(In)efficiency in Information Acquisition and Aggregation through Prices*

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Abstract

We study markets in which traders acquire private information before submitting their schedules. We characterize conditions under which traders over-invest (respectively, under-invest) in information and trade excessively (respectively, insufficiently) on their private signals. These inefficiencies arise from a novel interaction between learning and pecuniary externalities. We show that, generically, no policy based solely on equilibrium prices and individual trade volumes can simultaneously implement efficiency in information acquisition and trading. This impossibility result becomes a possibility when information acquisition is verifiable or when taxes can be conditioned on aggregate trading volume. Finally, we show that the optimal tax–subsidy scheme is progressive and attenuates traders' response to private information when pecuniary externalities dominate learning externalities, but regressive and amplifying the response otherwise.

Keywords: information acquisition, aggregation through prices, learning and pecuniary externalities, team efficiency, optimal policy

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1 Introduction

Improvements in information technology have dramatically reduced the cost of acquiring and processing information.[1] At the same time, there is growing concern about the social value of these improvements, particularly in asset markets, where trading reflects a mix of fundamental and speculative motives. Policy proposals span a wide range—from subsidies to information acquisition, motivated by the potential public-good value of information, to transaction taxes and other "sand-in-the-wheels" interventions aimed at curbing speculative trading based on asymmetric private information. A prominent example is the Tobin tax, an ad valorem levy on transactions. Despite their prevalence, the welfare implications of these policies remain largely unexplored.

In this paper, we develop a framework to study the positive and normative effects of information acquisition and trading in markets where prices only partially aggregate information. Our model is designed for welfare analysis in economies with endogenous private information in which traders compete in demand (or supply) schedules—often referred to as generalized limit orders. Competition in schedules is a central feature of many important markets, including stock exchanges with limit orders, central banks' liquidity auctions, Treasury auctions, and markets for pollution permits, electricity, and other commodities.[2] While these markets have been extensively studied from the perspective of price formation and information aggregation under exogenous private information, much less is known about how agents choose what private information to acquire prior to trading, and how inefficiencies in information acquisition interact with inefficiencies in its subsequent use. Addressing this interaction is the central objective of this paper.

Our contribution is twofold. First, we identify and characterize the inefficiencies that arise in the collection of private information in markets with schedule competition, and relate them to distortions in trading behavior. Second, we study policy interventions—subsidies and taxes with different informational and aggregate contingencies—designed to improve the efficiency of the market equilibrium. We study a linear–quadratic–Gaussian economy with a unit-mass continuum of traders who compete in schedules. Traders face uncertainty about both the asset's fundamental value and the elastic supply of the asset. Depending on the application, the latter captures the positions of liquidity or noisy traders in stock markets, the supply of bonds in Treasury auctions, the operations of a central bank in liquidity auctions, or the issuance of pollution permits by a regulator. Prior to trading, each trader endogenously

[1] See, for example, Nordhaus [2021] on the sharp decline in the cost of computation (and therefore of information processing).

[2] Both Li et al. [2023] and Anand et al. [2025] report a high percentage of use of limit orders in US stock markets.

acquires a private signal about the asset's value. Signal precision is a choice variable, and noise in the signals is correlated across traders, reflecting attention to common information sources subject to source-specific noise.

In order to identify the inefficiencies in the acquisition of information and relate to those in trading, we begin by analyzing an economy in which private information is exogenous. Our first result shows that, except in knife-edge cases, market equilibria are inefficient absent policy intervention. These inefficiencies arise from the interaction of two distinct externalities. The first is a familiar learning externality: individual traders do not internalize how collective changes in their schedules affect the informational content of prices and, consequently, other agents' ability to align their trades with fundamentals. The second, and more novel, is a pecuniary externality that arises from the interaction between noisy private information and schedule competition. Because traders submit price-elastic schedules, noise in their information affects equilibrium prices, which in turn alters other traders' asset holdings and payoffs.

To isolate this pecuniary externality, we consider a fictitious environment in which traders are "naive": they do not infer information from prices but instead observe an exogenous public signal with the same precision as the equilibrium price in the original economy. In this environment, traders fail to internalize how noise-driven fluctuations in their schedules affect the market-clearing price and, through it, other traders' asset holdings. These effects are welfare reducing, not because of changes in expenditures for given holdings, but because price movements induced by noise in the submitted schedules reallocate assets inefficiently across traders. This pecuniary externality differs fundamentally from those studied in settings with missing markets, collateral constraints, or other financial frictions. It disappears when information is symmetric (e.g., when the noise in information vanishes), or when schedules are price-inelastic. Notably, restricting traders to submit flat schedules ("market orders") eliminates both the learning and pecuniary externalities, but also weakens the response of allocations to fundamentals and therefore is not guaranteed to improve welfare (see also the discussion in the online appendix Pavan et al. [2026b]).

We show that the two externalities have opposite effects on the equilibrium schedules. The learning externality leads traders to under-react to private information, whereas the pecuniary externality induces them to over-react. When equilibrium schedules are downward sloping, the pecuniary externality dominates and trading responds too strongly to private information; when, instead, the schedules are upward sloping, the learning externality dominates and traders under-respond to private information. Correcting these distortions requires taxes and subsidies that are nonlinear in traded quantities.

Equipped with the above results, we then turn to the case where private information is endogenous. We show that policies that induce efficient trading when information is exogenous fail to induce efficient information acquisition.[3] As we explain in due time, correlation in the agents' noise, which is natural when traders rely on common information sources affected by noise at the source, plays a key role for this result.[4] In this case, we find that, generically, there exists no policy contingent solely on asset prices and individual trade volumes inducing efficiency in both information acquisition and trading.

This impossibility result can be overturned in two cases. When information acquisition is verifiable, efficiency can be achieved by combining standard taxes and subsidies on trading with separate taxes and subsidies on information expenditure. When information acquisition is not verifiable, conditioning taxes and subsidies on the aggregate volume of trade provides the planner with an additional contingency that allows efficiency to be restored. By introducing uncertainty about marginal tax rates, the new contingency permits the planner to align private and social incentives to acquire information without distorting trading behavior.

In both cases, the optimal tax–subsidy scheme is progressive when the pecuniary externality dominates, it is regressive when the learning externality dominates. Progressive schemes dampen traders' responses to private information, while regressive ones amplify them. Finally, we show that simple ad valorem transaction taxes perform poorly in our environment. By affecting the sensitivity of schedules to prices but not to private information or the traders' incentives to acquire information, they can move the equilibrium further away from efficiency and even reduce welfare relative to laissez-faire.[5] Improving upon the market outcome therefore requires more sophisticated, nonlinear policies that exploit aggregate contingencies.

Related literature This paper relates to several strands of the literature.

Information aggregation and acquisition.

A first strand studies inefficiencies in the equilibrium aggregation of dispersed information. Seminal contributions include Palfrey [1985], Vives [1988], Amador and Weill [2010], and Vives [2017]. Among these, the closest to ours are Amador and Weill [2010] and Vives [2017],

[3]This result is fundamentally different from the over-investment in information acquisition that arises in economies in which information undermines insurance (see Hisrhleifer [1971] for a seminal contribution). It is also different from the over-investment in information acquisition that arises in economies in which adverse selection precludes trade (see, e.g., Dang et al. [2017]).

[4]It is common in the literature to assume that private signals are independent, conditional on the state. The assumption is made for tractability but may mis-guide policy analysis when private information is endogenous.

