision is likely to be *wider*. Indeed, in these conditions traders need to hedge the most (due to the higher unpredictability of their endowment shock and their higher risk aversion), while dealers are less willing to supply liquidity (due to the higher volatility of the security's payoff). Interestingly, an increase in dealers' risk-bearing capacity has a non-monotonic impact on the magnitude of this region. This is because for given hedging aggressiveness ($|a_{21}|$ and $|a_{22}|$), an increase in γ lowers the cost of trading (see (7a) and (14)) which, for low levels of risk tolerance, induces more liquidity consumption on traders' side (see (15a) and (15b)). However, as γ grows large this effect becomes second order, and an increase in dealers' risk tolerance reduces the magnitude of the multiplicity region. Importantly, in the latter case, this implies that a decrease in dealers' risk bearing capacity can be responsible for an increase in market instability. Indeed for $\gamma > \gamma_H/2$ a lower γ enlarges the region of parameter values for which multiplicity obtains.

As argued in the proposition, the second period price sensitivities to the endowment shocks (Λ_2 and Λ_{21}) correspond to the two roots of the quadratic (20). This implies that at the second round the trading costs faced by traders in different cohorts are heterogeneous: the price impact of first and second period liquidity traders' endowment shocks are *negatively correlated*.

We denote by Λ_2^* and Λ_2^{***} the smallest and largest roots of (20), and with Λ_2^{**} the unique real root of the cubic (21). Correspondingly, Λ_{21}^{***}, Λ_{21}^*, and Λ_{21}^{**}, denote the smallest and largest roots of (20), and the unique real root of the cubic (21) (recall that in this case $\Lambda_2 = \Lambda_{21}$). Finally, Λ_1^{***}, Λ_1^* and Λ_1^{**} denote the first period price impact coefficient obtained via (18). Accordingly, we rank traders' hedging intensities in a similar way: a_2^* corresponds to the case where $\Lambda_2 = \Lambda_2^*$ (and $\Lambda_{21} = \Lambda_{21}^*$), and so on.[33] The next result characterizes the stability properties of the equilibrium and the hedging aggressiveness patterns arising with multiple equilibria.

Corollary 5. *When the market is fully opaque, with uniqueness, the equilibrium is stable. When multiple equilibria arise,*

1. *The two extreme equilibria are stable, while the intermediate equilibrium is unstable.*

2. *Equilibria can be ranked in terms of the price sensitivity to first and second period endowment shocks:*

$$\Lambda_2^* < \Lambda_2^{**} < \Lambda_2^{***}, \; \Lambda_{21}^{***} < \Lambda_{21}^{**} < \Lambda_{21}^*, \; \Lambda_1^{***} < \Lambda_1^{**} < \Lambda_1^*. \tag{22}$$

   Thus, at a stable equilibrium we have either that p_2 reacts more to u_2 than to u_1, or the opposite. Correspondingly, in the former (latter) case the first period market is more (less) liquid. Comparing liquidity across trading rounds, we have

$$\Lambda_1 < \Lambda_{21}^{***} < \Lambda_2^{***}, \; \textit{or} \; \Lambda_1 < \Lambda_2^* < \Lambda_{21}^*.$$

3. *Traders' hedging intensity is increasing in the cost of trading it induces: $-1 < a_2^{***} < a_2^{**} < a_2^* < 0$, $-1 < a_{21}^* < a_{21}^{**} < a_{21}^{***} < 0$, and $-1 < a_1^* < a_1^{**} < a_1^{***} < 0$.*

Therefore, only the extreme equilibria are stable.[34] Additionally, at equilibrium, the traders belonging to the cohort that faces the *highest market impact demand more liquidity.* This is because the price impact induced by the endowment shock (affecting traders in cohort) t, has a proportionally stronger effect on the execution risk faced by cohort $s \neq t$ traders than on the expected return obtained by traders in cohort t.

An important implication of Corollary 5 is that when multiple equilibria arise, at the first round of trade, dealers tend to speculate more aggressively (that is, "consume" more liquidity) when the market is more illiquid. Indeed, with opacity the equilibrium coefficient of the speculative component of the first period strategy of a dealer is given by $\gamma a_1/\gamma_H$. Given part 3 of the above corollary, it follows that dealers speculate more aggressively in the equilibrium with

[33] With multiple equilibria we have that the price impacts of hedgers' endowment shocks are symmetric at the second round. That is, at an equilibrium where Λ_2 takes on the high value Λ_2^{***}, Λ_{21} takes on the low value Λ_{21}^{***} and the opposite occurs at the equilibrium in which Λ_2 takes on the low value Λ_2^* with the result that $\Lambda_2^{***} = \Lambda_{21}^*$ and $\Lambda_2^* = \Lambda_{21}^{***}$. Similarly, hedging intensities a_2 and a_{21} are also symmetric: a_2 reaches its lowest value, and a_{21} its highest at when Λ_2 is the highest (Λ_2^{***}), and Λ_{21} the lowest (Λ_{21}^{***}) with the result that $a_2^{***} = a_{21}^*$ and $a_2^* = a_{21}^{***}$. At the intermediate illiquidity equilibrium $\Lambda_2 = \Lambda_{21}$ and $a_2 = a_{21}$.

[34] The price impacts associated with extremal equilibria can provide estimates for the "bid/ask extremes" used by portfolio managers when rebalancing, to assess the risks of worst-case scenarios (see, e.g., Liquidity risks in markets are not intractable, *Financial Times, December 2024.*)

the highest illiquidity. This prediction is consistent with the findings in Brogaard et al. (2018) and Bellia et al. (2022). The former show that when extreme price movements occur across different securities, high frequency traders step up their liquidity demand. The latter argue that HFT consume liquidity during flash crashes, contributing to triggering or exacerbating these events.

Recall that $p_2 = -\Lambda_2\theta_2$, where $\theta_2 = u_2 + \beta u_1$ and $\beta = \Lambda_{21}/\Lambda_2$. Then $\beta < 1$ ($\beta > 1$) when at extremal equilibria $\Lambda_2 = \Lambda_2^{***}$ ($\Lambda_2 = \Lambda_2^{*}$); at the intermediate equilibrium or when we have a unique equilibrium, $\beta = 1$ since then $\Lambda_{21} = \Lambda_2$. Therefore, with a unique equilibrium $\beta = 1$ and the AR(1) "noise" process is a random walk, while at extremal multiple equilibria it is either stable or explosive. In the transparent market we have that $\beta < 1$ and the process is stable (see Appendix C for other properties of the "noise" trading process). With multiple equilibria, we have either that dealers absorb most of the second period hedgers' risk exposure and little of the first period ones with Λ_2 large and β small, or that the opposite occurs. Therefore, our model predicts that $\beta > 1$ is an indicator of multiple equilibria and liquidity fragility as we show in the next section.

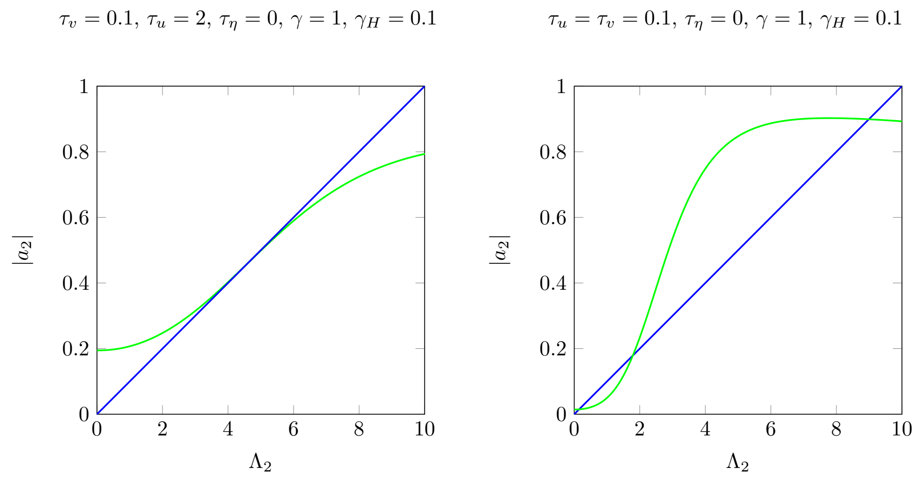
3.2 Liquidity demand and supply in a fully opaque market

The source of strategic complementarity discussed in the previous section suggests that when the market is opaque, liquidity demand can be an increasing function of the cost of trading it induces, that is, the *slope* of the function $|a_t(\Lambda_t)|$ can become positive, differently from what happens in the case where the market is fully transparent. To see this substitute (17a) and (17b) into (15a) and (15b) and take the absolute value of the resulting expressions to obtain the hedging aggressiveness. In the case of $|a_2|$, for example, this yields:

$$|a_2| = \left| \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \left(\tau_v^{-1} + \left(\frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2)\tau_v} \right)^2 \tau_u^{-1} \right)} \right|.$$

The above expression shows that an increase in Λ_2 has two countervailing effects on liquidity demand: a direct one (at the numerator), which lowers the returns from hedging and leads to a reduction in $|a_2|$ as in the fully transparent benchmark; an indirect one (at the denominator), which as illustrated in Figure 2 reduces execution risk and boosts liquidity demand. The impact of this second effect is displayed in Figure 4

In the figure, we plot $|a_2|$ (in green) as a function of the cost of trading it generates and the liquidity supply function (in blue) as a function of the hedging intensity it induces. The crossing points between the two curves occur at equilibrium. In Panel (a) and (b) we use the same parameterizations of the corresponding panels in Figure 3, and, respectively, a unique equilibrium and three equilibria obtain. As shown by the figure, and differently from what shown in Figure 1 with a fully transparent market, a higher Λ_2 leads second period traders to demand more liquidity ($|a_2|$ increases), which leads to the positive association between liquidity consumption and illiquidity when $\tau_\eta \to 0$.



(a) (b)

Figure 4: Liquidity demand and supply at the second round with a fully opaque market.

A liquidity "dry-up". Figure 4 also illustrates an important prediction of our model. Suppose the market is at a unique equilibrium and an unexpected shock boosts hedgers' endowment uncertainty. Then, the initial effect is that of reducing hedgers' liquidity demand. To see this, note that since $\varphi(\Lambda_2)$ is increasing in Λ_2, from (21) we obtain $\partial\varphi(\Lambda_2)/\partial\tau_u = (\gamma + \gamma_H)\Lambda_2\tau_v - 1$, which can be shown to be negative, implying that at the intermediate equilibrium, a decline in τ_u reduces Λ_2. Intuitively, a lower τ_u increases execution risk for 2nd period traders (the denominator in (15b)), lowering $|a_2|$, which reduces dealers' exposure to u_2 and thus Λ_2.[35]

Suppose now that, again starting at the unique equilibrium, the risk-bearing capacity of the market is unexpectedly lowered. Then, the initial effect is that of lowering dealers' liquidity supply, yielding a higher endowment shock price impact Λ_2: from (21) we obtain $\partial\varphi(\Lambda_2)/\partial\gamma > 0$, which via chain rule implies $\partial\Lambda_2/\partial\gamma < 0$.

Summing up, when τ_u or γ decline, the new aggregate best response becomes steeper at the intermediate equilibrium. This implies that:

Corollary 6. *An increase in the volatility of the endowment shock affecting liquidity traders or a decline in dealers' risk-bearing capacity heightens strategic complementarity at the intermediate equilibrium:*

$$\frac{\partial}{\partial\tau_u}\left(\frac{\partial\Phi(\Lambda_2)}{\partial\Lambda_2}\right)\bigg|_{\Lambda_2=\Lambda_2^{**}} < 0, \quad \frac{\partial}{\partial\gamma}\left(\frac{\partial\Phi(\Lambda_2)}{\partial\Lambda_2}\right)\bigg|_{\Lambda_2=\Lambda_2^{**}} < 0. \tag{23}$$

For a large enough shock (that fulfills condition condition (19)), the strengthening of strategic complementarity makes the effect on execution risk overpower that on expected returns, yielding multiple equilibria. When such a shock to τ_u occurs, all else equal, the old equilibrium

[35] At the unique equilibrium $\partial\Lambda_2/\partial\tau_u = -(\partial\varphi/\partial\tau_u)/(\partial\varphi/\partial\Lambda_2) > 0$, since the numerator in the expression is negative at the unique equilibrium.

value of illiquidity Λ_2 falls between Λ_2^{**} and Λ_2^{***}, and because of best-response adaptive dynamics, is attracted by the equilibrium with high Λ_2 (and low Λ_{21}). Similarly, when a large enough shock to γ occurs, all else equal, the old equilibrium value of illiquidity Λ_2 falls between Λ_2^{**} and Λ_2^{*}, and because of best-response adaptive dynamics, is attracted by the equilibrium with low Λ_2 (and high Λ_{21}). Furthermore, given the symmetry between the values for Λ_2 and Λ_{21} and for $|a_2|$ and $|a_{21}|$ at the extremal equilibria (see footnote 33), it is immediate that at those extremal equilibria total illiquidity ($WAPI$) and second period price volatility $\text{Var}[p_2] = (\Lambda_2^2 + \Lambda_{21}^2)\tau_u^{-1}$ are equal. This yields the following:

Corollary 7. *When the market is fully opaque and a unique equilibrium is obtained:*

1. *A shock increasing liquidity traders' endowment volatility which is large enough to induce multiple equilibria, leads the market to gravitate towards the extremal equilibrium with high Λ_2 (and low Λ_{21}) at the second round.*

2. *A shock increasing dealers' risk aversion which is large enough to induce multiple equilibria, leads the market to gravitate towards the extremal equilibrium with low Λ_2 (and high Λ_{21}) at the second round.*

3. *In both cases, the total illiquidity ($WAPI$) and second period price volatility $\text{Var}[p_2]$ are higher.*

Remark 1. *When endowment shock variances differ across periods, expressions (17a) and (17b) become:*

$$\Lambda_2 = \frac{\tau_{u_1}}{(\gamma_H \tau_{u_1} + \gamma(\tau_{u_1} + \Lambda_{21}^2 \tau_v))\tau_v} \tag{24a}$$

$$\Lambda_{21} = \frac{\tau_{u_2}}{(\gamma_H \tau_{u_2} + \gamma(\tau_{u_2} + \Lambda_2^2 \tau_v))\tau_v}, \tag{24b}$$

where $\tau_{u_t}^{-1}$ denotes the variance of endowment shock u_t. Expressions (24a) and (24b), confirm that the price impacts of endowment shocks are strategic substitutes: at the second round, an increase in Λ_2 lowers Λ_{21} (see (24b)); in turn, the reduction in Λ_{21} further increases Λ_2 (see (24a)). The aggregate best response (16), in this case is as follows:

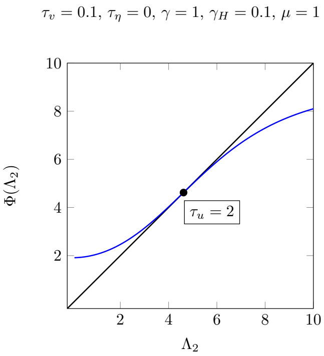
$$\Phi(\Lambda_2) \equiv \frac{((\gamma + \gamma_H)\tau_{u_2} + \gamma \Lambda_2^2 \tau_v)^2 \tau_{u_1}}{\gamma \tau_{u_2}^2 + ((\gamma + \gamma_H)\tau_{u_2} + \gamma \Lambda_2^2 \tau_v)^2 (\gamma + \gamma_H)\tau_{u_1}\tau_v}, \tag{25}$$

and is increasing in Λ_2, as (16), confirming the logic illustrated by Figure 2. Expression (25) can be used to see that, starting with the same parameters of Figure 4 (a), a shock that hits relatively more strongly the second period hedgers, yields a result qualitatively similar to Corollary 7.[36] *It is worth noting that if τ_{u_1} tends to infinity, then the Λ_2 tends to the transparent solution; indeed, in this case second period hedgers have no uncertainty about u_1.*

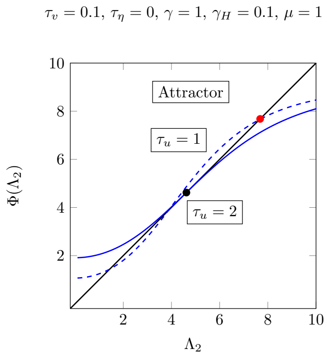
[36] For instance, if the status quo is like Figure 4, with $\gamma = 1$, $\gamma_H = 0.1$, $\tau_v = 0.1$, $\tau_{u_1} = \tau_{u_2} = 2$, then we obtain a unique equilibrium. Now, a heterogeneous increase in the variance of the endowment shocks (e.g., $\tau_{u_1} = 0.2$, $\tau_{u_2} = 0.1$, using (25), we see that multiplicity obtains and the equilibrium with high Λ_2 (and low Λ_{21}) is still the attractor.