[5]In the laissez-faire equilibrium, for given precision of private information and sensitivity of the schedules to the private signals, the sensitivity of the equilibrium schedules to the price is already welfare-maximizing. Hence policies that only manipulate the sensitivity of the schedules to prices bring the equilibrium further apart from the efficient allocation.

which examine inefficiencies in information aggregation when traders compete in schedules.[6] In all these papers, however, private information is exogenous and noise in information is uncorrelated across agents.

Earlier contributions studying information acquisition prior to trading include Diamond and Verrecchia [1981] and Verrecchia [1982]. More recent work on this topic includes Peress [2010], Manzano and Vives [2011], Kacperczyk et al. [2016] and Dávila and Parlatore [2021].[7] None of the papers in this literatures studies inefficiencies in the acquisition of private information, their relation to learning and pecuniary externalities, or the design of policies correcting distortions in both trading and information acquisition, which are the central contributions of this paper.[8]

Inefficiency in information usage and in information acquisition.

A second strand studies the relation between efficiency in information usage and in information acquisition. Vives [1988] shows that, in a Cournot economy with conditionally independent private signals, equilibrium trading and information acquisition are efficient when traders compete in market orders. We show that this result extends to environments with correlated noise. By contrast, when traders compete in schedules, both the acquisition and the use of information are generically inefficient.

Efficiency in the use of information implies efficiency in information acquisition in the macroeconomic environments of Angeletos et al. [2020]. In those economies, prices imperfectly aggregate information, as in our model, but agents have access to complete markets and can fully insure idiosyncratic risk. In contrast, our economy features incomplete markets, in the sense that traders bear the risk of their own asset positions. As a result, policies that correct inefficiencies in trading need not induce efficiency in information acquisition.

Colombo et al. [2025] study technology adoption with investment spillovers and show that state-invariant subsidies restore efficiency when information is exogenous but not when it is endogenous. In the latter case, more sophisticated Pigouvian policies contingent on aggregates are required. Their analysis abstracts from competition in schedules and information aggregation through prices, which are central in our setting. Colombo et al. [2014] study information acquisition in linear–quadratic–Gaussian economies without information aggre-

[6] See also Kyle [1989], Vives [2011], and Rostek and Weretka [2012] for related models of competition in schedules.

[7] See also Mondria et al. [2021] for a model in which traders' attention reduces the noise in the interpretation of price information.

[8] Related are also the literature on the Grossman-Stiglitz paradox, namely the lack of incentives to acquire information when prices are fully revealing (see, among others, Grossman and Stiglitz [1980], and Vives [2014] for a potential resolution of the paradox), and the literature on strategic complementarity/substitutability in information acquisition (see, among others, Ganguli and Yang [2009], Hellwig and Veldkamp [2009], Manzano and Vives [2011], Myatt and Wallace [2012], and Pavan and Tirole [2025]).

gation and show that, absent dispersion externalities, efficiency in use implies efficiency in acquisition. In contrast, in our model, traders compete in schedules and inefficiencies arise from information aggregation, even in the absence of dispersion externalities.

Hebert and La'O [2023] and Angeletos and Sastry [2025] study rational inattention in environments where agents learn from aggregate outcomes such as prices or macroeconomic statistics. While these papers share with ours the feature that agents learn from others' behavior, they do not consider schedule competition and therefore abstract from pecuniary externalities. Moreover, they do not characterize whether agents over- or under-invest in information, how they over- or under-respond to private information and prices, or which policies correct inefficiencies in both information acquisition and trading.

Transaction taxes and policy interventions.

A third strand examines taxation and regulation of financial transactions. Tobin [1978] famously advocates transaction taxes to curb trading and volatility, a view echoed in Stiglitz [1989] and Summers and Summers [1989]. We contribute to this literature by characterizing the contingencies—particularly aggregate contingencies—of the policy interventions and the progressivity/regressivity of the taxes/subsidies required to induce efficiency in both trading and information acquisition.

Correlated noise and rational inattention.

Finally, we assume that noise in endogenous information is correlated across agents. Recent work shows that rational inattention can generate correlated noise in beliefs (Woodford [2012a], Woodford [2012b], and Nimark and Sundaresan [2019]) and that such environments share features with models of biased information processing. Our paper shares with this literature the implication that investments in information acquisition affect agents' exposure to correlated noise, which may appear as biased behavior from an outside perspective. Our focus, however, is on how such correlation interacts with schedule competition to generate pecuniary externalities and inefficiencies in both information acquisition and trading.

Organization. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 identifies the inefficiencies in trading. Section 4 endogenizes the collection of private information, relates the inefficiencies in information acquisition to those in trading, and discusses policy interventions removing both inefficiencies. Section 5 concludes. All proofs are in the Appendix at the end of the document. The proofs are self-contained. The file Pavan et al. [2026a] on our webpages contains step-by-step detailed derivations of all results. The online appendix Pavan et al. [2026b], instead, discusses the case where the agents are restricted to compete in market orders (i.e., flat, price-invariant schedules).

2 Model

We first describe the environment, and then present the traders' problem of choosing how much private information to acquire and which schedule to submit.

2.1 Environment

The market is populated by a continuum of traders, indexed by $i \in [0,1]$, trading a homogenous and perfectly divisible asset. Depending on the application of interest, the asset can be a stock, a collateral-backed loan (as in central banks' liquidity auctions), a Treasury bill (as in Treasury auctions), or a pollution permit (as in auctions for emission rights).

Let x_i denote the quantity of the asset purchased (when x_i is positive) or short-sold (when x_i is negative) by trader i, and $\tilde{x} = \int x_i di$ the traders' aggregate net demand. Each trader i's payoff from trading x_i units of the asset at price p is given by

$$\pi_i \equiv (\theta - p)\, x_i - \lambda x_i^2/2,$$

where $\lambda \in \mathbb{R}_{++}$ is a positive scalar, and $\theta \sim N(0, \sigma_\theta^2)$. The variable θ proxies for the traders' gross common value for the asset (stemming from future cash flows, or the asset's resale value). The term $\lambda x_i^2/2$, instead, proxies for trading, or adjustment, costs whose role is to induce imperfectly elastic demands.[9]

Traders face an exogenous inverse net asset supply

$$p = \alpha - u + \beta\tilde{x},$$

where $\alpha, \beta \in \mathbb{R}_{++}$ are positive scalars, and where $u \sim N(0, \sigma_u^2)$ is an aggregate shock.[10] Depending on the application of interest, such a net supply may originate in the positions of small unsophisticated agents, the trade of collateral-backed loans by a central bank, the supply of T-bills in a Treasury auction, or the supply of pollution permits in auctions for emission rights. The shock u may then capture correlation in noise traders' positions, the central bank's (political and/or economic) cost of liquidity provision, the allocation of T-bills to a group of (un-modeled) institutional investors outside the market, or changes in the environment affecting a regulator's net benefit from issuing pollution permits.

[9]See also Vives [2011] and Rostek and Weretka [2012] for examples of models with a quadratic adjustment cost.

[10]As usual, the role of this shock is to prevent the price from being fully revealing of the information the traders collectively possess.

The planner believes the costs behind such a supply curve to be equal to $(\alpha - u)\,\tilde{x} + \beta\tilde{x}^2/2$. A positive shock u thus represents a reduction in the cost of unloading the asset, triggering an outward shift in the supply curve, whereas a negative u represents an increase in the cost, triggering an inward shift. This specification also permits us to interpret the supply of the asset as coming from a "representative supplier" with payoff $p\tilde{x} - (\alpha - u)\,\tilde{x} - \beta\tilde{x}^2/2$. In this case, the term $\alpha - u$ proxies for the opportunity cost for the representative supplier of unloading the asset, and $\beta\tilde{x}^2/2$ for a quadratic trading, or adjustment, cost.[11] Following the pertinent literature, we assume that both the traders and the planner treat such a supply as exogenous. Importantly, however, the planner accounts for the cost of unloading the asset (alternatively, for the utility of the representative supplier) when computing the efficient allocations.

To simplify the derivation of the equilibrium formulas, we assume that the variables θ and u are independently distributed. The results, however, extend to the case where they are imperfectly correlated. For notational purposes, given any Gaussian random variable h with variance σ_h^2, we denote by $\tau_h \equiv 1/\sigma_h^2$ the variable's precision.

The traders do not know θ. They privately collect information about it prior to submitting their schedules. When doing so, they also account for the fact that the equilibrium market-clearing price will imperfectly aggregate the traders' dispersed information about θ.

Formally, each trader i observes a signal $s_i \equiv \theta + \epsilon_i$, whose (endogenous) noise $\epsilon_i \equiv f(y_i)(\eta + e_i)$ is imperfectly correlated among the traders. Precisely, the variable $\eta \sim N(0, \sigma_\eta^2)$ is perfectly correlated whereas the variable $e_i \sim N(0, \sigma_e^2)$ is i.i.d. among the traders. The variables $(\theta, u, \eta, (e_i)_{i\in[0,1]})$ are jointly independent. The variable $y_i \in \mathbb{R}_+$ proxies for the trader's investment in information acquisition. The function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is decreasing which permits us to interpret a higher y_i as a larger investment. The cost of y_i is given by a twice-continuously differentiable function $\mathcal{C}$, satisfying $\mathcal{C}'(y) \geq 0$ and $\mathcal{C}''(y) > 0$ for all $y \geq 0$. All the main results hold for general cost functions. This is because inefficiencies in the acquisition of information originate in differences between the private and the social value of information which are invariant in $\mathcal{C}$. However, the existence of equilibria, both in the laissez-faire economy (Proposition 5) and under the optimal policies (Propositions 7 and 8) requires that individual payoffs be quasi-concave in y_i. As we show in the Appendix, this is the case

[11] For example, suppose the asset is a stock, and its net supply comes from a continuum of *noise* traders of measure 1, indexed by $j \in [0,1]$, each with payoff $\pi_j^{NT} = px_j - (\alpha - u_j)\,x_j - \beta x_j^2/2$, with x_j denoting the amount of shares sold (when x_j is positive) or bought (when x_j is negative) by trader j. Here $u_j = u + h_j$ is the trader's private opportunity cost of selling the asset, with $h_j \sim N(0, \sigma_h^2)$ drawn independently across the traders and with u_j known to the trader at the time he submits his limit orders. Each trader's net supply schedule is then equal to $x_j = [p - (\alpha - u_j)]/\beta$ implying that the inverse aggregate net supply is $p = \alpha - u + \beta\tilde{x}$. Given $(p, \tilde{x}, u)$, these traders' aggregate payoff is thus equal to $\int \pi_j^{NT} dj = p\tilde{x} - (\alpha - u)\tilde{x} - \beta\tilde{x}^2/2 + \sigma_h^2/2\beta$, which is the same as above, except for the constant term $\sigma_h^2/2\beta$, which plays no role in the analysis.

when $\mathcal{C}$ is sufficiently convex and $\mathcal{C}'(y)$ is small for small y. Specifically, there are constants $L, M \in \mathbb{R}_{++}$ (defined in the Appendix) such that payoffs are quasi-concave when $\mathcal{C}'(0) \leq L$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > M$. For example, when $\mathcal{C}(y) = By^2/2$, with $B \in \mathbb{R}_{++}$, these conditions are jointly satisfied for B large enough.

The idea behind the above information structure is that traders learn from a large number of information sources. We do not model such sources but instead assume that their content is summarized in the uni-dimensional statistic $s \equiv \theta + \epsilon$. The correlation in the statistic's noise may reflect error at the "source" as, e.g., in Myatt and Wallace [2012]. Such a correlation, in addition to being realistic, has implications for the (in)efficiency of the equilibrium acquisition and usage of information, as we discuss further on.

Finally, to simplify some of the derivations, we assume that the function f takes the form $f(y) = y^{-1/2}$. Such an assumption allows us to express the precision

$$\tau_\epsilon(y) \equiv \frac{\tau_e \tau_\eta}{\tau_e + \tau_\eta} y \tag{1}$$

of the endogenous noise term $\epsilon \equiv f(y)(\eta + e)$ in the agents' statistic as a linear function of y.

Timing. At $t = 0$, the traders simultaneously make their investments $(y_i)_{i \in [0,1]}$ in information acquisition. At $t = 1$, the traders observe their private signals $(s_i)_{i \in [0,1]}$. At $t = 2$, the traders simultaneously submit their schedules. At $t = 3$, the market clears, the equilibrium price is determined, the equilibrium trades are implemented, and payoffs are realized.

2.2 Traders' problem and equilibrium

Given $I_i \equiv (y_i, s_i)$, trader i chooses a schedule that maximizes, for each price p, the trader's expected payoff

$$\mathbb{E}\left[(\theta - p)\, x_i - \lambda \frac{x_i^2}{2} | I_i, p\right]$$

taking into account how the price p co-moves with the traders' fundamental value θ for the asset, the supply shock u, and the common noise η in the traders' information. The expectation is over θ. The solution to this problem is the demand schedule given by

$$X_i(p; I_i) = \frac{1}{\lambda}(\mathbb{E}[\theta | I_i, p] - p), \tag{2}$$

where $\mathbb{E}[\theta | I_i, p]$ denotes the trader's expectation of θ given the trader's investment y_i in information, the realization s_i of the trader's private signal, and the price p.[12]

[12] Our linear-quadratic model is close to the standard CARA-Normal one, except for the fact that, in the latter, the denominator of the asset demand is the product of the traders' constant risk aversion coefficient

At $t = 0$, each trader $i \in [0, 1]$ then selects y_i to maximize the expected profit

$$\mathbb{E}\left[\left(\theta - p - \frac{\lambda}{2} X_i(p; I_i)\right) X_i(p; I_i)\right] - \mathcal{C}(y_i),$$

where the expectation is over (s_i, θ, p), given y_i.

An equilibrium is a collection of investments in information and of schedules $(y_i, X_i(\cdot))$, one for each trader $i \in [0, 1]$, such that each $(y_i, X_i(\cdot))$ maximizes trader i's expected payoff given $(y_j, X_j(\cdot))_{j \neq i}$. Following the pertinent literature, we focus on equilibria (and on team-efficient allocations–defined below) in which (a) the market-clearing price p is an affine function of the aggregate variables (θ, u, η), and (b) all agents make the same investments in information acquisition and follow the same rule to map their private signals into the submitted schedules.

3 Inefficiency in trading

To identify the inefficiencies in trading, we fix the traders' investments in information acquisition and assume that $y_i = y$ for all i. We thus also fix the precision τ_ϵ, of the noise in the traders' private information s_i, as defined in (1). We first solve for the traders' equilibrium schedules and then compare them to their efficient counterparts. The analysis permits us to identify inefficiencies in trading, that is, in the usage of the agents' information, and policies correcting these inefficiencies. Because y is held fixed, to ease the notation, we drop it from the arguments of many of the functions below when there is no risk of confusion.