A "flash-crash". The above result implies that when the market is opaque, an unanticipated increase in traders' endowment shocks' dispersion is conducive to a liquidity crash with an increase in total illiquidity and price volatility. One example would be the case in which hedgers are investment banks with a position in the asset. If uncertainty over their endowments increases unexpectedly and is perceived permanent (e.g., because of an unanticipated macro event such as the Covid pandemic or the war in Ukraine), an opaque market triggers the loop we described above leading to a crash, characterized by a much higher illiquidity (see the upper part -a, b, c- of Figure 5). When the additional uncertainty dissipates, and traders think again that the change is permanent, the market recovers, returning to the status quo ante, as in a "flash crash" (see the lower part -d, e, f- of Figure 5). The flash crash increases our measure of total illiquidity $WAPI$ by 44 per cent (from 4.62 to 6.67) and price volatility by 70% (from 4.62 to 7.87).

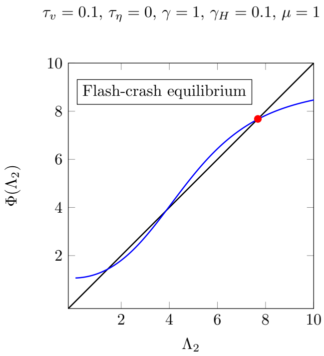
Similarly, an unanticipated increase in dealers' risk aversion or the volatility of the payoff, will lead to a liquidity crash. In Figure 6, we present the effect of a halving in τ_v (panel (a)) and an 11% decline in γ (panel (b)). In (a) the initial equilibrium moves to the low Λ_2 and high Λ_{21} equilibrium with an increase in total illiquidity $WAPI$ of 89% and price volatility increases by 138%. In (b) $WAPI$ increases by 20% and price volatility increases by 14%. It is worth noting that we have fragility (i.e., a large effect) also when the (small) γ parameter movement increases moderately strategic complementarity and preserves the uniqueness of equilibrium.



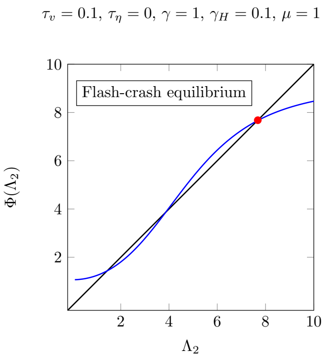
(a) Unique equilibrium



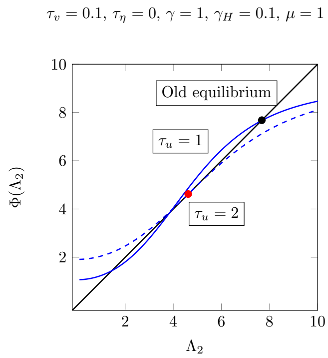
(b) Shock



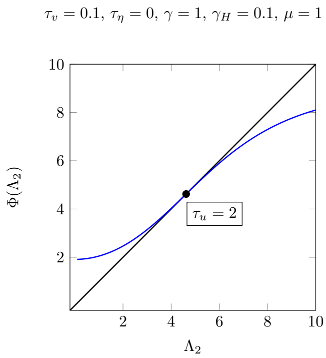
(c) Low liquidity eq.



(d) Low liquidity eq.



(e) Shock



(f) Unique equilibrium

Figure 5: An unanticipated, thought permanent, increase in endowment shock dispersion leading to a "flash crash." Starting from the unique stable equilibrium when $\tau_u = 2$ (panel (a)), an unanticipated increase in hedgers' endowment shock dispersion (with $\tau_u \downarrow 1$) increases the steepness of the best response (16) yielding three equilibrium points (panel (b)). Best response dynamics leads the market to temporarily gravitate towards the high illiquidity equilibrium (panel (c)). Once the endowment shock dispersion returns to its initial value ($\tau_u \uparrow 2$), the best response mapping moves to the right, and the market returns to its original equilibrium value (panels (d)–(f)).

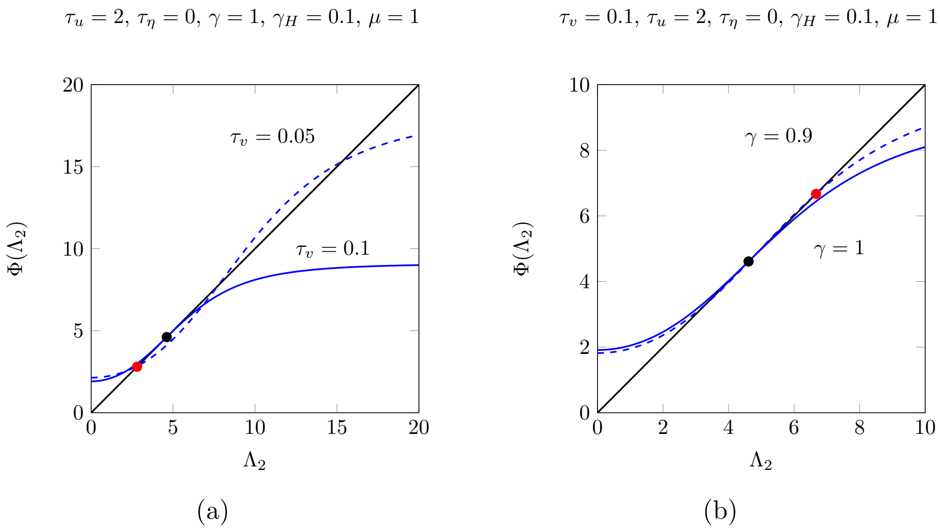


Figure 6: The effect of an unanticipated shock to the volatility of the risky security payoff (panel (a)) and dealers' risk tolerance (panel (b)). In panel (a), the shock to τ_v moves the market to the region with multiple equilibria. Due to the movement in the best response, the old equilibrium is unstable and the market gravitates towards the extremal equilibrium with low Λ_2 (and high Λ_{21}). In panel (b) the shock to γ shifts the best response preserving equilibrium uniqueness.

3.3 A partially opaque market

In a partially opaque market where second period hedgers receive a noisy signal with precision $\tau_\eta \in (0, \infty)$, prices are as in (1a) and (1b), and we have the following result:

Proposition 4. *With partial opacity, the equilibrium obtains as a solution to the system of non-linear, simultaneous equations* (A.21a)–(A.21c) *and* (A.33)*. The expressions for the equilibrium prices' coefficients* Λ_2, Λ_1, Λ_{21} *and* Λ_{22} *are as in* (A.33)*, and* (A.34a)-(A.34c)*. The coefficients of traders' strategies are as in Proposition 2, with* $\mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$ *and* $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$*, except for* b*, which is given by:*

$$b = \gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\mathrm{Var}_2[v - p_2]}. \tag{26}$$

At equilibrium, $\Lambda_2 > 0, \Lambda_{21} > \Lambda_1 > 0$*, and* $\Lambda_{22} < 0$*.*

Corollary 8. *As* $\tau_\eta \to \infty$*, we obtain the unique equilibrium of Proposition 1.*

For $\tau_\eta < \infty$, we are not able to analytically study the equilibrium due to complex nonlinearities and we resort to numerical simulations to investigate the properties of the model. As in the fully opaque case, we can have one or three equilibria. Multiple equilibria are obtained when transparency is low, in which case strategic complementarity, which also holds with partial opaqueness and is increasing in the degree of opaqueness, is high.

According to the above result, an informative signal about u_1 ($\tau_\eta \in (0, \infty)$) leads second period traders to speculate against the price pressure created by first period traders' liquidity demand, taking a contrarian position that increases in the signal's precision (in our simulations,

$b > 0$ and is increasing in τ_η), enhancing the risk-bearing capacity of the market. This dampens the strategic complementarity responsible for multiple equilibria and for τ_η large enough, leads to a unique equilibrium (see Figures 11 and 12).

In Figures 11 and 12 in the Appendix, we plot the price and strategy coefficients for one of our simulations. As shown in the figure, for τ_η small, three equilibria arise. We plot them using the colors green, blue and red to indicate the equilibrium that corresponds to the two extreme, stable price impacts (respectively in green and red) and the unstable one (in blue).

Importantly, when multiple equilibria obtain, order flow partial transparency does not modify an important conclusion we reached in Section 3.2: liquidity demand and illiquidity are positively related at equilibrium as in Corollary 4 with full opacity (see panels (a), (b), and (c) in Figures 11 and 12).

4 A market with restricted and (full) dealers and welfare

In this section, we consider an extension to the model where we assume that liquidity is also supplied by a class of dealers (of mass $1-\mu$) who can only trade in the first round and which we term "Restricted Dealers"–we denote them by RD and use D (of mass $0 < \mu < 1$) to denote the dealers we introduced in Section 1.1. Finally, we analyze the welfare properties of the model.

4.1 The market with restricted and full dealers

We assume here that in the first round, liquidity is provided by a mass $\mu \in (0, 1]$ of dealers D and a complementary mass $1 - \mu$ of RD (Restricted Dealers). An RD has CARA preferences with the same risk-tolerance γ as a D. However, as he is in the market only in the first period, he submits a price-contingent order x^{RD} to maximize the expected utility of his wealth $(v-p_1)x^{RD}$ which, as we show in the Appendix (see (A.18)), has the following expression: $x^{RD} = -\gamma\tau_v p_1$. The inability of RD to trade in the second period captures some liquidity suppliers' limited market participation. This friction could be due to technological reasons as in the case of dealers with impaired access to a technology that allows trading at high frequencies. Alternatively, it could arise from limited access to the trading venue, as in the case of those liquidity suppliers who in the 80s could not access the NYSE trading floor. Market clearing conditions are now:

$$\mu x_1^D + (1-\mu)x^{RD} + x_{11} = 0 \tag{27a}$$

$$(x_2^D - x_1^D)\mu + (x_{21} - x_{11}) + x_2 = 0 \iff \mu x_2^D + (1-\mu)x^{RD} + x_{21} + x_2 = 0, \tag{27b}$$

where in the latter we make use of the first period market clearing condition to obtain the expression at the right hand side of (27b). Figure 13 (in appendix D) displays the timeline of the model.

This version of the model is also analytically challenging, and we resort to numerical simulations to investigate its properties.

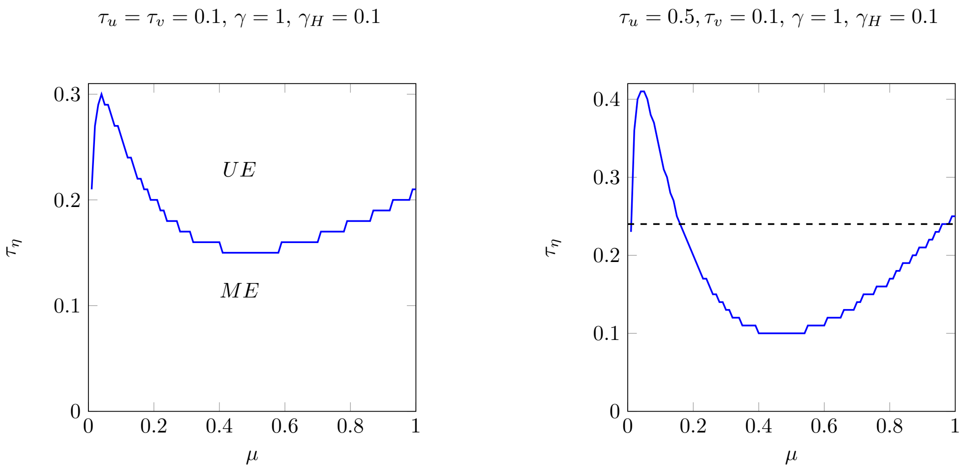
Unique and multiple equilibria. We first show that multiple equilibria also arise when second period traders observe an informative signal about u_1 and $\mu \in (0,1]$. In Figure 7 we partition the space $\mu \in (0,1], \tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium (three equilibria). According to the figure, uniqueness obtains when second period traders' signal is of sufficiently good quality, in line with the results of Sections 3 and 3.3. The effect of an increase in μ is less obvious. As the figure illustrates, we find that when τ_η is low, an increase in μ leads the market to switch from multiple equilibria to a unique equilibrium, and, eventually, back to multiple equilibria.[37] The bottom line is that with low order flow transparency, an increase in the mass of dealers that are continuously in the market has a non-monotonic effect on strategic complementarity. Thus, to eliminate fragility, enhancing transparency is key.

Effects on liquidity fragility of the dispersion of the endowment shock.[38] Comparing the areas below the blue curve in panel (a) and (b) in Figure 7 shows that that for $\mu \in (0,1)$, consistently with what we have found in Proposition 3, an increase in τ_u reduces the chances of liquidity fragility. For extreme values of μ (that is, for μ close to 0 or 1) the figure indicates that when τ_η is low an increase in τ_u increases the chances of liquidity fragility.[39]

[37]The intuition is as follows. Liquidity fragility is a byproduct of imperfect risk sharing and market opacity. When second period traders' information is noisy (τ_η low), as μ increases from zero, with multiple equilibria, initially risk sharing improves (as there are more dealers absorbing traders' liquidity demand). This stabilizes the market and improves liquidity for second period hedgers (i.e., lowering Λ_2). However the decline in Λ_2 means that first period traders face lower execution risk which boosts their liquidity demand, eventually heightening strategic complementarity and leading back to the region with multiple equilibria and liquidity fragility.

[38]The comparative statics results for τ_v and γ_H align with the intuition gained in Proposition 3 and are in Appendix D.

[39]The intuition is as follows. When the signal is not perfect 2nd period traders (1) may speculate in the "wrong" direction and (2) use p_2 and the signal to predict u_1. With a higher τ_u there is less noise in the price, which reduces second period traders speculative intensity. When μ is close to 0, almost only second period traders provide liquidity at the second round, and the reduction in speculation by these traders has a large impact on overall risk sharing. When μ is close to 1, almost only (full) dealers provide liquidity at 2 and the reduction in speculation by 2nd period traders means that dealers have less liquidity traders to share risk with. In either case this increases liquidity fragility. For intermediate values of μ, (full) dealers have a smaller exposure to the risky security, and the additional risk sharing provided by 2nd period traders is less important. In this case, the reduction in these traders' speculation rids the market of the "wrong" trades with a positive impact on liquidity fragility.



(a) (b)

Figure 7: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.

The effect of changes in the mass of dealers D (μ) and order flow transparency (τ_η) on liquidity fragility. It is highly non-linear. An increase in μ may be destabilizing for τ_η low (as shown in Figure 7 (a) we may move from a UE to the ME region), and a small decrease in μ may induce a liquidity crash for μ large and τ_η low as we see below.

A small shock to the mass of dealers. In Figure 8, when the market is initially at an equilibrium with a low Λ_2 (in panel (a) $\Lambda_2^* = 1.47$), a 11% reduction in the mass of D (from $\mu = 0.9$ to $\mu = 0.8$), plunges the market to the polar opposite equilibrium ($\Lambda_2 = 9.6$, corresponding to a 653% increase in second period price impact with total illiquidity going from $WAPI = 5.7$ to $WAPI = 10.3$, an 80% increase. This may explain how the disconnection of a small percentage of dealers from the market (say because of a technical problem) can cause a liquidity crash. We can show also when $\tau_v = 0.1$, $\tau_u = 1.9$, $\tau_\eta = 0$, $\gamma = 1$, $\gamma_H = .1$ that a 10% reduction in the mass of D (from $\mu = 1$ to $\mu = 0.9$), increases total illiquidity from 4.55 to 6.19, an 36% increase without generating multiple equilibria. This is so due to a moderate but sufficient increase in strategic complementarity (see Appendix D for the figure).

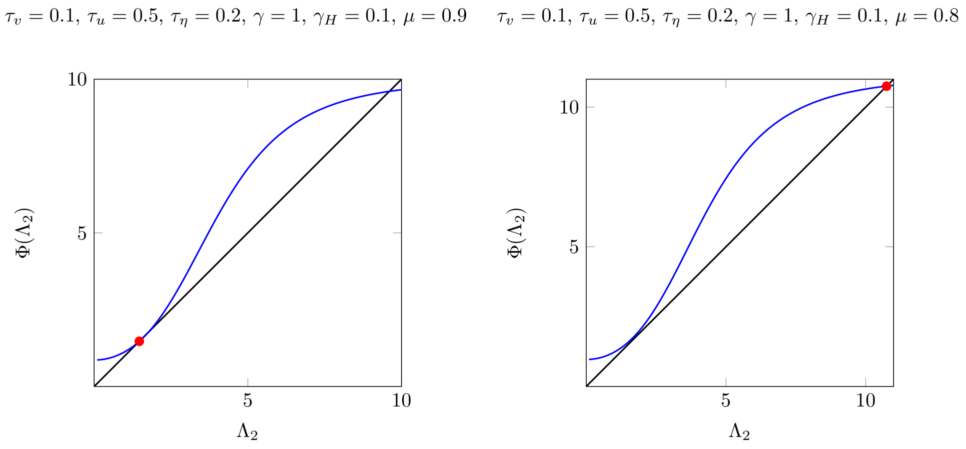


Figure 8: The effect of a small reduction in μ when τ_η is low. Total illiquidity increases.