3.1 Equilibrium usage of information

In any symmetric equilibrium in which the price is an affine function of the aggregate variables (θ, u, η), each trader's demand schedule is an affine function of her private signal s_i and the price p. That is,

$$X_i(p; I_i) = a^* s_i + \hat{b}^* - \hat{c}^* p,$$

for some scalars $(a^*, \hat{b}^*, \hat{c}^*)$ that depend on the exogenous parameters of the model, as well as on the investment y in information.[13] Aggregating across traders, we then have that

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and the conditional variance of the asset value.

[13]The reason why we denote the sensitivity $\hat{c}^*$ of the equilibrium schedules to the price, and the constant term $\hat{b}^*$ in the equilibrium demand schedules, with the "$\wedge$" symbol is that, in the Appendix, we use the notation $x_i = a^* s_i + b^* + c^* z$ to denote the induced trades (that is, the volume of the asset purchased/sold by each trader i) as a function of the trader's private information and the endogenous signal z contained in the equilibrium price. We do not use "$\wedge$" for the sensitivity a^* of the equilibrium schedules to the traders' private information s_i because it is the same no matter whether one looks at the submitted schedules or the induced

the aggregate demand is equal to $\int X_i(p; I_i)di = a^*(\theta + f(y)\eta) + \hat{b}^* - \hat{c}^* p$. As usual, the idiosyncratic errors in the traders' signals wash out in the aggregate demand.[14] However, the agents' investments in information acquisition (parametrized by y) impacts the aggregate demand through its effect on the traders' exposure to common noise η. This property has important implications for the positive and normative results we discuss below. Letting

$$z \equiv \theta + \underbrace{f(y)\eta - \frac{u}{\beta a^*}}_{\omega}, \tag{3}$$

we then have that the equilibrium price must satisfy

$$p = \frac{\alpha + \beta \hat{b}^*}{1 + \beta \hat{c}^*} + \frac{\beta a^*}{1 + \beta \hat{c}^*} z. \tag{4}$$

The information about θ contained in the market-clearing price is thus the same as the one contained in the endogenous public signal z whose noise $\omega \equiv f(y)\eta - u/\beta a^*$ is a combination of the common noise η in the traders' private information and the shock u to the supply of the asset. The precision of ω is equal to $\tau_\omega(a^*)$, with the function τ_ω given by

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 \tau_u y \tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta} \tag{5}$$

for all $a \in \mathbb{R}$. Let Λ be the function defined, for all τ_ω, by

$$\Lambda(\tau_\omega) \equiv \frac{y^2 \tau_\eta^2 \left(\tau_\omega + \tau_\epsilon + \tau_\theta\right) - \tau_\omega \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right)}{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\omega\right)}, \tag{6}$$

and, for any a, let $\hat{C}$ and $\hat{B}$ be the functions defined by

$$\hat{C}(a) \equiv \frac{\beta(\lambda + \beta)a^2 \tau_\theta \tau_u + y\tau_\eta \tau_\theta - \beta a y \tau_\eta \tau_u \left[1 - (\lambda + \beta)a\right]}{y\tau_\eta \left(\lambda \tau_\theta + \beta^2 a \tau_u\right)}, \tag{7}$$

and

$$\hat{B}(a) \equiv \frac{\alpha}{\beta + \lambda} \left(\lambda \hat{C}(a) - 1\right). \tag{8}$$

trades.

[14] This is because we make the convention that the analog of the strong law of large numbers holds for a continuum of independent random variables with uniformly bounded variances. The last property holds as long as y is bounded away from 0.

We then have the following result:

Proposition 1 **(equilibrium trading).** *Suppose $y_i = y$ for all i, with y exogenous. There exits a unique symmetric equilibrium. The sensitivity of the traders' equilibrium schedules to their private information, a^*, is given by the unique real root to the equation*

$$a^* = \frac{1}{\lambda\Lambda(\tau_\omega(a^*))}, \tag{9}$$

and is such that $0 < a^* < 1/\lambda$. Given a^*, the equilibrium values of the other two parameters $\hat{c}^*$ and $\hat{b}^*$ defining the equilibrium schedules are given by the functions (7) and (8).

Fixing the quality of the traders' private information, y, the equilibrium schedules thus solve a familiar fixed-point problem in which the traders correctly account for the information contained in the market-clearing price, and the latter is consistent with the submitted schedules. As anticipated above, the novelty relative to previous work is the presence of common noise in the traders' information, η, which is present in both the aggregate demand schedule and the market-clearing price.

As we show in the Appendix, $1/\Lambda(\tau_\omega(a))$ is the coefficient of the projection of θ on (s_i, p) accounting for the endogenous information z contained in the market-clearing price. Using the formulas for τ_ϵ and τ_ω, respectively in (1) and (5) one can show that, for any a,

$$\Lambda(\tau_\omega(a)) = 1 + \frac{\beta^2 a^2 \tau_u + \tau_\theta}{\tau_e y} + \frac{\beta^2 a^2 \tau_u \tau_\theta}{\tau_e y^2 \tau_\eta} + \frac{\tau_\theta}{y\tau_\eta}. \tag{10}$$

It is easy to see that, when $y \to \infty$, for any a, $\Lambda(\tau_\omega(a)) \to 1$, in which case $a^* \to 1/\lambda$, $\hat{c}^* \to 1/\lambda$, and $\hat{b}^* \to 0$. That is, as the noise in the agents' signals s_i vanishes, the equilibrium schedules converge to the complete-information ones $(s_i - p)/\lambda$, for all p and s_i. Similarly, as $\tau_e \to \infty$, for any a, $\Lambda(\tau_\omega(a)) \to 1 + \tau_\theta/y\tau_\eta$, in which case $a^* \to y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$, $\hat{c}^* \to 1/\lambda$ and $\hat{b}^* \to 0$. In other words, when the idiosyncratic noise in the agents' signals vanishes, the price does not contain any information about θ that the traders do not know already through their signals s_i, in which case

$$\mathbb{E}\left[\theta | I_i, p\right] = \frac{y\tau_\eta}{y\tau_\eta + \tau_\theta} s_i$$

where $y\tau_\eta$ is the precision of the signal s_i when $\tau_e \to \infty$. On the other hand, when either $y \to 0$, or $\tau_e \to 0$, or $\tau_\eta \to 0$, for any a, $\Lambda(\tau_\omega(a)) \to \infty$, implying that $a^* \to 0$, $\hat{c}^* \to 1/\lambda$, and $\hat{b}^* \to 0$. When the noise in the agents' signals diverges to infinity, the agents' stop responding to their signals and their schedules converge to $-p/\lambda$; that is, the agents' short-sell the asset when its price is positive.

3.2 Efficient usage of information

To isolate the inefficiencies in the equilibrium usage of information, we first identify the welfare losses (relative to the full-information benchmark) under any symmetric profile of affine schedules. We then characterize the schedules that minimize these losses (equivalently, that maximize ex-ante welfare) over the relevant class. The comparison between the equilibrium and the efficient schedules permits us to identify the inefficiency in equilibrium trading. By considering a fictitious environment in which traders do not learn from prices, we then identity the pecuniary externalities that, jointly with the familiar learning externalities that are present when agents learn from prices, are responsible for the inefficiency. Finally, we show how the interaction between the two externalities relates to the slope of the schedules. We discuss policies correcting the inefficiency in trading at the end of the section.

3.2.1 Welfare losses

Ex-post welfare is given by

$$W \equiv \int \left(\theta x_i - \frac{\lambda}{2} x_i^2\right) di - \left(\alpha - u + \beta \frac{\tilde{x}}{2}\right) \tilde{x}.$$

The integral term is the total payoff that the traders derive from purchasing the asset. The remaining term is the supply cost. The traders' payoffs are net of the expenses they incur to purchase the asset. These expenses do not appear in the welfare function because they are a transfer between the traders and the relevant asset suppliers and all agents' payoffs are linear in consumption. Importantly, note that W accounts for the cost of unloading the asset (equivalently, for the utility $p\tilde{x} - (\alpha - u)\tilde{x} - \beta\tilde{x}^2/2$ of the representative supplier).

It is easy to see that the trades that maximize ex-post welfare are given by $x_i = x^o$ for all i, with

$$x^o \equiv (\theta + u - \alpha) / (\beta + \lambda). \tag{11}$$

When traders know θ, these trades coincide with those sustained in equilibrium, which is a manifestation of the First Welfare Theorem.[15]

Next, let W^o denote welfare under the first-best allocation, and $WL \equiv \mathbb{E}[W^o] - \mathbb{E}[W]$ denote the ex-ante expected welfare losses that arise when the traders purchase the asset in a quantity different from the first-best level, due to asymmetric information. When each schedule $X_i(p; I_i)$

[15]Clearly, the theorem does not require that the traders know u. In fact, it does not even require that they know θ. It suffices that they have no way of learning about their payoffs beyond what they know prior to trading.

is affine in s_i and p, the welfare losses can be expressed as follows (the derivations are in the Appendix):

$$WL = \frac{1}{2}(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2]. \tag{12}$$

The term $\mathbb{E}[(\tilde{x} - x^o)^2]$ captures the losses due to the discrepancy between the aggregate level of trade $\tilde{x}$ and its first-best counterpart, x^o. The term $\mathbb{E}[(x_i - \tilde{x})^2]$, instead, captures the losses due to the dispersion of the individual trades around the average level.

3.2.2 Efficient schedules

Consistently with the rest of the literature (see, among others, Vives [1988], Angeletos and Pavan [2007], Amador and Weill [2012], and Vives [2017]), we define the efficient use of information as the schedules that minimize the ex-ante welfare losses over the set of schedules that are affine in the private signals and the price. While the welfare definition accounts for the costs of supplying the asset, the optimization is over the traders' schedules, respecting the exogeneity of the supply of the asset. This definition permits us to isolate the inefficiencies in the traders' usage of information. Accordingly, we say that $(a^T, \hat{b}^T, \hat{c}^T)$ identifies the efficient use of information if, and only if, when all traders submit the schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$, the welfare losses are as small as under any other affine schedule $x_i = a' s_i + \hat{b}' - \hat{c}' p$.[16]

Lemma 1 (efficiency of demands for given sensitivity to private information). *For any sensitivity of the schedules to the traders' private information, a, the values of $\hat{c}$ and $\hat{b}$ in the schedules that minimize the welfare losses are given by the same functions (7) and (8) that define the equilibrium schedules.*

Inefficiencies in equilibrium trading thus originate in the discrepancy between the sensitivity of the equilibrium schedules to private information a and the sensitivity a^T that maximizes welfare. The other terms $\hat{c}$ and $\hat{b}$ defining the equilibrium schedules are also different from their efficient counterparts $\hat{c}^T$ and $\hat{b}^T$, but only because $a \neq a^T$: given a, the response of the equilibrium schedules to the price and the unconditional level of trade are efficient.

Using Lemma 1, we can express the welfare losses as a function $WL(a, \tau_\omega(a))$ of the sensitivity of the traders' schedules to their private information, a, and the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price (the expression for $WL(a, \tau_\omega(a))$ is in the Appendix – proof of Proposition 2). The efficient level of a, which we denote by a^T,

[16] Again, we use the symbol "ˆ" to distinguish the efficient schedules from the efficient trades.

is the value of a that minimizes $WL(a, \tau_\omega(a))$. Let

$$\Delta(a) \equiv -\frac{\beta^2 y^3 \tau_\eta^3 \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 \left(\beta^2 a^2 \tau_u + y\tau_\eta\right)^2 \left(\tau_\omega(a) + \tau_\theta\right)\left(y\tau_\eta - \tau_\omega(a)\right)},$$

and

$$\Xi(a) \equiv \frac{\tau_\eta \beta \left(\tau_\omega(a) + \tau_\theta\right)}{\lambda \tau_e \left(y\tau_\eta - \tau_\omega(a)\right)}.$$

We then have the following result:

Proposition 2 (efficient trading). *Suppose that $y_i = y$ for all i, with y exogenous. The planner's problem has a unique solution. The sensitivity a^T of the traders' schedules to their private information is implicitly given by the solution to*

$$a^T = \frac{1}{\lambda} \frac{1}{\Lambda(\tau_\omega(a^T)) + \Delta(a^T) + \Xi(a^T)} \tag{13}$$

and is such that $0 < a^T < 1/\lambda$. Given a^T, the other two parameters defining the efficient schedules, $\hat{c}^T$ and $\hat{b}^T$, are given by the same functions in (7) and (8) that describe the corresponding coefficients under the equilibrium use of information.

When, for any a, $\hat{b}$ and $\hat{c}$ are set optimally, the welfare losses $WL(a, \tau_\omega(a))$ are a convex function of a reaching a minimum at $a = a^T$, with $0 < a^T < 1/\lambda$. Note that the equation (13) that determines the value of a^T differs from the one in (9) yielding the equilibrium value of a^* only by the two terms $\Delta(a)$ and $\Xi(a)$ in the denominator of the right-hand side of (13). The term $\Delta(a)$ is essentially a scaling of

$$\frac{\partial WL(a, \tau_\omega(a))}{\partial \tau_\omega(a)} \frac{\partial \tau_\omega(a)}{\partial a}.$$

Therefore, this term can be thought of as a proxy for the familiar learning externality originating in the fact that traders do not internalize that the sensitivity of their schedules to their private information determines the informativeness of the equilibrium price and hence the possibility for other traders to use the price as an endogenous signal about θ when choosing how much to trade. Note that, after replacing the formula for $\tau_\omega(a)$ into the formula for $\Delta(a)$, we have that

$$\Delta(a) \quad = \quad -\frac{\beta^2 y \tau_\eta \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 \left[\beta^2 a^2 y \tau_\eta \tau_u + \tau_\theta \left(\beta^2 a^2 \tau_u + y\tau_\eta\right)\right]}.$$

This term is always negative reflecting the under-response of the equilibrium schedules to private information. Essentially, traders do not consider that, when they all respond more to their private signals, they make the price more informative which leads to more efficient trades. The social planner, instead, internalizes this effect and asks that the traders respond more to their private signals.

Naturally, this learning externality vanishes when $\tau_e \to \infty$, for, in this case, there is no idiosyncratic noise in the agents' signals. Recall that, in this case, $a^* \to y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$, and, $\Delta(a^*) = 0$ when $a^* = y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$. Likewise, this externality vanishes when $y \to \infty$, i.e., when $s_i = \theta$ almost surely. In this case, $a^* \to 1/\lambda$, and $\Delta(1/\lambda) \to 0$ as $y \to \infty$. In the same vein, the externality vanishes when $\tau_u \to 0$ for, in this case, the price becomes uninformative (because of the high volatility of the supply schedule), no matter the sensitivity of the traders' schedules to their private information. On the other hand, the learning externality does not vanish when $\tau_\eta \to \infty$, i.e., when the noise in the traders' information is independent; in this case, the externality converges to $\beta^2\tau_u (1 - \lambda a)^2 / [\lambda^2 (\beta^2 a^2 \tau_u + \tau_\theta)]$.