4.2 Welfare analysis

In this section, we study the welfare implications of the general version of the model of Section 4.1. Denoting by EU^D, EU^{RD}, and EU^H_t, respectively the unconditional expected utilities of D, RD and round $t \in \{1, 2\}$ hedgers, we measure traders' payoffs by computing their certainty equivalents:

$$CE^D = -\gamma \ln(-EU^D),\ CE^{RD} = -\gamma \ln(-EU^{RD}),\ CE^H_t = -\gamma_H \ln(-EU^H_t).$$

The next result provides a condition for traders' payoffs to be well-defined.

Proposition 5. *If*

$$\gamma_H^2 \tau_u \tau_v > 1, \tag{28}$$

traders' payoffs are well defined and their expressions are given in the appendix (see (A.57), (A.60), (A.63), and (A.66)). When $\tau_\eta = 0$ and $\mu = 1$, (28) implies that the equilibrium is unique.

Using (A.57), (A.60), (A.63), and (A.66), we define the total (utilitarian) welfare of market participants as follows:

$$TW(\mu; \tau_\eta) = \mu CE^D + (1 - \mu)CE^{RD} + CE^H_1 + CE^H_2. \tag{29}$$

We then numerically evaluate (29) to assess the welfare properties of the unique equilibrium as either the market becomes less opaque (τ_η increases), or the mass of D increases (μ increases). In this case, we assume $\gamma = \gamma_H = 1$, $\tau_v = 1$, $\tau_u = 2$. With this set of parameters, we solve for the equilibrium of the market and compute traders' payoffs and $TW(\mu; \tau_\eta)$, for $\mu \in \{0.1, 0.2, \ldots, 1\}$ and $\tau_\eta \in \{0.1, 25, 50, 75, 100\}$.

Regarding the welfare ranking with multiplicity, whenever multiple equilibria obtain, hedgers' payoffs (that is (A.63), and (A.66)) are complex-valued functions, which prevents obtaining a general welfare ranking result across equilibria. Turning now to the welfare properties of the unique equilibrium, our numerical simulations yield the following result:

Numerical Result 1. *When a unique equilibrium obtains, $TW(\mu;\tau_\eta)$ is increasing in μ and τ_η. The total welfare improvement is driven by the increase of CE_t^H, $t = 1, 2$, with μ and τ_η. CE^{RD} decreases with τ_η as well as CE^D when τ_η is not too small and CE^D decreases in μ.*

Therefore, policies aimed at increasing market transparency and/or increase the mass of dealers who are always in the market to supply liquidity, achieve a higher total welfare by effecting a welfare transfer from liquidity providers to liquidity consumers.

A trade-off between transparency and dealer participation As we argued in Section 3.3, transparency also spurs second period hedgers' speculative activity, reducing the rewards to liquidity provision enjoyed by dealers. This suggests that an increase in transparency may come at the expense of a reduced dealer participation to the market. Entry (or exit) of dealers will be affected by their prospective profits.[40] To characterize such potential trade off, we have simulated the version of our model with "Restricted dealers."

In Figure 9 we present the results of one such simulation, where we set $\tau_u = 2$, $\tau_\eta = 10$, $\gamma = \gamma_H = 1$, and $\mu = 0.5$ (corresponding to a volatility of about $\sqrt{\text{Var}[p_2 - p_1]} \approx 47\%$), and look for the reduction in the mass of dealers that is needed to keep total welfare constant when transparency increases in the two regimes of high and low payoff volatility (respectively, $\tau_v = 1$ and $\tau_v = 3$). The plots in the figure illustrate the negative relationship between transparency and dealers' market participation that is implied by our model. Interestingly, such relationship is steeper and tighter when the payoff is *less* volatile (blue curve). This suggests that the effect of transparency on dealers' market participation depends on the riskiness of the security being traded. For riskier securities ($\tau_v = 1$), to keep total welfare constant an increase in market transparency calls for a smaller dealers' participation reduction compared to safer securities ($\tau_v = 3$). All else equal, a riskier security augments hedgers' need to share risk, boosting their liquidity demand and making their payoffs lower and the payoff of dealers higher. Therefore, in this case the competitive effect due to second period hedgers' increased speculation spurred by an increase in transparency has a smaller impact on dealers' prospective profits.

[40]Significant concerns have arisen from the exit of dealers or from the high costs of inducing them to supply liquidity (e.g., O'Hara and Zhou (2021), and Allen et al. (2024)).

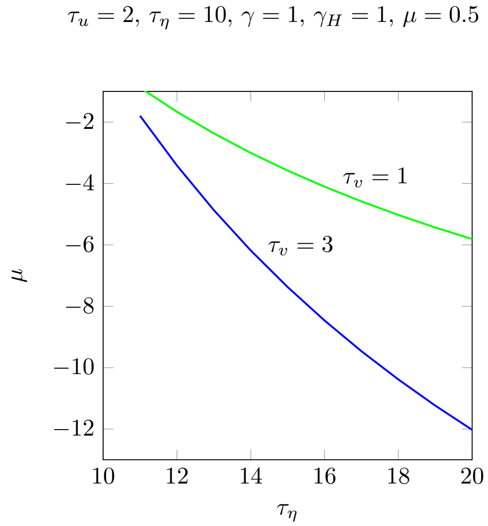


Figure 9: The percentage change in μ needed to keep total welfare constant against an increase in transparency when payoff volatility is high ($\tau_v = 1$) and low ($\tau_v = 3$).

This indicates that moderate increases in transparency may induce a reduction in the mass μ of dealers always present in the market which may hurt welfare, in particular for riskier securities. This is so since the smaller is the impact on μ of a given increase in transparency to keep welfare constant, the easier it is to be achieved with a moderate increase in the costs of entry of dealers and therefore for welfare to decrease.[41] Non-dealer liquidity provision may partially compensate for this exit (see the evidence in Hendershott et al. (2021) for the corporate bond market). Overall, our model's prediction is consistent with Bessembinder and Maxwell (2008) who find that the introduction of the TRACE system in the US corporate bond market in 2002 reduced trading costs for investors and negatively impacted dealers, who experienced a reduction in employment and compensation.

5 Opacity vs. transparency

In this section we take stock of our model's features, and present a detailed comparison of the polar cases of transparency (Section 2) and opacity (Section 3) for two scenarios: a "normal" volatility scenario, in which we set $\tau_v = 1$, $\tau_u = 2$, $\gamma = \gamma_H = 1$, and $\mu = 1$ (with opacity these yield a return volatility of $\sqrt{\text{Var}[p_2 - p_1]} \approx 30\%$, which is consistent with Yuan (2005)), and a "liquidity crisis" scenario, in which we set $\tau_v = \tau_u = .1$, $\gamma = \gamma_H = 1$, and $\mu = 1$. In this case, instead, return volatility is way higher.

In tables 1 and 2, we collect the results of two simulations in which we report the equilibrium strategy and price coefficients, the payoffs and total welfare, as well as $WAPI$ in the two scenarios of normal volatility and liquidity crisis. Note that second period hedgers' ag-

[41] In our setup, an increase in transparency boosts second period hedgers' speculation which erodes dealers' profits and increases total welfare. To keep total welfare constant, we thus need μ to go down, that is some dealers' exit is required. In a model with free entry, this is equivalent to an increase in entry cost, which also leads dealers to exit. So, the effect (on μ) of the increase in transparency is equivalent in our model to the effect of the increase in entry cost on μ in a model with free entry.

gressiveness $|a_2|$ is higher in a transparent market in the normal volatility scenario (Table 1). The same ranking also applies to the price impact of the second period endowment shock (Λ_2). This is consistent with what said above. However, when contrasting $WAPI$ between the two regimes, with opacity the market is *more illiquid* than with transparency. This reflects the fact that in a transparent market second period hedgers speculate more aggressively against u_1. This reduces the net liquidity demand absorbed by dealers, lowering Λ_{21} and leading to a lower value for $WAPI$ compared to the case with opacity. With the "liquidity crisis" parameterization the effect of second period speculation is much weaker (compare the values of b for the transparency case across tables), which explains why $WAPI$ is higher with transparency than with opacity for such parameterization (see Table 2 where with opacity there are multiple equilibria). Figure 10 provides evidence that for parameter values encompassing the normal volatility scenario, opacity makes total illiquidity higher while in the "liquidity crisis" scenario the opposite happens.

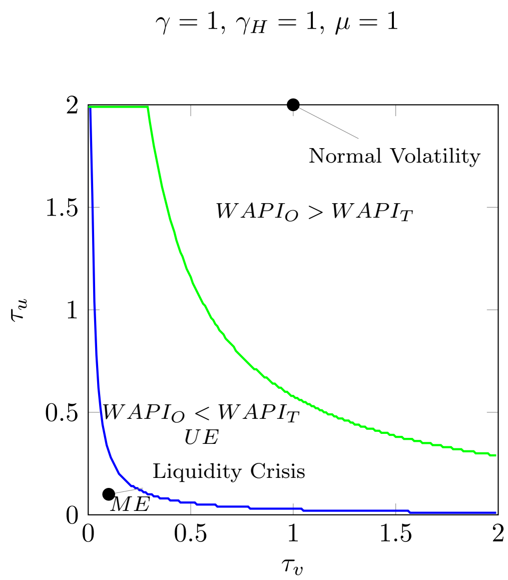


Figure 10: The figure partitions the parameter space into three regions: below (above) the blue curve, multiple equilibria (a unique equilibrium) obtain with opacity, and below (above) the green curve $WAPI$ is lower (higher) with opacity compared to transparency. The two black dots correspond to the parameter values that identify our calibration exercises.

Table 3 displays the results of a simulation in which we take as baseline parameters the ones of the "normal" volatility scenario, assuming some transparency ($\tau_\eta = 0.5$, corresponding to $\text{Var}[p_2 - p_1] \approx 30\%$) and investigate the effect of increasing τ_η. The results show that an increase in transparency increases the aggressiveness of the second period hedgers, and relatedly, the exposure of dealers to u_2, which increases the second period price impact of the endowment shock Λ_2. However, greater transparency also leads to stronger speculation (higher b), which reduces the net liquidity demand of first-period hedgers that is cleared by the dealers, and moderates the impact of signal noise trades on illiquidity, resulting in a reduction in $WAPI$.

From Numerical Result 1, we know that TW is monotonically increasing in τ_η, but this is not the case for $WAPI$. In Figure 16 (see Appendix D), we plot TW and $WAPI$ as functions

$\lvert a_1\rvert$	$\lvert a_{21}\rvert$	$\lvert a_2\rvert$	b	CE_1^H	CE_2^H	CE^D	CE^{RD}	TW	Λ_1	Λ_{21}	Λ_2	$WAPI$
						Transparency						
0.144	0.615	0.5	0.308	−0.168	−0.216	0.100	0.020	−0.284	0.290	0.308	0.499	0.423
						Opacity						
0.224	0.473	0.473	0	−0.233	−0.235	0.150	0.047	−0.318	0.448	0.473	0.473	0.473

Table 1: Comparing the case with transparency and opacity in the "normal" volatility scenario ($\tau_v = 1$, $\tau_u = 2$, $\gamma = \gamma_H = 1$, $\mu = 1$).

$\lvert a_1\rvert$	$\lvert a_{21}\rvert$	$\lvert a_2\rvert$	b	CE_1^H	CE_2^H	CE^D	CE^{RD}	TW	Λ_1	Λ_{21}	Λ_2	$WAPI$
						Transparency						
0	0.038	0.5	0.019	n/a	n/a	1.630	0	n/a	0.014	0.189	5	4.824
						Opacity						
0.001	0.043	0.456	0	n/a	n/a	1.55	0	n/a	0.038	0.438	4.561	4.2
0.034	0.184	0.184	0	n/a	n/a	1.33	0.19	n/a	0.682	1.847	1.847	1.847
0.208	0.456	0.043	0	n/a	n/a	3.02	1.453	n/a	4.161	4.561	0.438	4.2

Table 2: Comparing the case with transparency and opacity in the "liquidity crisis" scenario ($\tau_v = \tau_u = 0.1$, $\gamma = \gamma_H = 1$, $\mu = 1$). Note that with this parametrization, condition (19) is satisfied, which yields multiple equilibria with opacity.

of τ_η. The figure confirms that TW is monotonically increasing in τ_η, and shows that $WAPI$ is U-shaped in transparency. As τ_η increases second period hedgers demand more liquidity and speculate more aggressively against u_1. These two effects have an opposite impact on $WAPI$. Furthermore, as μ increases, more dealers enter the market and hedgers benefit from improved intermediation (TW increases), while the positive effect of speculation becomes less important, which exacerbates the U-shaped behavior of $WAPI$. Specifically, for τ_η small, $WAPI$ is smaller for $\mu = 0.8$ than for $\mu = 0.1$. However, this ranking reverses as τ_η grows larger.

$\lvert a_1\rvert$	$\lvert a_{21}\rvert$	$\lvert a_2\rvert$	b	CE_1^H	CE_2^H	CE^D	CE^{RD}	TW	Λ_1	Λ_{21}	Λ_2	$WAPI$
						$\tau_\eta = 0.5$						
0.211	0.494	0.477	0.047	−0.224	−0.233	0.143	0.042	−0.314	0.422	0.447	0.477	0.415
						$\tau_\eta = 1$						
0.201	0.510	0.479	0.081	−0.217	−0.231	0.137	0.039	−0.311	0.403	0.428	0.479	0.394
						$\tau_\eta = 1.5$						
0.195	0.522	0.481	0.108	−0.211	−0.230	0.133	0.037	−0.308	0.390	0.414	0.481	0.382
						$\tau_\eta = 2$						
0.190	0.531	0.483	0.128	−0.207	−0.229	0.129	0.035	−0.306	0.379	0.403	0.483	0.374

Table 3: The effect of an increase in transparency in the normal volatility scenario ($\tau_v = 1$, $\tau_u = 2$, $\gamma = \gamma_H = 1$, $\mu = 1$, and $\tau_\eta \in \{0.5, 1, \ldots, 2\}$).

6 Concluding remarks

Our model predicts that market opacity can make markets fragile (with multiple equilibria) and impair the rationing function of the cost of trading (i.e., illiquidity). Furthermore, it also predicts that trading costs are heterogeneous when the market is fragile. The model provides a plausible explanation for several recent events in which market liquidity "crashes" without any observable change in the value of the risky asset. In these events, it looks as if traders chased liquidity while liquidity suppliers withdrew from the market.[42] We argue that opacity of the trading process can be responsible for this effect, as it can severely impair the market participation of "non-standard" liquidity suppliers. Similarly, our model is also consistent with the narrative of the impact of the COVID-19 pandemic on the US Treasury market. On March 12, 2020, the World Health Organization declared COVID-19 a global pandemic, and liquidity deteriorated in the US Treasury market, with spreads increasing tenfold compared to their normal level and depth virtually disappearing at times (Duffie (2023)).

We also find that for high enough transparency, we have always equilibrium uniqueness, and total welfare is increasing in both the mass of dealers always present in the market and the degree of transparency. This offers a justification for policies aimed at enhancing access to order flow information and for enhanced continuous presence of dealers in the market.

There is a particular worry about fragility in the US Treasury market, where liquidity has deteriorated while the market has enlarged substantially, increasing the odds of a financial accident.[43] Given the documented decline in quoted depth over the past twenty years, this should reinforce regulatory concerns over the paucity of *public, affordable* order flow information in current markets. There is also concern about the quality of the liquidity provided by non-standard liquidity suppliers such as hedge funds and HFT firms, particularly due to their lack of transparency and potential discontinuity in their liquidity provision. In February 2024, the SEC passed the so-called "dealer rule" to force those liquidity providers to register as dealers and to make public the transactions for "on-the run" bonds.[44] Our model is thus in line with the calls to increase the resilience and price transparency in the US Treasury market.[45]

[42]In a somewhat related manner Menkveld and Yueshen (2019) attribute the flash-crash of May 6, 2010 to the fleeing of cross-market arbitrageurs from the E-mini market.

[43]See, e.g. "Fed Frets About Shadow Banks and Eyes Treasury Liquidity in New Report," New York Times, November 4, 2022, and https://www.ft.com/content/632411eb-c3fa-4351-a3b6-b0e30bdc0ef7 A recent financial stability report of the Fed states: "The continued low level of market depth means that liquidity remains more sensitive to the actions of liquidity providers that use high-frequency trading strategies to replenish the order book rapidly." "Greater concentration of liquidity provision among firms that may follow similar strategies can be a source of fragility, making it more likely that liquidity could further deteriorate sharply in response to future shocks." (November 2002, https://www.federalreserve.gov/publications/files/financial-stability-report-20221104.pdf).

[44]See "Bond market liquidity squeeze keeps regulators alert to risks", H. Clarfelt, *Financial Times, 2/05/2024* and https://www.sec.gov/files/rules/sro/finra/2024/34-99487.pdf

[45]For example, Duffie (2023) states that "Improving post-trade price transparency with the real-time publication of Treasuries transactions would also improve market intermediation capacity through a more efficient matching of specific types of trades to specific dealer balance sheets". PIMCO states: "[I]n our view, an effective all-to-all platform for Treasuries would function similarly to a utility and would 1) include all legitimate, professional market participants; 2) require that participants trade under the same rules with the same access to price, information . . . " https://www.pimco.com/en-us/insights/viewpoints/in-depth/how-can-policymakers-improve-the-functioning-of-the-us-treasury-market

Our work also offers an additional argument in support of the introduction of a "consolidated tape" in the EU trading venues. Indeed, the level of stock market fragmentation in the EU is higher than in the US. However, differently from their US peers, traders in the EU cannot rely on a common signal displaying the best quotes available across trading venues. To obtain such a "consolidated" market view, they need to piece together the more expensive feeds offered by each exchange, which creates a suboptimal two-tiered market (Brogaard et al. (2021)). In an attempt to level the playing field, the European Commission is seeking to introduce the supply of a consolidated tape, at a reasonable price. However, this effort is facing fierce resistance from exchanges arguing that handing over data will reduce their revenues and may hurt the small ones.[46] We do not see the consolidated tape as a sure remedy against flash events (e.g., the US market has had a tape since the introduction of RegNMS and has had flash events) but we view the availability of reliable and prompt market information as an important ingredient that can help reducing the likelihood of market disruptions.

[46]See "EU faces last-ditch challenge from exchanges over trading reforms", N. Asgari, *Financial Times, April 2023.*

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Appendices

A Proofs

The following is a standard results (see, e.g. Vives (2008), Technical Appendix, pp. 382–383) that allows us to compute the unconditional expected utility of market participants.

Lemma 1. *Let the n-dimensional random vector $z \sim N(0, \Sigma)$, and $w = c + b'z + z'Az$, where $c \in \mathbb{R}$, $b \in \mathbb{R}^n$, and A is a $n \times n$ matrix. If the matrix $\Sigma^{-1} + 2\rho A$ is positive definite, and $\rho > 0$, then*

$$E[-\exp\{-\rho w\}] = -|I + 2\rho\Sigma A|^{-1/2} \exp\{-\rho(c - \rho b'(\Sigma + 2\rho A)^{-1} b)\}.$$

We now derive the equilibrium for the general case in which $\tau_\eta \in (0, \infty)$ and $\mu \in (0, 1]$ that we discuss in Section 4.1. The two benchmarks with full transparency ($\mu = 1$ and $\tau_\eta \to \infty$) and full opacity ($\mu = 1$ and $\tau_\eta \to 0$) obtain as special cases of this result.

Proposition A.1. *When $\mu \in (0, 1]$ and $\tau_\eta \in (0, \infty)$, at a linear equilibrium:*

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22} \eta \tag{A.1a}$$

$$p_1 = -\Lambda_1 u_1 \tag{A.1b}$$

where the coefficients in the above expressions obtain as a solution to the following system of non-linear, simultaneous equations:

$$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v} \tag{A.2a}$$

$$\Lambda_{21} = -\frac{b + a_{21} + (1-\mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v} \tag{A.2b}$$

$$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v} \tag{A.2c}$$

$$\Lambda_1 = -\frac{\mu\gamma + \gamma_H}{\gamma\gamma_H\tau_v} a_1, \tag{A.2d}$$

and expressions for a_2, b, a_{21}, and a_1 are respectively given by

$$a_2 = \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \mathrm{Var}_2[v - p_2]} \tag{A.3a}$$

$$b = \gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\mathrm{Var}_2[v - p_2]} \tag{A.3b}$$

$$a_{21} = \frac{\gamma_H \Lambda_{21} \tau_v - 1}{\tau_v \mathrm{Var}_1[v - p_2]} \tag{A.3c}$$

$$a_1 = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]}, \tag{A.3d}$$

and

$$\text{Var}_2[v - p_2] = \frac{1}{\tau_v} + \frac{(\Lambda_{21} - \Lambda_{22})^2}{\tau_\eta + \tau_u} \tag{A.4a}$$

$$\text{Var}_1[v - p_2] = \frac{1}{\tau_v} + \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta} \tag{A.4b}$$

$$\text{Var}_1[p_2] = \Lambda_2^2 \tau_u^{-1} + \Lambda_{22}^2 \tau_\eta^{-1}. \tag{A.4c}$$

At equilibrium, $\Lambda_2 > 0, \Lambda_{21} > \Lambda_1 > 0$*, and* $\Lambda_{22} < 0$.

Proof. Based on the market clearing condition (3), to pin down p_2 we need the strategies of first and second period traders, and dealers. We work by backward induction. In the second period, CARA and normality assumptions imply that the objective function of a liquidity trader is given by

$$E_2[-\exp\{-\pi_2/\gamma_H\}] = -\exp\left\{ -\frac{1}{\gamma_H}\left(E_2[\pi_2] - \frac{1}{2\gamma_H}\text{Var}_2[\pi_2] \right) \right\}, \tag{A.5}$$

where $\pi_2 \equiv (v - p_2)x_2 + u_2 v$. Therefore, a second period hedger's utility maximization problem reads as follows:

$$E_2[-\exp\{-\pi_2/\gamma_H\}] = \tag{A.6}$$
$$-\exp\left\{ -\frac{1}{\gamma_H}\left(E_2[v - p_2]x_2 - \frac{1}{2\gamma_H}\left(\text{Var}_2[v - p_2]x_2^2 + u_2^2\tau_v^{-1} + 2x_2u_2\text{Cov}_2[v, v - p_2] \right) \right) \right\}.$$

Maximizing (A.6) with respect to x_2, and solving for the optimal strategy yields:

$$x_2 = \gamma_H \frac{E_2[v - p_2]}{\text{Var}_2[v - p_2]} - \frac{\text{Cov}_2[v - p_2, v]}{\text{Var}_2[v - p_2]} u_2, \tag{A.7}$$

where,

$$E_2[v - p_2] = \Lambda_2 u_2 + \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{\tau_\eta + \tau_u} s_{u_1} \tag{A.8a}$$

$$\text{Var}_2[v - p_2] = \frac{1}{\tau_v} + \frac{(\Lambda_{21} - \Lambda_{22})^2}{\tau_\eta + \tau_u} \tag{A.8b}$$

$$\text{Cov}_2[v - p_2, v] = \frac{1}{\tau_v}, \tag{A.8c}$$

and the second order condition reads as follows:

$$-\frac{\text{Var}_2[v - p_2]}{\gamma_H} < 0, \tag{A.9}$$

which is always satisfied since $\gamma_H > 0$. Substituting (A.8a) and (A.8c) in (A.7), and rearranging yields:

$$X_2(u_2, s_{u_1}) = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_v \mathrm{Var}_2[v - p_2]}}_{a_2} u_2 + \underbrace{\gamma_H \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\mathrm{Var}_2[v - p_2]}}_{b} s_{u_1}. \tag{A.10}$$

A dealer maximizes the expected utility of his second period wealth:

$$E_2^D\Big[-\exp\Big\{-\frac{1}{\gamma}\Big((p_2 - p_1)x_1^D + (v - p_2)x_2^D\Big)\Big\}\Big] = \tag{A.11}$$
$$= \exp\Big\{-\frac{1}{\gamma}(p_2 - p_1)x_1^D\Big\}\Big(-\exp\Big\{-\frac{1}{\gamma}\Big(E_2^D[v - p_2]x_2^D - \frac{(x_2^D)^2}{2\gamma}\mathrm{Var}_2^D[v - p_2]\Big)\Big\}\Big).$$

For given x_1^D the above is a concave function of the second period strategy x_2^D. Solving the first order condition, yields that a second period D's strategy is given by:

$$X_2^D(p_1, p_2) = \gamma \frac{E_2^D[v - p_2]}{\mathrm{Var}_2^D[v - p_2]}. \tag{A.12}$$

Computing expectation and variance in the above expression:

$$E_2^D[v - p_2] = -p_2 \tag{A.13a}$$
$$\mathrm{Var}_2^D[v - p_2] = \frac{1}{\tau_v}, \tag{A.13b}$$

and substituting these in x_2^D yields:

$$X_2^D(p_1, p_2) = -\gamma \tau_v p_2. \tag{A.14}$$

Similarly, due to CARA and normality, in the first period a Restricted Dealer (RD) maximizes

$$E_1^{RD}\Big[-\exp\Big\{-\frac{1}{\gamma}(v - p_1)x_{11}^{RD}\Big\}\Big] = \tag{A.15}$$
$$-\exp\Big\{-\frac{1}{\gamma}\Big(E_1^{RD}[v - p_1]x_{11}^{RD} - \frac{(x_{11}^{RD})^2}{2\gamma}\mathrm{Var}_1^{RD}[v - p_1]\Big)\Big\}.$$

Maximizing the above and solving for x_{11}^{RD} yields:

$$x_{11}^{RD}(p_1) = \gamma \frac{E_1^{RD}[v - p_1]}{\mathrm{Var}_1^{RD}[v - p_1]}. \tag{A.16}$$

Computing the conditional expectation and variance:

$$E_1^{RD}[v - p_1] = -p_1 \tag{A.17a}$$
$$\mathrm{Var}_1^{RD}[v - p_1] = \frac{1}{\tau_v}, \tag{A.17b}$$

so that

$$X_1^{RD}(p_1) = -\gamma\tau_v p_1. \tag{A.18}$$

At the second round, first and second period traders face the same utility maximization problem. This is because they both need to hedge the endowment shock, and have only one round to go. As a consequence, the second order condition looks similar to (A.9), and a first period trader's strategy reads as follows:

$$\begin{aligned} X_{21}(u_1) &= \gamma_H \frac{E_1[v - p_2]}{\text{Var}_1[v - p_2]} - \frac{\text{Cov}_1[v, v - p_2]}{\text{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{\gamma_H \Lambda_{21}\tau_v - 1}{\tau_v \text{Var}_1[v - p_2]}}_{a_{21}} u_1, \end{aligned} \tag{A.19}$$

where

$$\text{Var}_1[v - p_2] = \frac{1}{\tau_v} + \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta}. \tag{A.20}$$

aubstituting (A.10), (A.14), (A.18), and (A.19) in (3), solving for p_2 and identifying the equilibrium price coefficients yields:

$$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v} \tag{A.21a}$$

$$\Lambda_{21} = -\frac{b + a_{21} + (1-\mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v} \tag{A.21b}$$

$$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v} \tag{A.21c}$$

According to (A.21a), at an equilibrium

$$\Lambda_2 = \frac{1}{(\gamma_H + \mu\gamma\tau_v \text{Var}_2[v - p_2])\tau_v}, \tag{A.22}$$

so that at equilibrium $\Lambda_2 > 0$, and $\gamma_H \Lambda_2 \tau_v < 1$. Based on the expression for a_2 in (A.10), this implies that

$$a_2 \in (-1, 0). \tag{A.23}$$

To obtain the first period equilibrium price, we need to pin down the expressions for dealers' and liquidity traders' first period strategies. Starting from the latter, we obtain the second period value function of a first period trader by substituting (A.19) into the trader's objective function:

$$E_1[-\exp\{-((v - p_2)x_{21} + vu_1)/\gamma_H\}] = -\exp\{-(\text{Var}_1[v - p_2]x_{21}^2 - \text{Var}[v]u_1^2)/2\gamma_H^2\}. \tag{A.24}$$

As a consequence, at the first round, the trader's objective function reads as follows:

$$
\begin{aligned}
& E_1[-\exp\{-\pi_1/\gamma_H\}] \qquad \text{(A.25)} \\
&\quad = E_1[-\exp\{-((p_2-p_1)x_{11} + (\text{Var}_1[v-p_2]a_{21}^2 u_1^2 - \text{Var}[v]u_1^2)/2\gamma_H)/\gamma_H\}] \\
&\quad = E_1[-\exp\{-((p_2-p_1)x_{11} + \underbrace{((\text{Var}_1[v-p_2]a_{21}^2 - \text{Var}[v])/2\gamma_H)}_{C} u_1^2)/\gamma_H\}],
\end{aligned}
$$

where $\pi_1 = vu_1 + (v-p_2)x_{21} + (p_2-p_1)x_{11}$. Using the expression for p_2 in (1b), the argument of the exponential in the latter expression of (A.25) can be written as follows:

$$
(p_2-p_1)x_{11} + Cu_1^2 = -(\Lambda_{21}-\Lambda_1)u_1x_{11} + Cu_1^2 - (\Lambda_2u_2 + \Lambda_{22}\eta)x_{11}, \qquad \text{(A.26)}
$$

which is a quadratic form of the normal random variable $Z \equiv -(\Lambda_2u_2+\Lambda_{22}\eta)|u_1 \sim N(0, \text{Var}_1[p_2-p_1])$ (the constant multiplying the squared term of Z in the quadratic form is in this case null), where

$$
\text{Var}_1[p_2-p_1] = \text{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}. \qquad \text{(A.27)}
$$

We can then write the objective function of a trader at the first round as follows:

$$
E[-\exp\{-\pi_1/\gamma_1\}|u_1] = -\exp\{-(-(\Lambda_{21}-\Lambda_1)u_1x_{11}+Cu_1^2-x_{11}^2\text{Var}_1[p_2-p_1]/2\gamma_H)/\gamma_H\}. \qquad \text{(A.28)}
$$

Maximizing the above function with respect to x_{11} yields

$$
X_{11}(u_1) = \underbrace{-\gamma_H\frac{\Lambda_{21}-\Lambda_1}{\text{Var}_1[p_2]}}_{a_1} u_1, \qquad \text{(A.29)}
$$

where the second order condition reads as follows: $-\text{Var}_1[p_2-p_1]/\gamma_H < 0$. We now obtain the strategy of a liquidity provider. Substituting a D's second period strategy (A.12) in (A.11), rearranging and applying Lemma 1 yields the following expression for the first period objective function of a D:

$$
\begin{aligned}
& E_1^D[U((p_2-p_1)x_1^D + (v-p_2)x_2^D)] = -\left(1+\frac{\text{Var}_1^D[p_2]}{\text{Var}[v]}\right)^{-1/2} \times \qquad \text{(A.30)} \\
& \exp\left\{-\frac{1}{\gamma}\left(\frac{\gamma\tau_v}{2}(E_1^D[p_2])^2 + (E_1^D[p_2]-p_1)x_1^D - \frac{(x_1^D+\gamma\tau_vE_1^D[p_2])^2}{2\gamma}\left(\frac{1}{\text{Var}_1^D[p_2]}+\frac{1}{\text{Var}[v]}\right)^{-1}\right)\right\},
\end{aligned}
$$

where

$$
E_1^D[p_2] = -\Lambda_{21}u_1 \qquad \text{(A.31a)}
$$

$$
\text{Var}_1^D[p_2] = \frac{\Lambda_{21}^2}{\tau_u} + \frac{\Lambda_2^2}{\tau_\eta}. \qquad \text{(A.31b)}
$$

Maximizing (B.3) with respect to x_1^D and solving for the first period strategy yields

$$\begin{aligned} x_1^D(p_1) &= \frac{\gamma}{\text{Var}_1^D[p_2]}E_1^D[p_2] - \gamma\left(\frac{1}{\text{Var}_1^D[p_2]} + \frac{1}{\text{Var}[v]}\right)p_1 \\ &= -\gamma\frac{\Lambda_{21}-\Lambda_1}{\text{Var}_1^D[p_2]}u_1 - \gamma\tau_v p_1. \end{aligned} \tag{A.32}$$

Comparing (A.32) with (A.29) shows that in this model at the first round D and liquidity traders submit the same type of market order. That is, we can think of the strategy of a liquidity trader as being similar to the "directional bet" part of the D strategy (more on this in section 2).