The term $\Xi(a)$, instead, is a pecuniary externality. When the traders respond to their private signals, they do not internalize that variations in their schedules due to noise in their signals impact other traders' asset holdings through the dependence of other traders' schedules on the price. Being noise-driven, such variations are not justified in the planner's eyes. The planner thus asks that the traders respond less to their private signals to reduce the welfare losses of such noise-driven variations. Note that, after replacing the formula for $\tau_\omega(a)$ into the formula for $\Xi(a)$, we have that

$$\Xi(a) = \frac{\beta\left[\beta^2 a^2 \tau_u y \tau_\eta + \tau_\theta\left(\beta^2 a^2 \tau_u + y\tau_\eta\right)\right]}{\lambda \tau_e y^2 \tau_\eta}.$$

The term $\Xi(a)$ is thus always positive, reflecting the over-response of the equilibrium schedules to private information. This pecuniary externality is different from those that originate in financial frictions in other incomplete-market economies; see, e.g., Lorenzoni [2008]. It is closer in spirit to the distributive pecuniary externality of Davila and Korinek [2018], although the mechanics and origins are different. When there is no noise in the agents' information (i.e., when $y \to \infty$), this externality vanishes, contrary to what happens in these other papers. More interestingly, this externality vanishes also when $\tau_e \to \infty$, i.e., when the only noise in the agents' signals is perfectly correlated across the agents. This is because the economy is competitive and, when there is no private information, the First Welfare Theorem applies. On the contrary, this externality does not vanish when the common component of the noise in the agents' information vanishes, i.e., when $\tau_\eta \to \infty$; in this case, the externality converges to $\beta\left[\beta^2 a^2 \tau_u + \tau_\theta\right]/\lambda \tau_e y$: Trading in the laissez-faire economy is inefficient even in the absence of correlation in the noise in the agents' information.

Importantly, both externalities arise because of the combination of the following properties: (1) asymmetric information, (2) competition in schedules, and (3) incomplete markets. When information is symmetric, the First Welfare Theorem applies. Similarly, when the traders' positions do not condition on the price, there is nothing the planner can do to improve upon the traders' ability to tell apart variations in their expectations of θ driven by the fundamental value θ of the asset from those driven by noise; ex-ante welfare is below the complete-information level, but there is no inefficiency in the equilibrium usage of information (see the online appendix Pavan et al. [2026b] for a formal proof of this result, as well as the discussion in Subsection 3.2.4). Finally, when markets are complete, traders can fully insure against ex-post idiosyncratic variations in their consumption due to interim idiosyncratic variations in their perceptions of the fundamental value of the asset at the trading stage; again, the welfare theorems then guarantee efficiency of the equilibrium trades.[17]

That total welfare is higher under the efficient schedules than the equilibrium ones does not mean that moving from the latter to the former yields a Pareto improvement: either the traders or the representative investor (but not both) can be worse off under the efficient schedules. This reflects the fact that the efficient allocations maximize the sum of all agents' payoffs, with each agent receiving equal weight by the planner.

3.2.3 Fictitious environment

To shed more light on the two externalities introduced above, consider a fictitious environment in which the traders are naive in that they do not recognize the information contained in the market-clearing price. Such a benchmark is similar in spirit to the (fully) cursed equilibrium of Eyster and Rabin [2005]. See also Bastianello and Fontanier [2025] for recent work discussing mis-learning from prices and its interaction with mis-learning from fundamentals. The reason for considering such an environment is that it permits us to isolate the pecuniary externality by shutting down the more familiar learning externality. To facilitate the comparison to the true economy, assume that, in this fictitious environment, each trader, in addition to observing the private signal

$$s_i = \theta + \underbrace{f(y)(\eta + e_i)}_{\equiv \epsilon_i}$$

[17] See, for example, Angeletos et al. [2020] for a model in which the completeness of the market occurs via consumption sharing – all traders are members of the same family and share consumption equally ex-post.

as in the true economy, also observes an exogenous public signal

$$z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$$

whose structure is the same as the one contained in the market-clearing price, but with the endogenous noise $-u/\beta a$ replaced by the exogenous one χ, with the latter drawn from a Normal distribution with mean zero and variance τ_χ^{-1} independently of all other variables (this shock is the same for all traders). Let $\tau_\zeta \equiv y\tau_\eta\tau_\chi / (\tau_\chi + y\tau_\eta)$ denote the precision of the total noise $\zeta \equiv f(y)\eta + \chi$ in the exogenous signal z. As we show in the Appendix, in the cursed equilibrium of this fictitious economy, traders submit affine schedules $x_i = a^*_{exo}s_i + \hat{b}^*_{exo} - \hat{c}^*_{exo}p + \hat{d}^*_{exo}z$, where the sensitivity of the traders' demands to their private information is given by

$$a^*_{exo} = \frac{1}{\lambda\Lambda(\tau_\zeta)}, \tag{14}$$

with the function Λ as defined in (6). Note that the formula in (14) is similar to the one in (9) in the true economy, except for the fact that the precision $\tau_\omega(a)$ of the endogenous public signal contained in the market-clearing price is replaced by the precision τ_ζ of the exogenous public signal about θ.

Now suppose that, in this fictitious economy, the planner can control the sensitivity a of the traders' demands to their private information. However, given a, the planner must choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a and $(\hat{b}, \hat{c}, \hat{d})$ as between a^*_{exo} and $(\hat{b}^*_{exo}, \hat{c}^*_{exo}, \hat{d}^*_{exo})$ in the cursed equilibrium.[18] The level of a that maximizes ex-ante welfare is then equal to

$$a^T_{exo} = \frac{1}{\lambda} \frac{1}{\Lambda(\tau_\zeta) + \frac{\tau_\eta\beta(\tau_\zeta + \tau_\theta)}{\lambda\tau_e(y\tau_\eta - \tau_\zeta)}}. \tag{15}$$

Again, the formula for a^T_{exo} is similar to the one for a^T defining the efficient sensitivity to private information in the true economy, except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ and the term $\Delta(a)$ in the denominator of the expression giving the socially-optimal level of a in the true economy is equal to zero, reflecting the fact that the planner recognizes that the

[18]In the true economy, maintaining the same relationship between a and $(\hat{b}, \hat{c})$ is without loss of optimality for the planner (see Lemma 1 above). This need not be the case in the fictitious economy. However, imposing the restriction permits us to isolate the relevant effects.

agents do not learn from the price. Note that

$$\frac{\tau_\eta \beta \left(\tau_\zeta + \tau_\theta\right)}{\lambda \tau_e \left(y\tau_\eta - \tau_\zeta\right)}$$

has exactly the same form as the pecuniary externality $\Xi(a)$ in the true economy (except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ). Hence, in this fictitious economy, the cursed-equilibrium schedules unambiguously feature an excessively high sensitivity to private information: $a^*_{exo} > a^T_{exo}$. Furthermore, when the precision of the exogenous public signal in the cursed economy is the same as the precision of the endogenous public signal under the efficient schedules of the true economy (that is, when $\tau_\zeta = \tau_\omega(a^T)$), a^T_{exo} coincides with the solution to the equation $\partial WL(a^T_{exo}, \tau_\omega(a^T))/\partial a = 0$ and $a^T_{exo} < a^T$:[19] in the true economy, the planner recognizes the value of increasing the precision of the endogenous signal contained in the market-clearing price and thus demands that traders respond more to their private information.