Substituting (A.18), (A.29) and (A.32) into the first period market clearing condition (2) and identifying the equilibrium price coefficient yields:

$$\Lambda_1 = -\frac{\mu\gamma+\gamma_H}{\gamma\gamma_H\tau_v}a_1. \tag{A.33}$$

We have already signed Λ_2. To sign the remaining price coefficients, we substitute the expressions for the strategy coefficients into (A.21b), (A.21c), and (A.33), obtaining:

$$\begin{aligned} &\Lambda_{21} \\ &= \frac{(\tau_u+\tau_\eta)\text{Var}_2[v-p_2] - (\gamma_H\Lambda_{22}\tau_u + (\tau_u+\tau_\eta)(1-\mu)\gamma\tau_v\Lambda_1\text{Var}_2[v-p_2])\tau_v\text{Var}_1[v-p_2]}{\gamma_H\tau_v\tau_\eta\text{Var}_1[v-p_2] + (\tau_u+\tau_\eta)(\gamma_H+\mu\gamma\tau_v\text{Var}_1[v-p_2])\tau_v\text{Var}_2[v-p_2]} \end{aligned} \tag{A.34a}$$

$$\Lambda_{22} = -\frac{\gamma_H\Lambda_{21}\tau_\eta}{\mu\gamma\tau_v(\tau_u+\tau_\eta)\text{Var}_2[v-p_2]+\gamma_H\tau_u} \tag{A.34b}$$

$$\Lambda_1 = \frac{(\mu\gamma+\gamma_H)\Lambda_{21}\tau_u\tau_\eta}{(\mu\gamma+\gamma_H)\tau_u\tau_\eta + (\Lambda_{22}^2\tau_u+\Lambda_2^2\tau_\eta)\gamma\tau_v}. \tag{A.34c}$$

Note that from (A.34c), the sign of Λ_{21} coincides with that of Λ_1. Now, suppose that $\Lambda_{21} \le 0$, then this implies that $\Lambda_1 \le 0$. However, because of (A.34a), we then have that $\Lambda_{21} > 0$, which is a contradiction. Once we have signed Λ_{21}, because of (A.34b), we know that $\Lambda_{22} < 0$, and by computing $\Lambda_{21} - \Lambda_1$ with (A.34c), we obtain $\Lambda_{21} - \Lambda_1 > 0$. □

Proof of Proposition 1

We prove here that that when second period traders observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$), the equilibrium obtained in Proposition 4, is unique. Note that this assumption has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period traders' is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second period price only reflects endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21}u_1,$$

and, using (A.7), second period traders' position reads as follows:

$$x_2 = \gamma_H \frac{E_2[v - p_2]}{\text{Var}_2[v - p_2]} - \frac{\text{Cov}_2[v, v - p_2]}{\text{Var}_2[v - p_2]} u_2$$
$$= \underbrace{(\gamma_H \tau_v \Lambda_2 - 1)}_{= a_2} u_2 + \underbrace{\gamma_H \tau_v \Lambda_{21}}_{= b} u_1, \tag{A.35}$$

where we note that since second period traders perfectly observe u_1, $\text{Var}_2[v - p_2] = \tau_v^{-1}$. First period traders, trading at the second round, can only anticipate the impact of u_1 on p_2. Thus, using (A.19), we obtain:

$$x_{21} = \gamma_H \frac{E_1[v - p_2]}{\text{Var}_1[v - p_2]} - \frac{\text{Cov}_1[v, v - p_2]}{\text{Var}_1[v - p_2]} u_1$$
$$= \underbrace{\frac{(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}}_{= a_{21}} u_1. \tag{A.36}$$

The strategy for D is as in (A.14), so that plugging it in the second period market clearing condition yields:

$$x_2^D + x_2 + x_{21} = 0 \iff p_2 = \underbrace{\frac{a_2}{\gamma \tau_v}}_{= -\Lambda_2} u_2 + \underbrace{\frac{b + a_{21}}{\gamma \tau_v}}_{= -\Lambda_{21}} u_1. \tag{A.37}$$

Based on the above, we can immediately identify the second period price impact coefficients:

$$\Lambda_2 = \frac{1}{(\gamma + \gamma_H)\tau_v} \tag{A.38a}$$
$$\Lambda_{21} = \frac{\tau_u}{((\gamma + \gamma_H)(\tau_u + \Lambda_2^2 \tau_v) + \gamma_H \tau_u)\tau_v}. \tag{A.38b}$$

Finally, turning to the first period market, we have the following expression for the market clearing equation:

$$x_1^D + x_{11} = 0.$$

Replacing the expressions for traders and dealers' strategies (see, respectively (A.29), (A.32), and (A.18)), taking the limit for $\tau_\eta \to \infty$ and identifying the endowment shock price coefficient yields

$$p_1 = \underbrace{\frac{(\gamma + \gamma_H)\tau_u \Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2}}_{= -\Lambda_1} u_1. \tag{A.39}$$

The equilibrium is uniquely pinned down by the solution to the linear system given by the expressions for the price coefficients of u_1 at the two trading rounds:

$$\Lambda_1 = \frac{(\gamma+\gamma_H)^4\tau_u^2\tau_v}{(\gamma+(\gamma+\gamma_H)^3\tau_u\tau_v)(1+(\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v)} \tag{A.40a}$$

$$\Lambda_{21} = \frac{(\gamma+\gamma_H)\tau_u}{1+(\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v}, \tag{A.40b}$$

which possesses the unique solution illustrated in the text of the proposition. The ranking across the price impact coefficients follows immediately from the comparison of (A.40b) and (A.40a):

$$\Lambda_{21} - \Lambda_1 = \frac{\gamma\tau_v\Lambda_{21}\Lambda_2^2}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2} > 0.$$

□

Proof of Proposition 2

We obtain the equilibrium in the case with full opacity by setting $\mu = 1$ and taking the limit for $\tau_\eta \to 0$ of the equilibrium price coefficients obtained in the proof of Proposition 4.

Starting from Λ_{22}:

$$\Lambda_{22} = \lim_{\tau_\eta\to 0} -\frac{\gamma_H\Lambda_2\Lambda_{21}\tau_v\tau_\eta}{\tau_u + (1-\gamma_2\tau_v\Lambda_2)} = 0. \tag{A.41a}$$

Based on (A.41a) we then have

$$\Lambda_2 = \lim_{\tau_\eta\to 0} \frac{1}{(\gamma_H + \gamma\tau_v \mathrm{Var}_2[v-p_2])\tau_v} = \frac{\tau_u}{((\mu\gamma+\gamma_H)\tau_u + \gamma\tau_v(\Lambda_{21}-\Lambda_{22})^2)\tau_v} \tag{A.41b}$$
$$= \frac{\tau_u}{((\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v}$$

and

$$\begin{aligned}
&\Lambda_{21} \\
&= \lim_{\tau_\eta\to 0} -\frac{(\tau_v \mathrm{Var}_1[v-p_2])^{-1}(\gamma_H\tau_v\Lambda_{21}-1) + \gamma_H((\tau_u+\tau_\eta)\mathrm{Var}_2[v-p_2])^{-1}(\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u)}{\mu\gamma\tau_v} \\
&= -\frac{(\gamma_H\tau_v\Lambda_{21}-1)\tau_u}{(\tau_u+\Lambda_2^2\tau_v)\gamma\tau_v}.
\end{aligned} \tag{A.41c}$$

Also,

$$\lim_{\tau_\eta\to 0}\frac{\Lambda_{22}^2}{\tau_\eta} = \lim_{\tau_\eta\to 0}\left(\frac{\gamma_H\Lambda_2\Lambda_{21}\tau_v}{(\tau_u/\tau_\eta^{1/2}) + (1-\gamma_H\tau_v\Lambda_2)\tau_\eta^{1/2}}\right)^2$$
$$= 0,$$

which implies that

$$\Lambda_1 = \lim_{\tau_\eta \to 0} \frac{(\gamma_H + \gamma)\tau_u \Lambda_{21}}{\gamma_H \tau_u + \gamma(((\Lambda_{22}^2/\tau_\eta)\tau_u + \Lambda_2^2)\tau_v + \tau_u)} \\ = \frac{(\gamma_H + \gamma)\tau_u \Lambda_{21}}{\gamma_H \tau_u + \gamma(\Lambda_2^2 \tau_v + \tau_u)}. \tag{A.41d}$$

Based on the limits (A.41a)-(A.41d), the coefficients of traders' strategies are given by

$$a_1 = -\gamma_H \tau_u \frac{\Lambda_{21} - \Lambda_1}{\Lambda_2^2} < 0 \tag{A.42a}$$

$$a_{21} = \tau_u \frac{\gamma_H \tau_v \Lambda_{21} - 1}{\tau_u + \Lambda_2^2 \tau_v} \in (-1, 0) \tag{A.42b}$$

$$a_2 = \tau_u \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_u + \Lambda_{21}^2 \tau_v} \in (-1, 0) \tag{A.42c}$$

$$b = 0. \tag{A.42d}$$

Additionally, an equilibrium is pinned down by solving the following system of simultaneous equations:

$$\Lambda_2 = \Phi_1(\Lambda_{21}) \equiv \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_{21}^2)\tau_v} \tag{A.43a}$$

$$\Lambda_{21} = \Phi_2(\Lambda_2) \equiv \frac{\tau_u}{((\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2)\tau_v} \tag{A.43b}$$

$$\Lambda_1 = \frac{(\gamma + \gamma_H)\tau_u \Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2}. \tag{A.43c}$$

An equilibrium obtains via the solution of the system (A.43a)–(A.43b). Replacing (A.43a) into (A.43b) and rearranging yields:

$$\Lambda_{21} = \Phi_2(\Lambda_{21}) \equiv \frac{(\gamma \tau_u + (\gamma + \gamma_H)B^2 \tau_v)B^2}{(\gamma + \gamma_H)(\gamma + \gamma_H)B^4 \tau_v^2 + 2(\gamma + \gamma_H)\gamma B^2 \tau_u \tau_v + \gamma^2 \tau_u^2}, \tag{A.44}$$

where $B \equiv (\gamma + \gamma_H)\tau_u + \gamma \Lambda_{21}^2 \tau_v$. Inspection of (A.44) reveals (i) that $\Phi_2(\Lambda_{21}) > 0$, (ii) that $\Phi_2(0) > 0$, and (iii) that $\Lambda_{21} - \Phi_2(\Lambda_{21})$ is proportional to a 9-the degree polynomial in Λ_{21}, which thus always possesses at least one positive root Λ_{21}^*. Recursive substitution of such root first in (A.43a) and then in (A.43c) allows to pin down the set of equilibrium coefficients for p_1 and p_2.

Comparison of (A.43c) and (A.43b) shows that $\Lambda_{21}, \Lambda_1 > 0$ and $\Lambda_1 < \Lambda_{21}$. To see this, suppose $\Lambda_{21} \leq 0$. Then, because of (A.43c), $\Lambda_1 \leq 0$. However, because of (A.43b) this implies that $\Lambda_{21} > 0$, contradicting the initial assumption. Next, using (A.43c)

$$\Lambda_{21} - \Lambda_1 = \Lambda_{21} - \frac{(\gamma + \gamma_H)\tau_u \Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2} \\ = \frac{\gamma \tau_v \Lambda_2^2 \Lambda_{21}}{(\gamma + \gamma_H)\tau_u + \gamma \tau_v \Lambda_2^2},$$

which is positive. □

Proof of Proposition 3

Divide (17a) by (17b) to obtain

$$\frac{\Lambda_2}{\Lambda_{21}} = \frac{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2}{(\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_{21}^2}.$$

Rearranging the above, yields the following equation

$$(\Lambda_2 - \Lambda_{21})((\gamma+\gamma_H)\tau_u - \gamma\tau_v\Lambda_{21}\Lambda_2) = 0. \tag{A.45}$$

One solution to the above equation is $\Lambda_2 = \Lambda_{21}$ which, substituted into (17a) after rearranging yields the following cubic in Λ_2:

$$\varphi(\Lambda_2) \equiv ((\gamma+\gamma_H)\tau_u + \gamma\tau_v\Lambda_2^2)\Lambda_2\tau_v - \tau_u, \tag{A.46}$$

which, since $\varphi(0) < 0$ and $\varphi'(\Lambda_2) > 0$, is easily seen to posses a unique, positive root. Suppose instead that $\Lambda_{21} \neq \Lambda_2$. In this case, for (A.45) to be satisfied, we need

$$\Lambda_{21}\Lambda_2 = \frac{(\gamma+\gamma_H)\tau_u}{\gamma\tau_v}. \tag{A.47}$$

Solving the above for Λ_{21} and substituting the result into (17a), yields the following quadratic in Λ_2:

$$(\gamma+\gamma_H)\gamma\tau_v\Lambda_2^2 - \gamma\Lambda_2 + (\gamma+\gamma_H)^2\tau_u = 0. \tag{A.48}$$

The roots of the equation are given by

$$\Lambda_2^{*,***} = \frac{\gamma \pm \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}. \tag{A.49}$$

Both roots are positive, which implies that, provided

$$0 < \tau_u\tau_v \leq \frac{\gamma}{4(\gamma+\gamma_H)^3},$$

there are two additional equilibria of the model which are either distinct, and the corresponding value of Λ_{21} obtains by substituting either root into (A.47), or identical:

$$\Lambda_2 = \Lambda_{21} = \frac{1}{2(\gamma+\gamma_H)\tau_v},$$

when $\gamma/(4(\gamma+\gamma_H)^3) = \tau_u\tau_v$. Finally, note that when

$$\frac{\gamma}{4(\gamma+\gamma_H)^3} < \tau_u\tau_v,$$

the quadratic (A.48) does not have a real solution, and only the equilibrium with $\Lambda_{21} = \Lambda_2$ obtains.

□

Proof of Corollary 5

To analyze the stability properties of the equilibrium in this case, we use the aggregate best response function (A.44) which for $\mu = 1$ has the following expression:

$$\Phi_2(\Lambda_{21}) = \frac{((\gamma+\gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2}{\gamma\tau_u + ((\gamma+\gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma+\gamma_H)\tau_v}. \tag{A.50}$$

1. First, based on the above expression, $\Phi_2(0) > 0$ and differentiating (A.50) with respect to Λ_2 yields:

$$\Phi_2'(\Lambda_{21}) = \frac{4((\gamma+\gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)\gamma^2\Lambda_{21}\tau_u\tau_v}{(\gamma\tau_u + ((\gamma+\gamma_H)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma+\gamma_H)\tau_v)^2} > 0, \tag{A.51}$$

   implying that the best response is always upward sloping. Thus, with uniqueness $\Phi_2(\Lambda_{21})$ cuts the 45-degree line from "above" implying that the equilibrium is stable. When multiple equilibria arise, it instead crosses the 45-degree line at three points, with a slope smaller (larger) than one at the two extreme (intermediate) crossings, which correspond to the three equilibria of the market. Hence, with multiplicity, the two extreme equilibria are stable, while the intermediate one is unstable.

2. Second, evaluating the cubic (A.46) at the low and high roots of the quadratic (A.48) yields

$$\varphi\left(\frac{\gamma - \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}\right) = \frac{\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3 - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}}{2\tau_v(\gamma+\gamma_H)^3} < 0 \tag{A.52a}$$

$$\varphi\left(\frac{\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}\right) = \frac{\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3 + \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_H)^3\right)}}{2\tau_v(\gamma+\gamma_H)^3} > 0, \tag{A.52b}$$

   for $0 < \tau_u\tau_v < \gamma/(4(\gamma+\gamma_H)^3)$. Hence, when multiple equilibria arise, the roots of the quadratic equation (A.48) "straddle" the root of the cubic (A.46).

3. Third, taking the product of the two extreme equilibrium values for Λ_2 yields:

$$\frac{\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v} \times \frac{\gamma - \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v} = \frac{(\gamma+\gamma_H)\tau_u}{\gamma\tau_v}.$$

Thus, in view of the second expression in (A.13b), at a stable equilibrium we have either that the price reacts more to u_2 than to u_1, or the opposite. Additionally, because of (18), when p_2 reacts more to u_1 than to u_2, the market is also less liquid at the first round.

4. Fourth, evaluating a_2 at the two extreme equilibria, we obtain:

$$a_2|_{\Lambda_2=\Lambda_2^{***}} = -\frac{\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)} >$$
$$a_2|_{\Lambda_2=\Lambda_2^{*}} = \frac{-\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)},$$

which always holds within the parameter restriction needed for multiple equilibria to obtain. Given the symmetry of the equilibrium solution, this result also implies that when second period traders **consume** more liquidity, first period traders **consume** less of it. Finally, replacing (18) and (A.47) in the expression for a_1 yields:

$$\begin{aligned} a_1 &= -\gamma_H \frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2\tau_u^{-1}} \\ &= -\gamma_H \frac{(1-(\gamma+\gamma_H)\tau_v\Lambda_{21})\gamma^2\tau_v^2\Lambda_{21}^3}{(\gamma+\gamma_H)^2\tau_u}, \end{aligned} \tag{A.53}$$

implying that also at the first round, liquidity consumption increases in the cost of trading generated.

□

Proof of Corollary 6

To prove our claim, we need to show that when evaluated at the intermediate equilibrium (A.54) is negative:

$$\begin{aligned} &\frac{\partial}{\partial \tau_u}\left(\frac{\partial \Phi(\Lambda_2)}{\partial \Lambda_2}\right) \\ &\quad = \frac{4\gamma^2\Lambda_2\tau_v^2\left(\gamma^3\Lambda_2^6\tau_v^3(\gamma+\gamma_H) - \gamma^2\Lambda_2^2\tau_u - 3\gamma\Lambda_2^2\tau_u^2\tau_v(\gamma+\gamma_H)^3 - 2\tau_u^3(\gamma+\gamma_H)^4\right)}{\left(\gamma^2\Lambda_2^4\tau_v^3(\gamma+\gamma_H) + 2\gamma\Lambda_2^2\tau_u\tau_v^2(\gamma+\gamma_H)^2 + \tau_u^2\tau_v(\gamma+\gamma_H)^3 + \gamma\tau_u\right)^3}, \end{aligned} \tag{A.54}$$

To do this, it suffices to check that the cubic that pins down the intermediate equilibrium (see (21)), satisfies the following condition:

$$\varphi(\hat{\Lambda}_2) > 0, \text{for } \hat{\Lambda}_2 = \frac{1}{\gamma_H\tau_v},$$

which implies that $\Lambda_2^{**} < \hat{\Lambda}_2$. Next, verify that the numerator in (A.54)

$$\gamma^3\Lambda_2^6\tau_v^3(\gamma+\gamma_H) - \gamma^2\Lambda_2^2\tau_u - 3\gamma\Lambda_2^2\tau_u^2\tau_v(\gamma+\gamma_H)^3 - 2\tau_u^3(\gamma+\gamma_H)^4 < 0,$$

for $\Lambda_2 < \hat{\Lambda}_2$. □

Proof of Proposition 4

The result follows from Proposition A.1 setting $\mu = 1$. If we take the limit for $\tau_\eta \to \infty$ in (A.22), (A.34a), and (A.34c), we obtain the unique linear equilibrium of Proposition 1.

□

Proof of Proposition 5

We start by obtaining an expression for the unconditional expected utility of RD and D. Because of CARA and normality, an RD conditional expected utility evaluated at the optimal strategy is given by

$$E[U((v-p_1)x_1^{RD})|p_1] = -\exp\left\{-\frac{(E[v|p_1]-p_1)^2}{2\text{Var}[v]}\right\}$$
$$= -\exp\left\{-\frac{\tau_v\Lambda_1^2}{2}u_1^2\right\}. \tag{A.55}$$

Thus, RD derive utility from the expected, long-term capital gain obtained supplying liquidity to first-period hedgers.

$$EU^{RD} \equiv E\left[U\left((v-p_1)x_1^{RD}\right)\right] = -\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}\right)^{-1/2}$$
$$= -\left(\frac{\tau_{u_1}}{\tau_{u_1}+\tau_v\Lambda_1^2}\right)^{1/2}, \tag{A.56}$$

and

$$CE^{RD} = \frac{\gamma}{2}\ln\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}\right). \tag{A.57}$$

Turning to D, replacing the optimal x_1^D in (B.3) and rearranging yields

$$E[U((p_2-p_1)x_1^D+(v-p_2)x_2^D)|u_1] = -\left(1+\frac{\text{Var}[p_2|u_1]}{\text{Var}[v]}\right)^{-1/2}\times\exp\left\{-\frac{g(u_1)}{\gamma}\right\}, \tag{A.58}$$

where

$$g(u_1) = \frac{\gamma}{2}\left(\frac{(E[p_2|p_1]-p_1)^2}{\text{Var}[p_2|p_1]}+\frac{(E[v|p_1]-p_1)^2}{\text{Var}[v]}\right).$$

The argument of the exponential in (A.58) is a quadratic form of the first-period endowment shock. We can therefore apply Lemma 1 and obtain

$$EU^D \equiv E[U((p_2-p_1)x_1^D+(v-p_2)x_2^D)] =$$
$$= -\left(1+\frac{\text{Var}[p_2|p_1]}{\text{Var}[v]}\right)^{-1/2}\left(1+\frac{\text{Var}[p_1]}{\text{Var}[v]}+\frac{\text{Var}[E[p_2|p_1]-p_1]}{\text{Var}[p_2|p_1]}\right)^{-1/2}. \tag{A.59}$$

Computing the certainty equivalent yields:

$$CE^D = \frac{\gamma}{2}\Bigg(\ln\left(1 + \frac{\text{Var}[E[v-p_1|p_1]]}{\text{Var}[v-p_1|p_1]} + \frac{\text{Var}[E[p_2-p_1|p_1]]}{\text{Var}[p_2-p_1|p_1]}\right) + \ln\left(1+\frac{\text{Var}[E[v-p_2|p_1,p_2]]}{\text{Var}[v-p_2|p_1,p_2]}\right)\Bigg). \tag{A.60}$$

To obtain the expression for first period hedgers' payoff we replace the strategy (A.29) into the objective function (A.28) and rearrange the result, to obtain

$$E[-\exp\{-\pi_1/\gamma_H\}|u_1] = -E\left[\exp\left\{-\frac{u_1^2}{\gamma_H}\left(\frac{\gamma_H(\Lambda_1-\Lambda_{21})^2}{2\text{Var}_1[p_2]} + \frac{a_{21}^2\tau_v\text{Var}_1[v-p_2]-1}{2\gamma_H\tau_v}\right)\right\}\right]. \tag{A.61}$$

The argument in the above expression is a quadratic form of the normal random variable $u_1 \sim N(0,\tau_u^{-1})$. Thus, to compute the unconditional expectation of (A.61), we apply Lemma 1 to obtain

$$E[-\exp\{-\pi_1/\gamma_H\}] = -\left(\frac{\gamma_H^2\tau_u\tau_v}{\gamma_H^2\tau_u\tau_v - 1 + (a_1^2\text{Var}_1[p_2] + a_{21}^2\text{Var}_1[v-p_2])\tau_v}\right)^{1/2}. \tag{A.62}$$

To obtain the certainty equivalent, we compute

$$\begin{aligned} CE_1^H &= -\gamma_H \ln\left(-E[-\exp\{-\pi_1/\gamma_1\}]\right) \\ &= \frac{\gamma_H}{2}\ln\left(1+\frac{(a_1^2\text{Var}_1[p_2]+a_{21}^2\text{Var}_1[v-p_2])\tau_v - 1}{\gamma_H^2\tau_u\tau_v}\right). \end{aligned} \tag{A.63}$$

To obtain the payoff of second period liquidity traders we proceed similarly by replacing their equilibrium strategy into their objective function:

$$E_2[-\exp\{-(1/\gamma_H)((v-p_2)x_2+vu_2)\}] = -\exp\left\{-\frac{1}{\gamma_H}\left(\frac{\text{Var}_2[v-p_2]x_2^2 - \text{Var}[v]u_2^2}{2\gamma_H}\right)\right\}. \tag{A.64}$$

The argument of the exponential at the right hand side of the above expression is a quadratic form of the normally distributed random vector

$$\begin{pmatrix} x_2 \\ u_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underbrace{\begin{pmatrix} \text{Var}[x_2] & a_2/\tau_u \\ a_2/\tau_u & \tau_u^{-1} \end{pmatrix}}_{\Sigma}\right).$$

Indeed, we have

$$\frac{\text{Var}_2[v-p_2]x_2^2 - \text{Var}[v]u_2^2}{2\gamma_H} = \frac{1}{2\gamma_H}\begin{pmatrix} x_2 & u_2 \end{pmatrix}\underbrace{\begin{pmatrix} \text{Var}_2[v-p_2] & 0 \\ 0 & -\text{Var}[v] \end{pmatrix}}_{A}\begin{pmatrix} x_2 \\ u_2 \end{pmatrix}.$$

Applying again Lemma 1, we then have

$$E\bigg[-\exp\bigg\{-\frac{1}{\gamma_H}\bigg(\frac{\text{Var}_2[v-p_2]x_2^2-\text{Var}[v]u_2^2}{2\gamma_H}\bigg)\bigg\}\bigg] = -\left|I+(2/\gamma_H)\Sigma A\right|^{-1/2} \tag{A.65}$$
$$= -\left(\frac{\gamma_H^4\tau_u^2\tau_v}{a_2^2\text{Var}_2[v-p_2]+(\text{Var}_2[v-p_2]\text{Var}[x_2]+\gamma_H^2)(\gamma_H^2\tau_u\tau_v-1)\tau_u}\right)^{1/2}.$$

Finally, the certainty equivalent obtains by computing

$$CE_2^H = -\gamma_H\ln\left(-E[-\exp\{-\pi_2/\gamma_H\}]\right) \tag{A.66}$$
$$= \frac{\gamma_H}{2}\ln\left(1+\frac{a_2^2\text{Var}_2[v-p_2]\tau_v-1}{\gamma_H^2\tau_u\tau_v}+\frac{b^2\text{Var}[s_{u_1}](\gamma_H^2\tau_u\tau_v-1)}{\gamma_H^4\tau_u\tau_v}\right).$$

We check here that when $\tau_\eta=0$ and $\mu=1$, $\tau_u\tau_v>1/\gamma_H^2$ ((28)) implies that the equilibrium is unique (see (19)). Based on (28) when $\tau_u\tau_v>1/\gamma_H^2$ the payoffs of the market participants are well defined, and it is immediate that

$$\frac{1}{\gamma_H^2}>\frac{\gamma}{4(\gamma+\gamma_H)^3},$$

which yields the desired result.

□

Proof of Corollary 11

Direct comparison of Λ_2^{***} with $\Lambda_2^T(\equiv 1/((\gamma+\gamma_H)\tau_v))$, yields

$$\Lambda_2^{***}\equiv\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2\gamma\tau_v(\gamma+\gamma_H)}<\Lambda_2^T\equiv\frac{1}{(\gamma+\gamma_H)\tau_v},$$

for $\tau_u\tau_v\leq\gamma/(4(\gamma+\gamma_H)^3)$. Also, comparing

$$a_2^{***}=-\gamma\tau_v\Lambda_2^{***}=-\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)}>a_2^T\equiv-\frac{\gamma}{\gamma+\gamma_H},$$

again for $\tau_u\tau_v\leq\gamma/(4(\gamma+\gamma_H)^3)$. Therefore, when multiple equilibria arise with opacity the price impact of u_2 is larger and second period liquidity traders hedge more with transparency. When uniqueness obtains with opacity, we evaluate the resolving cubic (21) at Λ_2^T obtaining:

$$\varphi\left(\Lambda_2^T\right)\equiv\frac{\gamma}{(\gamma+\gamma_H)^3\tau_v}>0,$$

which proves our result. □

B Dealers' strategies with full transparency

In this appendix, we provide a more detailed derivation of the strategies adopted by dealers in the fully transparent benchmark of Section 2.

The liquidity supply that accommodates the demand of hedgers is offered by dealers who submit price contingent orders (generalized limit orders) at both rounds.

A dealer's strategy at $t = 1$ is given by (see Appendix (A.32)):

$$\begin{aligned} X_1^D(p_1) &= \frac{\gamma}{\text{Var}_1^D[p_2]} E_1^D[p_2] - \gamma \left(\frac{1}{\text{Var}_1^D[p_2]} + \frac{1}{\text{Var}[v]} \right) p_1 \\ &= -\gamma \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1^D[p_2]} u_1 - \gamma \tau_v p_1. \end{aligned} \tag{B.1}$$

According to (B.1), $X_1^D(p_1)$ reflects two trading motives: short-term return speculation (captured by the component $-u_1\gamma(\Lambda_{21} - \Lambda_1)/\text{Var}_1[p_2]$) and liquidity supply (captured by the price dependent component in (B.1), $-\gamma\tau_v p_1$). We now explain in more detail these components, starting from the short-term return speculation one.

Due to their ability to infer traders' endowment shock and the fact that they know these traders split their hedging order, dealers exploit the anticipated effect the shock has on expected returns. To see this, note that at the second round dealers in aggregate hold (see (A.14))

$$\begin{aligned} X_2^D(p_1, p_2) &= \gamma \frac{E_2^D[v - p_2]}{\text{Var}_2^D[v - p_2]} = \\ &= -\gamma\tau_v p_2 \\ &= \gamma\tau_v\Lambda_{21}u_1 + \gamma\tau_v\Lambda_2 u_2, \end{aligned} \tag{B.2}$$

where the expression at the third line in (B.2) originates from substituting (1b) in dealers' second period aggregate position. Expression (B.2) implies that at the second round dealers hold $\gamma\tau_v\Lambda_{21}$ of the first period endowment shock. Based on (B.1), at the first round their position is given by

$$X_1^D(p_1) = \gamma \left(\tau_u \frac{\Lambda_1 - \Lambda_{21}}{\Lambda_2^2} + \tau_v \Lambda_1 \right) u_1.$$

Hence, if $u_1 > 0$, at the first round dealers provide liquidity by absorbing part of first period traders' endowment shock ($\Lambda_1 > 0$). Additionally, they *consume* liquidity by taking a *short position* in the risky security ($\Lambda_1 - \Lambda_{21} < 0$).

At the second round, based on what said above, they provide liquidity to the incremental hedging trade of first period traders: their trade with respect to the latter is given by

$$\gamma\tau_v\Lambda_{21}u_1 - X_1^D(p_1) = \gamma \frac{\tau_u + \Lambda_2^2\tau_v}{\Lambda_2^2} (\Lambda_{21} - \Lambda_1) u_1,$$

i.e., a buy order (if $u_1 > 0$). Thus, because of their ability to anticipate returns, dealers gain from short term speculation at the first round (moderating their buying at the first round and deferring it since in expectation the price at the second round is lower: $E_1^D[p_2 - p_1] < 0$ if $u_1 > 0$).

Due to risk aversion, dealers have a limited capacity to bear risk. This implies that the price coefficients in (7a)–(7c) capture the risk-tolerance weighted risk compensation dealers require to absorb the aggregate liquidity demand.

We derive Λ_1 here. At the first round a_1 reflects the marginal position of liquidity traders:

$$a_1 = \frac{\partial x_{11}}{\partial u_1} = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]}. \tag{B.3}$$

As observed above, dealers also demand liquidity, since they speculate on the price impact of u_1 and their aggregate liquidity demand is given by

$$-\gamma \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]} = \gamma \frac{a_1}{\gamma_H}.$$

Aggregating across liquidity traders' and dealers' demands yields the aggregate liquidity demand at the first round:

$$a_1 + \gamma \frac{a_1}{\gamma_H} = \frac{\gamma + \gamma_H}{\gamma_H} a_1.$$

At equilibrium, replacing dealers and liquidity traders' equilibrium strategies (respectively, (B.1) and the first in (8a)) in the first period equilibrium condition (2), we have:

$$\begin{aligned} x_1^D + x_{11} = 0 \iff & \gamma \frac{a_1}{\gamma_H} u_1 - \gamma \tau_v p_1 + a_1 u_1 = 0 \\ \iff & \frac{\gamma + \gamma_H}{\gamma_H} a_1 u_1 = \gamma \tau_v p_1 \end{aligned} \tag{B.4}$$

At a linear equilibrium the price is proportional to the aggregate endowment shock u_1: $p_1 = -\Lambda_1 u_1$. Identifying $-\Lambda_1$ in the latter expression yields:

$$\underbrace{\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_H}{\gamma_H} a_1}_{-\Lambda_1} u_1 = p_1. \tag{B.5}$$

Thus, $-\Lambda_1$ measures the price impact of a marginal increase in the endowment shock hitting the first period cohort. Since this covers a "cost" incurred to supply immediacy, we interpret (somewhat loosely) Λ_1 as the first period "liquidity supply" function.

At the second round, liquidity demand comes from first and second period traders coefficients a_{21} and a_2:

$$\begin{aligned} x_{21} &= \gamma_H \frac{E_1[v - p_2]}{\text{Var}_1[v - p_2]} - \frac{\text{Cov}_1[v, v - p_2]}{\text{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}}_{= a_{21}} u_1. \end{aligned} \tag{B.6}$$

and

$$
\begin{aligned}
x_2 &= \gamma_H \frac{E_2[v-p_2]}{\mathrm{Var}_2[v-p_2]} - \frac{\mathrm{Cov}_2[v, v-p_2]}{\mathrm{Var}_2[v-p_2]} u_2 \\
&= \underbrace{(\gamma_H \tau_v \Lambda_2 - 1)}_{=a_2} u_2 + \underbrace{\gamma_H \tau_v \Lambda_{21}}_{=b} u_1.
\end{aligned} \tag{B.7}
$$

We can interpret the expressions for a_{21} and a_2 in the following way. A liquidity trader hedges a larger fraction of his shock (demands more liquidity), the lower is the impact the endowment shock has on p_2 (as a larger price impact reduces a trader's expected return from hedging), and the lower is the return uncertainty he faces (as a higher return variance dents his utility since he is risk averse). Consider now the second period market clearing condition:

$$
\begin{aligned}
(x_2^D - x_1^D) + x_{21} - x_{11} + x_2 = 0 &\iff x_2^D + x_{21} + x_2 = 0 \\
&\iff -\gamma\tau_v p_2 + a_2 u_2 + (a_{21} + b) u_1 = 0 \\
&\iff p_2 = \underbrace{\frac{a_2}{\gamma\tau_v}}_{=-\Lambda_2} u_2 + \underbrace{\frac{a_{21} + b}{\gamma\tau_v}}_{=-\Lambda_{21}} u_1.
\end{aligned} \tag{B.8}
$$

At the second line of the above expression we make use of the first period market clearing equation: $x_1^D + x_{11} = 0$. We then replace strategies with their equilibrium expressions and finally solve for p_2, identifying the price coefficients.

C The effects of a partially informative signal

In this appendix, we illustrate the effect of increasing the precision of second period hedgers' information on price impacts (Figure 11) and strategy coefficients (Figure 12).