3.2.4 Sign of externalities and slope of the schedules

We now return to the economy in which both the traders and the planner account for the information contained in the market-clearing price. Whether the sensitivity of the equilibrium schedules to the traders' private information is excessively high or low (compared to the efficient level a^T) then depends on which of the two externalities prevails.

Proposition 3 (externalities and slope of schedules). *Suppose that $y_i = y$ for all i, with y exogenous. The following equalities hold:*

$$a^* - a^T \overset{sgn}{=} \Xi(a^T) + \Delta(a^T) \overset{sgn}{=} \Xi(a^*) + \Delta(a^*) \overset{sgn}{=} \hat{c}^* \overset{sgn}{=} \hat{c}^T.$$

Hence, whether the sensitivity of the equilibrium schedules to private information is excessively high or low, compared to what is efficient, depends on which of the two externalities prevails. When the two externalities cancel each other out, the schedules are price-inelastic (i.e., $\hat{c}^T = \hat{c}^* = 0$) and $a^* = a^T$. When, instead, the pecuniary externality dominates, $\hat{c}^T, \hat{c}^* > 0$ (both the efficient and the equilibrium schedules are downward sloping) and the equilibrium schedules feature an excessive response to the traders' private information, i.e., $a^* > a^T$. Finally, when the learning externality dominates, $\hat{c}^T, \hat{c}^* < 0$ (both the efficient and the equilibrium schedules are upward sloping) and the equilibrium response to private information is insufficiently low, i.e., $a^* < a^T$.

[19] The notation $\partial WL(a^T_{exo}, \tau_\omega(a^T))/\partial a$ stands for the partial derivative of the function $WL(\cdot, \tau_\omega(a^T))$ with respect to a, evaluated at $a = a^T_{exo}$.

It is worth noting that if the traders were restricted to submitting market orders (like in a Cournot model), then the usage of information would be efficient since the two externalities would not be present (See the online appendix Pavan et al. [2026b] for a formal proof of this result).

We conclude this subsection by highlighting the role that the common noise η in the traders' private information plays for the sign and magnitude of the two externalities identified above. Unsurprisingly, both a^* and a^T are increasing in the precision τ_η of the noise η, reflecting the fact that responding to private information is more valuable (both for the traders and for the planner) when it is affected less by correlated noise and hence more precise. Similarly, holding a fixed, we have that the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price naturally increases with τ_η, reflecting the fact that the noise in the traders' signals becomes less correlated when τ_η increases and, as a result, washes out more at the aggregate level, making the price more informative, for given schedules. Furthermore, fixing a^T, the absolute value of both $\Xi(a^T)$ and $\Delta(a^T)$ increases with τ_η, reflecting the larger role that either externality plays when the noise in the private signals is less correlated. However, whereas the pecuniary externality $\Xi(a^T)$ increases with τ_η, the learning externality $\Delta(a^T)$ decreases with it. Combining all of the above effects, we then have that the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ can be non-monotonic in τ_η.

3.3 Policies inducing efficient trading with exogenous information

Next, we discuss policies that correct the inefficiencies in trading, once again holding fixed the quality of the traders' private information y for the time being.

Proposition 4 (policy inducing efficient trading with exogenous information). *Suppose that $y_i = y$ for all i, with y exogenous. There exist policies that induce the traders to submit the efficient schedules. Such policies are non-linear in the traders' expenditures on the asset. Formally, there exist $\delta, t_p, t_0 \in \mathbb{R}$ such that efficient trading can be implemented with a policy that charges the traders a total tax bill equal to $T(x_i, p) = \frac{\delta}{2}x_i^2 - t_0 x_i + t_p p x_i$ where t_0, t_p and δ are functions of all exogenous parameters. The optimal tax/subsidy scheme is progressive and dampens the traders' response to private information when the pecuniary externality dominates over the learning externality, whereas it is regressive and boosts the traders' response to private information otherwise: $[\delta^*] \overset{sgn}{=} [\Xi(a^T) + \Delta(a^T)]$.*

The efficient use of information can thus be induced through a combination of a linear-quadratic tax $\frac{\delta}{2}x_i^2 - t_0 x_i$ on the individual volume of trade (equivalently on the quantity of the asset traded), along with an ad valorem tax $t_p p x_i$. The role of δ is to manipulate the traders' adjustment cost (from λ to $\lambda + \delta$). This manipulation suffices to induce the traders to submit schedules whose sensitivity to their exogenous private information is equal to the efficient level a^T. The sign of δ depends on whether the pecuniary or the learning externality prevails and determines whether the optimal policy is progressive or regressive (i.e., whether the marginal tax/subsidy increases or decreases with x_i).

The role of the linear ad valorem tax is to guarantee that, once the sensitivity a coincides with the efficient level a^T, the sensitivity $\hat{c}$ of the equilibrium schedules to the price coincides with the efficient level $\hat{c}^T$. In the absence of such a correction, the traders fail to submit the efficient schedules, even if they respond efficiently to their private information. Finally, the role of the linear tax $t_0 x_i$ on the individual volume of trade is to guarantee that the fixed part $\hat{b}$ of the schedules (equivalently, the unconditional volume of trade) also coincides with its efficient counterpart $\hat{b}^T$. As one can anticipate from the discussion above, when the two externalities cancel each other out, the use of information is efficient under the laissez-faire equilibrium, in which case there is no need to tax the trades: $\delta = t_p = t_0 = 0$.

4 Inefficiency in information acquisition and policy corrections

We now endogenize the traders' private information. We start by establishing existence of an equilibrium in the full game of the laissez-faire economy (and its uniqueness in the family of equilibria in affine strategies). We then turn to the relation between the inefficiency in information acquisition and those in trading identified in the previous section and investigate policy interventions correcting each of the two inefficiencies.

Proposition 5 (equilibrium in full game). *There exist constants $L, M \in \mathbb{R}_{++}$ (defined in the Appendix) such that, when $\mathcal{C}'(0) \leq L$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > M$, in the laissez-faire game with endogenous information acquisition, there exists one, and only one, symmetric equilibrium in affine trading strategies.*

The conditions guaranteeing existence say that (a) the marginal cost of information is small for small y, and (b) the cost is sufficiently convex. The first condition guarantees existence and uniqueness of a y^* such that, when all other traders acquire information of quality y^* and submit the equilibrium schedules for information of quality y^*, each agent's net private marginal benefit of increasing the quality of his information at $y_i = y^*$ is zero. The second condition guarantees that, fixing the other traders' strategies, each trader's payoff is strictly quasi-concave in y_i, accounting for the optimal usage of the trader's information. When $\mathcal{C}(y) = By^2/2$, the conditions in the proposition jointly hold for B sufficiently large.

We now address the question of whether efficiency in information acquisition can be induced through an appropriate policy design. In the previous section, we showed that, when private information is exogenous, efficiency in trading can be induced through a combination of a linear-quadratic tax on the individual volume of trade paired with an ad valorem tax (both rebated in a lump-sum manner, if desired). Based on other results in the literature, one may conjecture that the same policy mix also induces efficiency in information acquisition. The next result shows that this is not the case. If the planner were to use the tax/subsidy scheme $T(p, x_i)$ of Proposition 4 (applied to $y = y^T$), the traders would acquire information of quality different from y^T and then submit schedules different from the efficient ones. More generally, the proposition shows that there exists no policy measurable in the individual volume of trade, x_i, and the price of the asset, p, that induces efficiency in both trading and information acquisition.