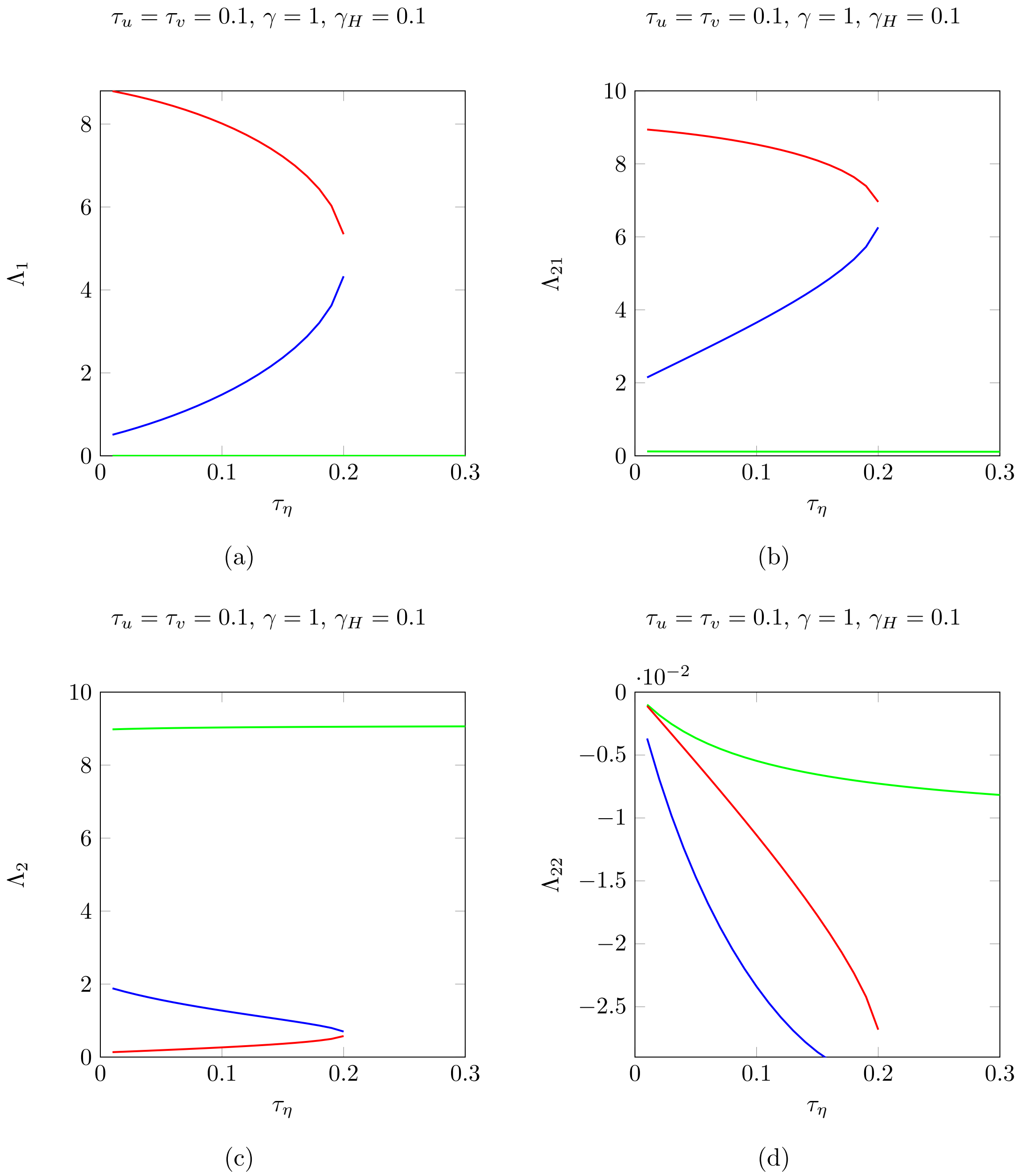

Figure 11: Price impact coefficients with a partially informative signal ($\tau_\eta < \infty$). As shown in the figure, for τ_η small, three equilibria arise. We plot them using the colors green, blue and red to indicate the equilibrium that corresponds to the two extreme, stable price impacts (respectively in green and red) and the unstable one (in blue). Parameter values are as in Figure 4 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$: we analyze the increase in transparency letting τ_η increase by 0.01.

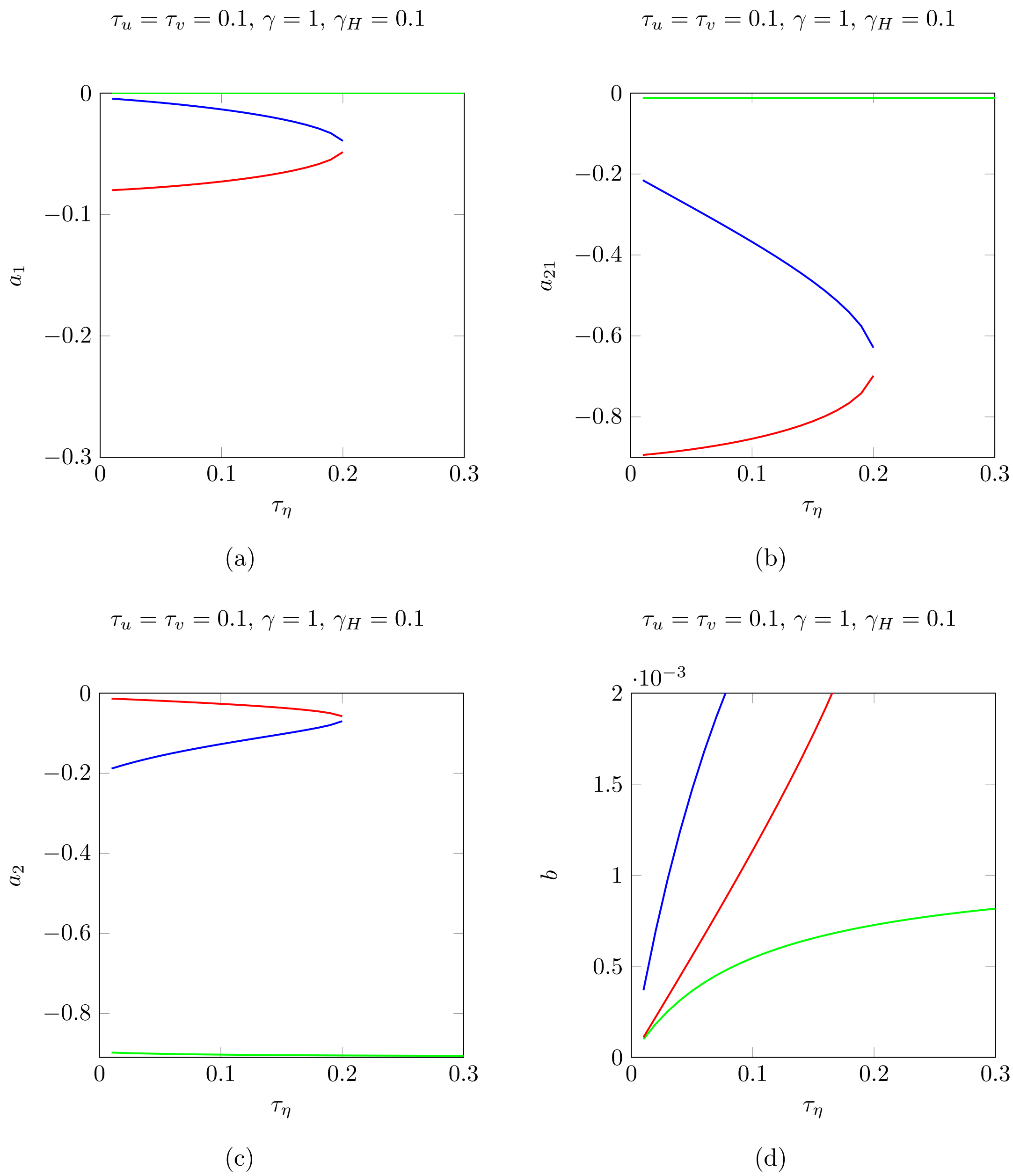

Figure 12: Strategy coefficients with a partially informative signal ($\tau_\eta < \infty$). As shown in the figure, for τ_η small, three equilibria arise. We plot them using the colors green, blue and red to indicate the equilibrium that corresponds to the two extreme, stable price impacts (respectively in green and red) and the unstable one (in blue). Parameter values are as in Figure 4 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$: we analyze the increase in transparency letting τ_η increase by 0.01.

D Comparative statics with restricted dealers

In this appendix we present the full set of comparative statics results about the effects on liquidity fragility of traders' risk aversion (γ_H), payoff volatility (τ_v^{-1}) and endowment shock dispersion (τ_u^{-1}) in the model with restricted dealers (see Figure 13 for a timeline of events).

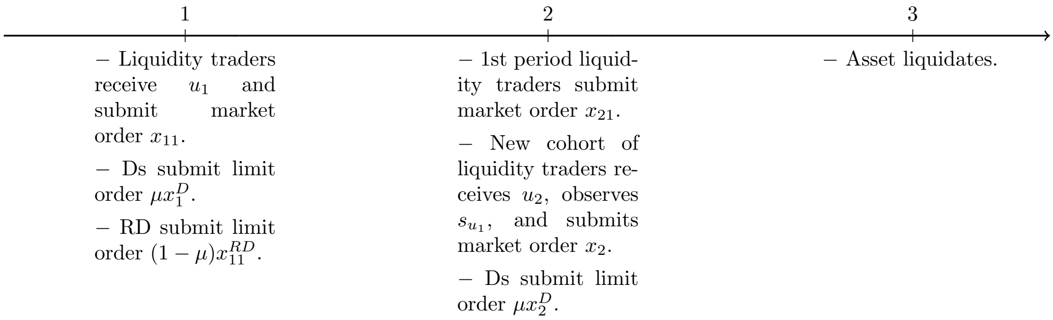

Figure 13: The timeline with restricted dealers.

In Figure 14, we partition the space $\mu \in (0, 1]$, $\tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium (three equilibria).

Consistently with what we have found in Proposition 3, an increase in γ_H or τ_v tends to reduce the chances of liquidity fragility (compare the areas below the blue curve in panel (a) and panels (c) and (d)). The effect of an increase in τ_u is more complicated. Recall that when the signal is not perfect second period traders (1) may speculate in the "wrong" direction and (2) use p_2 and the signal to predict u_1. With a higher τ_u there is less noise in the price, which reduces second period traders speculative intensity. When μ is close to 0, almost only second period traders provide liquidity at the second round, and the reduction in speculation by these traders has a large impact on overall risk sharing. Conversely, when μ is close to 1, almost only (full) dealers provide liquidity at 2 and the reduction in speculation by 2nd period traders means that dealers have less liquidity traders to share risk with. In either case this increases liquidity fragility. For intermediate values of μ, (full) dealers have a smaller exposure to the risky security, and the additional risk sharing provided by 2nd period traders is less important. In this case, the reduction in these traders' speculation rids the market of the "wrong" trades with a positive impact on liquidity fragility.

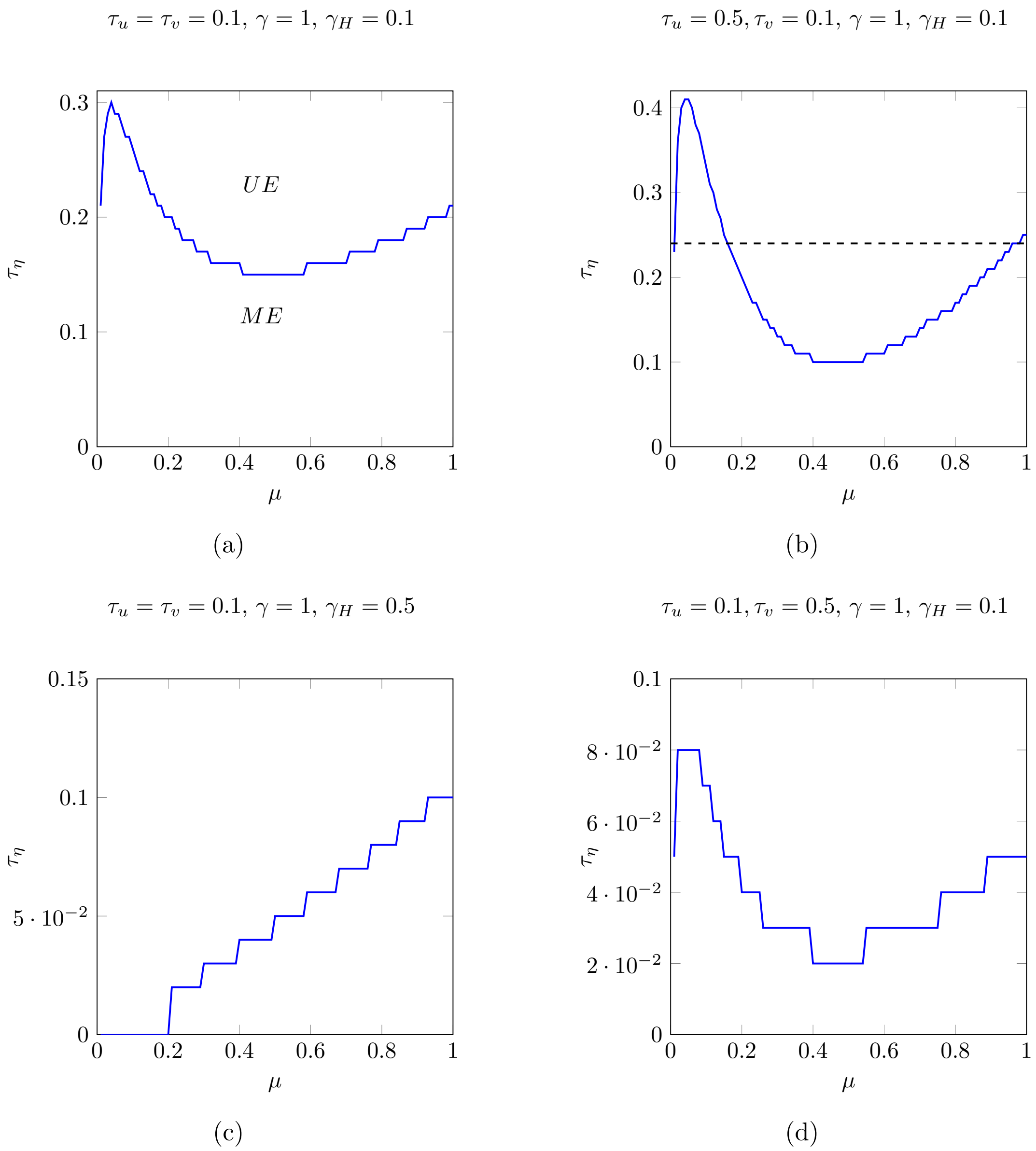

Figure 14: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.

Figure 15 displays the effect of a small unanticipated change in μ on the equilibrium when parameters are such that a unique equilibrium is obtained.

$\tau_v = 0.1,\ \tau_u = 1.9,\ \tau_\eta = 0,\ \gamma = 1,\ \gamma_H = 0.1$

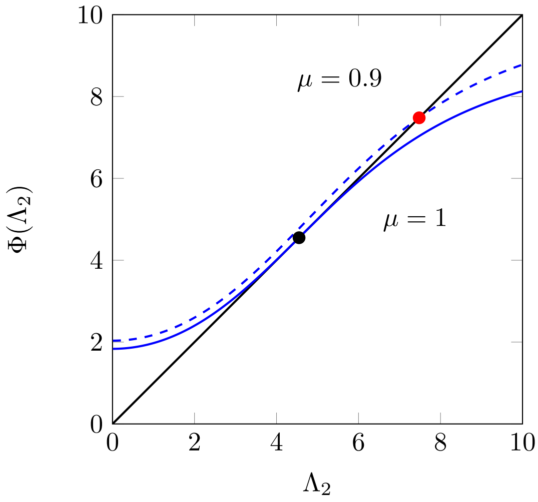

Figure 15: The effect of a small reduction in μ when τ_η is low. Total illiquidity increases.

Figure 16 illustrates the effect of increasing transparency and μ on total welfare and $WAPI$.

$\tau_v = 1,\ \tau_u = 2,\ \gamma = 1,\ \gamma_H = 1$

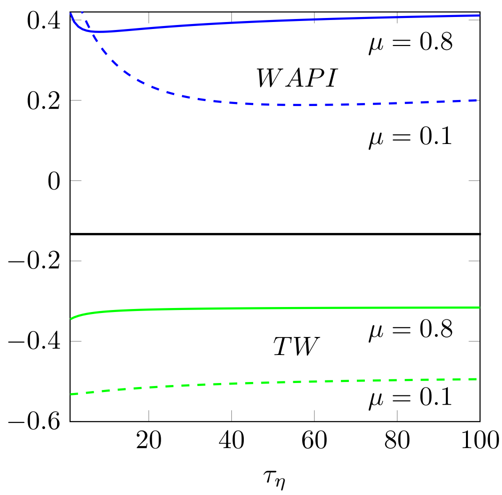

Figure 16: TW and $WAPI$ as a function of transparency.

E Liquidity trading and noise trading

In this appendix, we consider the implications of our analysis for the time series properties of noise trading and returns. First, note that, based on Proposition 1 we can say that with transparency, at the second round dealers absorb a smaller portion of the first period endowment shock (compared to the second period one), and the noise process is stable: $\beta \equiv \Lambda_{21}/\Lambda_2 < 1$. That is, the second period endowment shock impacts p_2 more than u_1.

Second, the first and second period returns are *positively* serially correlated. That is, the model displays momentum, in the absence of any fundamentals information:

$$\begin{aligned}\text{Cov}[p_2 - p_1, p_1] &= \text{Cov}[-(\Lambda_2 u_2 + (\Lambda_{21} - \Lambda_1)u_1), -\Lambda_1 u_1] \\ &= (\Lambda_{21} - \Lambda_1)\Lambda_1 \tau_u^{-1} > 0, \end{aligned} \tag{E.1}$$

due to Proposition 1. At the first round hedgers can count on the additional liquidity supplied by second period traders, which implies that they will increase their first period hedging position, inducing $\Lambda_{21} > \Lambda_1$.

We collect these results in the following corollary:

Corollary 9. *When the market is transparent: (1) liquidity trading behaves as a stable AR(1) process; (2) first and second period returns are positively serially correlated.*

The following result states the implications for the time series properties of noise trades and returns autocovariance when the market is fully opaque:

Corollary 10. *With multiple equilibria, the autocovariance of first and second period returns increases in Λ_{21} and also increases compared to the case with full transparency at both equilibria; (3) at the intermediate equilibrium or when we have a unique equilibrium we have that $\beta = 1$ since $\Lambda_{21} = \Lambda_2$.*

We evaluate the expression for returns autocovariance at the equilibrium with full transparency (and $\mu = 1$):

$$\text{Cov}[p_2 - p_1, p_1] = \frac{\gamma \tau_u^2 \tau_v (\gamma + \gamma_H)^5}{\left(\tau_u \tau_v \left(2\gamma^2 + 4\gamma\gamma_H + \gamma_H^2\right)(\gamma + \gamma_H) + \tau_u^2 \tau_v^2 (\gamma + 2\gamma_H)(\gamma + \gamma_H)^4 + \gamma\right)^2} \tag{E.2}$$

and at both the equilibria that obtain under the parameter restriction ensuring multiplicity, when the market is fully opaque, when $\Lambda_{21} = \Lambda_{21}^*$ we have

$$\text{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)}\right)^3 \left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)} + \gamma\right)}{16\gamma^4 \tau_u \tau_v^2 (\gamma + \gamma_H)^2}, \tag{E.3}$$

and when $\Lambda_{21} = \Lambda_{21}^{***}$ we have instead

$$\text{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)}\right) \left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)} + \gamma\right)^3}{16\gamma^4 \tau_u \tau_v^2 (\gamma + \gamma_H)^2}. \tag{E.4}$$

Comparing the two latter expressions shows that return autocovariance is higher when $\Lambda_{21} = \Lambda_{21}^{*}$. While comparing the latter expression above with (E.2) shows that it increases with respect to the case with full transparency.

F First period hedgers observing u_2

Suppose that at the second round first period traders perfectly observe u_2, while second period traders do not know u_1.

Assume that prices are linear in the endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 \tag{F.1}$$

$$p_1 = -\Lambda_1 u_1. \tag{F.2}$$

To characterize the equilibrium, we start from second period traders whose position is given by:

$$x_2 = \gamma_H \frac{E_2[v - p_2]}{\text{Var}_2[v - p_2]} - \frac{\text{Cov}_2[v, v - p_2]}{\text{Var}_2[v - p_2]} u_2, \tag{F.3}$$

where

$$E_2[v - p_2] = \Lambda_2 u_2 \tag{F.4}$$

$$\text{Var}_2[v - p_2] = (\tau_u + \Lambda_{21}^2 \tau_v)\tau_u^{-1}\tau_v^{-1} \tag{F.5}$$

$$\text{Cov}_2[v, v - p_2] = \tau_v^{-1}. \tag{F.6}$$

Replacing the latter expressions into (F.3) and rearranging yields

$$x_2 = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_u + \Lambda_{21}^2 \tau_v} \tau_u}_{a_2} u_2. \tag{F.7}$$

First period traders, when they re-trade at the second round have a position given by:

$$x_{21} = \gamma_H \frac{E_{21}[v - p_2]}{\text{Var}_{21}[v - p_2]} - \frac{\text{Cov}_{21}[v, v - p_2]}{\text{Var}_{21}[v - p_2]} u_1, \tag{F.8}$$

where

$$E_{21}[v - p_2] = \Lambda_2 u_2 + \Lambda_{21} u_1 \tag{F.9}$$

$$\text{Var}_{21}[v - p_2] = \tau_v^{-1} \tag{F.10}$$

$$\text{Cov}_{21}[v, v - p_2] = \tau_v^{-1}. \tag{F.11}$$

Replacing the latter expressions into (F.8) and rearranging yields:

$$x_{21} = \underbrace{(\gamma_H \tau_v \Lambda_{21} - 1)}_{a_{21}} u_1 + \underbrace{\gamma_L \tau_v \Lambda_2}_{b} u_2 \tag{F.12}$$
$$= -\gamma_H \tau_v p_2 - u_1.$$

Because dealers observe u_1 and u_2, and submit limit orders, at the second round their position is given by

$$x_2^D = -\gamma\tau_v p_2. \tag{F.13}$$

Replacing (F.3), (F.12) and (F.13) in the second period market clearing condition yields

$$x_2^D + x_{21} + x_2 = 0 \iff -\gamma\tau_v p_2 + (\gamma_H\tau_u\Lambda_{21} - 1)u_1 + \gamma_H\tau_v\Lambda_2 u_2 + \frac{\gamma_H\tau_v\Lambda_2 - 1}{\tau_u + \Lambda_{21}^2\tau_v}\tau_u u_2 = 0. \tag{F.14}$$

Solving for p_2 and identifying the price coefficients we obtain (F.1) with:

$$\Lambda_2 = \frac{(\gamma + \gamma_H)\tau_u}{1 + (\gamma + 2\gamma_H)(\gamma + \gamma_H)\tau_u\tau_v} \tag{F.15}$$

$$\Lambda_{21} = \frac{1}{(\gamma + \gamma_H)\tau_v}. \tag{F.16}$$

Based on (F.7), (F.12), and the expressions for the price coefficients above, at the second round second period traders hedge their endowment shock (selling the risky security if $u_2 > 0$ and buying it otherwise), while first period traders hedge and speculate on the imbalance due to second period traders' order. Therefore, the fact that information on order imbalances is observed by first period traders implies that the additional source of risk sharing dealers rely upon comes from them.

At the first round, the strategy of a dealer is like in the current benchmark of the paper, that is:

$$x_1^D = -\gamma\tau_u\frac{\Lambda_{21} - \Lambda_1}{\Lambda_2^2}u_1 - \gamma\tau_v p_1. \tag{F.17}$$

Denoting by $\pi_1 = (p_2 - p_1)x_1 + (v - p_2)x_{21} + u_1 v$, first period traders' profit, we pin down their strategy maximizing the following value function, obtained by substituting first period traders' equilibrium strategy into the second period objective function and rearranging:

$$-E[\exp\{-\pi_1/\gamma_H\}|u_1] = -E\left[\exp\{-((p_2 - p_1)x_1 + \frac{1}{2\gamma_L\tau_v}(x_{21}^2 - u_1^2))/\gamma_H\}|u_1\right]. \tag{F.18}$$

Applying the usual transformation to the expression at the exponent of dealers' objective function yields:

$$\begin{aligned} -E&\left[\exp\{-((p_2 - p_1)x_1 + \frac{1}{2\gamma_L\tau_v}(x_{21}^2 - u_1^2))/\gamma_H\}|u_1\right] \qquad \text{(F.19)} \\ &= -\exp\left\{-\left((\Lambda_1 - \Lambda_{21})u_1 x_1 + \frac{(a_{21}^2 - 1)}{2\gamma_L\tau_v}u_1^2 - \frac{1}{2}\left(\frac{a_{21}b}{\gamma_L\tau_v}u_1 - \Lambda_2 x_1\right)^2(\tau_u^{-1} + b^2/\gamma_H\tau_v)\right)/\gamma_H\right\}. \end{aligned}$$

Differentiating the argument of the objective function and equating the result to zero, we solve for first period traders' optimal strategy at the first round obtaining:

$$x_1 = \left(\frac{a_{21} b}{\gamma_H \Lambda_2 \tau_v} + \frac{\gamma_H (\Lambda_1 - \Lambda_{21}) \tau_u \tau_v}{(b^2 \tau_u + \gamma_H \tau_v) \Lambda_2^2} \right) u_1 \\ = \underbrace{\left(\gamma_H \Lambda_{21} \tau_v - 1 + \frac{(\Lambda_1 - \Lambda_{21}) \tau_u}{(1 + \gamma_H \Lambda_2^2 \tau_u \tau_v) \Lambda_2^2} \right)}_{a_1} u_1. \tag{F.20}$$

Finally, we replace (F.17) and (F.20) in the first period market clearing condition:

$$-\gamma \tau_u \frac{\Lambda_{21} - \Lambda_1}{\Lambda_2^2} u_1 - \gamma \tau_v p_1 + a_1 u_1 = 0, \tag{F.21}$$

solve for p_1 and identify the first period price coefficient Λ_1:

$$\Lambda_1 = \frac{\Lambda_2^2 \left(\gamma_H \Lambda_{21} \tau_v \left(\gamma \tau_u^2 - 1 \right) + 1 \right) + (1 + \gamma) \Lambda_{21} \tau_u + \gamma_H \Lambda_2^4 \tau_u \tau_v (1 - \gamma_H \Lambda_{21} \tau_v)}{\gamma \gamma_H \Lambda_2^4 \tau_u \tau_v^2 + \gamma \Lambda_2^2 \tau_v \left(1 + \gamma_H \tau_u^2 \right) + (1 + \gamma) \tau_u}. \tag{F.22}$$

Substituting (F.15) and (F.16) in the above expression and simplifying yields:

$$\Lambda_1 = \Lambda_{21} = \frac{1}{(\gamma + \gamma_H) \tau_v}. \tag{F.23}$$

Therefore, when first period traders observe u_2:

1. Again, the case with transparency has a unique equilibrium.
2. However, now it's the 1st period traders who, at the second round, "speculate" on u_2, posting a contrarian market order which represents the only change in their position. That is, first period traders' exposure to their endowment shock does not change across trading rounds. The reason for this effect is that according to (F.20) first period traders at the first round hedge the same fraction they will hedge at the second round modified to take advantage of differences in their price impact across rounds. However, the only reason why Λ_{21} may differ from Λ_1 is a change in liquidity providers' exposure to u_1, which depends on traders' liquidity demand at the second round. But liquidity traders' have no reason to change their position, since market conditions have not changed compared to the first trading round: they are not learning anything new about v, and they can fully control the execution risk due to second period traders' order. The consequence of this is that $\Lambda_{21} = \Lambda_1$ (dealers' exposure to u_1 does not change across trading rounds).
3. In turn, this implies that the autocovariance of 1st and 2nd period returns is null:

$$\text{Cov}[p_2 - p_1, p_1] = 0.$$

4. Noise trading persistence. It is still the case that $p_2 = -\Lambda_2 \theta_2$, $p_1 = -\Lambda_1 \theta_1$, with $\theta_1 \equiv u_1$ and $\theta_2 \equiv u_2 + \beta \theta_1$,

$$\beta \equiv \frac{\Lambda_{21}}{\Lambda_2} > 1.$$

G Hedgers' aggressiveness under opacity and transparency

In this appendix, we analytically compare Λ_2, Λ_{21} and a_2, a_{21} across the two regimes of Section 2 and Section 3.1. We denote by a_2^T (a_{21}^T) and Λ_2^T (Λ_{21}^T), and a_2^O (a_{21}^O) and Λ_2^O (a_{21}^O), respectively, the hedging intensity of second (first) period traders and second (first) period endowment shock price impact with transparency, and opacity.

Corollary 11. *With transparency:*

1. *Second-period liquidity traders hedge more and Λ_2 is higher than with opacity:*

$$|a_2^O| < |a_2^T|, \;\; \Lambda_2^O < \Lambda_2^T.$$

2. *When there are multiple equilibria, first-period liquidity traders hedge more and Λ_{21} is higher with opacity:*

$$|a_{21}^O| > |a_{21}^T|, \;\; \Lambda_{21}^O > \Lambda_{21}^T.$$

When a unique equilibrium arises, the result is ambiguous.

Proof. It is immediate to check that $\Lambda_2^{***} < \Lambda_2^T \equiv \gamma/(\gamma+\gamma_H)\tau_v$. Recall that with opacity $-1 < a_2^{***} < a_2^{**} < a_2^* < 0$. Thus, it is enough to check that

$$a_2^T \equiv -\frac{\gamma}{\gamma+\gamma_H} < a_2^{***} \equiv -\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)},$$

for $0 < \tau_u\tau_v \leq \gamma/(\gamma+\gamma_H)^3$. Next, for $4\tau_u\tau_v > \gamma/(\gamma+\gamma_H)^3$, with opacity, the unique equilibrium obtains as the unique real root of the cubic (21), which is strictly increasing in Λ_2 and negative at $\Lambda_2 = 0$. Evaluating it at Λ_2^T yields

$$\varphi(\Lambda_2^T) \equiv \frac{\gamma}{(\gamma+\gamma_H)^3\tau_v} > 0,$$

which proves our result for Λ_2. To see that $|a_2^O|$ is lower than $|a_2^T|$ in this case too, recall that, independently of the information regime, at equilibrium $\Lambda_2 = -a_2/\gamma\tau_v$.

Turning to a_{21}, recall that in either regime, we have

$$a_{21} = \frac{\gamma_H\tau_v\Lambda_{21}-1}{\tau_u+\Lambda_2^2\tau_v}\tau_u.$$

Substituting the values for the price impact coefficients under transparency in the above expression yields:

$$a_{21}^T = -\frac{(\gamma+\gamma_H)^2\tau_u\tau_v}{1+(\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v}. \tag{G.1}$$

With opacity, we need to distinguish between the two parameter regions identified in Proposition 3. When multiple equilibria arise, at the equilibrium with high Λ_2, we have

$$a_{21}^O = -\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)}. \tag{G.2}$$

Comparing (G.2) with (G.1), yields that

$$|a_{21}^O| > |a_{21}^T|,$$

for $0 < \tau_u\tau_v \leq (\gamma/(4(\gamma+\gamma_H)^3)$ (because of the ranking we established in Corollary 5, this is sufficient). Additionally, when multiple equilibria obtain with opacity, we have

$$\Lambda_{21}^T \equiv \frac{(\gamma+\gamma_H)\tau_u}{1+(\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v} < \Lambda_{21}^{***} \equiv \frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}.$$

When a unique equilibrium obtains with opacity our numerical simulations show that the relationship between a_{21}^T and a_{21}^O is ambiguous. See Figure 17 where for parameter values such that we are well into the region with a unique equilibrium under opacity, hedging aggressiveness is larger with transparency. For the illiquidity ranking, we evaluate $\varphi(\cdot)$ at Λ_{21}^T and obtain $\varphi(\Lambda_{21}^T) < 0$, implying that with opacity even when a unique equilibrium obtains $\Lambda_{21}^O > \Lambda_{21}^T$. □

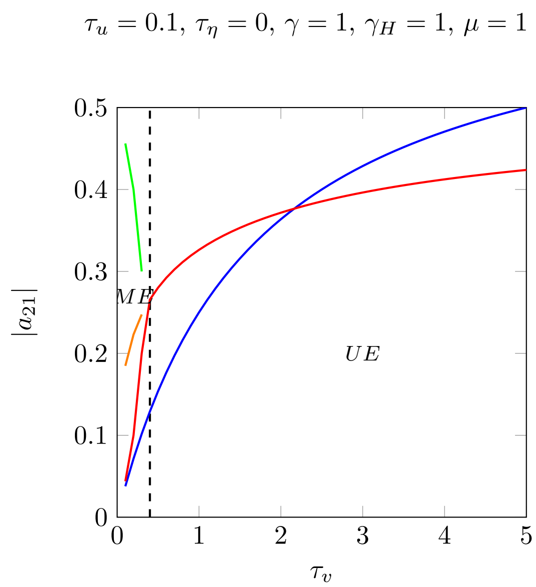

Figure 17: Comparing hedging aggressiveness across the two regimes. The blue curve illustrates $|a_{21}^T|$, while the green, orange and red curves $|a_{21}^O|$; we denote by ME and UE respectively the parameter region for which with opacity, multiple equilibria or a unique equilibrium obtain.