Proposition 6 (impossibility to induce efficiency in both information acquisition and trading with standard contingencies). *Generically (i.e., with the exception of a set of parameters of zero Lebesgue measure), there exists no policy $T(x_i, p)$ that induces efficiency in both information acquisition and trading.*

The proof in the Appendix shows that any policy $T(x_i, p)$ that induces the traders to submit the efficient schedules (for quality of information y^T) coincides with the one in Proposition 4 (applied to $y = y^T$), except for terms that play no role for incentives. Any such a policy induces the traders to misperceive the value of private information and hence fails to induce them to collect information efficiently.

Importantly, the above conclusions hinge on the traders being exposed to correlated noise in their information sources, that is, on $\tau_\eta \in (0, +\infty)$. When $\tau_\eta = 0$, the noise in the agents' private signals is infinite, making the signals worthless both for the individual traders and for the planner. When, instead, $\tau_\eta \to +\infty$, the correlated noise in the agents' private signals disappears, in which case, fixing the agents' schedules, we have that the aggregate volume of trade is invariant to the quality of private information. This is the case considered in the previous literature. The only effect of an increase in the quality of private information on welfare is through the reduction in the dispersion of individual trades around the aggregate level of trade. Because this effect is weighted equally by the planner and by each individual trader, the private and the social value of information coincide, which guarantees efficiency in information acquisition under efficient trading. Hence, in the absence of correlated noise in the agents' information, efficiency in both information acquisition and trading can be induced through the policies of Proposition 4 (applied to $y = y^T$).

We conclude with two possibility results.

Proposition 7 (**policy inducing efficiency in both information acquisition and trading when acquisition is verifiable**). *Under conditions similar to those in Proposition 5,*[20] *efficiency in both information acquisition and trading can be induced through a non-linear policy*

$$T^{tot}(x_i, p, y_i) = \frac{\delta}{2}x_i^2 - t_0 x_i + t_p p x_i - A y_i$$

where (δ, t_p, t_0) *are as in Proposition 4 for* $y = y^T$. *Information acquisition is taxed (i.e.,* $A < 0$*) when* $\hat{c}^T > 0$ *and subsidized when* $\hat{c}^T < 0$.

That information acquisition is taxed when the efficient schedules are downward sloping and subsidized when they are upward sloping reflects the fact that, under the policy of Proposition 4, agents over-invest in information acquisition in the former case whereas they under-invest in the latter one. Combining the two propositions, we thus have that the optimal policy is progressive and information acquisition is taxed when the pecuniary externality prevails over the learning externality (i.e., when $\Xi(a^T) + \Delta(a^T) > 0$), whereas it is regressive and information acquisition is subsidized when the learning externality prevails (i.e., when $\Xi(a^T) + \Delta(a^T) < 0$).

Proposition 8 (**policy inducing efficiency in both information acquisition and trading when acquisition is non-verifiable**). *Suppose that the acquisition of information is not verifiable. Under conditions similar to those in Proposition 5,*[21] *there exist constants* $\delta^*, t^*_{\tilde{x}}, t^*_0, t^*_p \in \mathbb{R}$ *such that efficiency in both information acquisition and trading can be induced through a policy*

$$T^*(x_i, \tilde{x}, p) = \frac{\delta^*}{2}x_i^2 + (t^*_{\tilde{x}}\tilde{x} - t^*_0)\, x_i + t^*_p p x_i$$

that conditions the marginal tax/subsidy rate $\partial T^*(x_i, \tilde{x}, p)/\partial x_i$ *on the aggregate volume of trade* $\tilde{x}$.

When information acquisition is not verifiable, conditioning the marginal tax/subsidy rate on the aggregate volume of trade $\tilde{x}$ is essential to induce efficiency in both information acquisition and trading. As we show in the Appendix, the policy of Proposition 8 permits the

[20] Formally, there exist constants $\tilde{L}, \tilde{M} \in \mathbb{R}_{++}$ such that the result in the proposition holds when $\mathcal{C}'(0) \leq \tilde{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \tilde{M}$. As in Proposition 5, these conditions guarantee global quasi-concavity of the payoffs under the proposed policy.

[21] Formally, there exist constants $\hat{L}, \hat{M} \in \mathbb{R}_{++}$ such that the statement in the proposition holds when $\mathcal{C}'(0) \leq \hat{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \hat{M}$.

planner to equalize the expected marginal tax rate

$$\mathbb{E}\left[\frac{\partial}{\partial x_i} T^*(x_i, \tilde{x}, p) | x_i, p; y_i, y^T\right]\Big|_{y_i = y^T;\, x_i = a^T s_i + \hat{b}^T - \hat{c}^T p}$$

of each trader acquiring information efficiently and submitting the efficient schedule with the discrepancy

$$\mathbb{E}\left[\theta - p - \lambda x_i | x_i, p; y_i, y^T\right]\Big|_{y_i = y^T;\, x_i = a^T s_i + \hat{b}^T - \hat{c}^T p}$$

between the private marginal benefit and the private marginal cost of expanding the individual volume of trade around the efficient level. Eliminating such a discrepancy is essential to inducing efficiency in trading. Importantly, the new contingency provides the planner with flexibility in how to eliminate such a discrepancy. When, instead, the policy depends only on x_i and p, there exists a unique way of eliminating such a discrepancy, as shown in the proof of Proposition 6. The extra flexibility in turn can be used to realign the marginal private value of more precise private information to its social counterpart, something that is not possible when the policy depends only on x_i and p. This can be done by exploiting the fact that, when the marginal tax rate depends on the aggregate volume of trade, because the latter is not known to the individual trader, the value of information also accounts for the benefit of forecasting the marginal tax rate. In the proof in the Appendix, we show how to exploit this extra benefit to realign incentives. We also show that, when information acquisition is not verifiable, the policy that implements efficiency in both information acquisition and trading is in fact unique, up to terms that do not matter for incentives.

The schemes of the last two propositions require a non-linear dependence of the total bill on the individual quantity traded as well as a dependence of marginal tax/subsidy rates on aggregate trade volume (when information acquisition is not verifiable). Such schemes, while conceptually straight-forward, are sometimes perceived as difficult to implement in practice. The question of interest is then whether a planner who is restricted to simple ad valorem taxes such as those often discussed in the policy debate can still improve upon the laissez-faire equilibrium by choosing t_p optimally.

Proposition 9 (sub-optimality of ad valorem taxes). *Suppose that the planner is restricted to use ad valorem taxes of the form $T(x_i, p) = t_p p x_i$, for some $t_p \in \mathbb{R}$. Then the optimal t_p is zero.*

The reason why simple ad valorem taxes are suboptimal is that they fail to change the sensitivity of the equilibrium schedules to the private signals. They also do not affect the marginal value that each trader assigns to acquiring private information. These taxes only affect $\hat{c}$ and $\hat{b}$, that is, the sensitivity of the equilibrium schedules to the price and the unconditional volume of trade. However, in the laissez-faire equilibrium, given y and a, $\hat{c}$ and $\hat{b}$ are welfare-maximizing. (see Lemma 1). As a result, these taxes bring the equilibrium allocation further away from the efficient one, and hence reduce welfare.

In a similar vein, it is often suggested that governments can improve over the laissez-faire equilibrium by manipulating prices through asset purchases. One can show that, in our economy, such policies are also welfare detrimental. The reason is essentially the same as for ad valorem taxes.[22]

5 Conclusions

We show that, in markets in which traders compete in schedules, the acquisition, usage, and aggregation of private information are inefficient due to learning and pecuniary externalities. The former induce the agents to collect too little private information and then submit schedules that are not sufficiently responsive to it. The latter contribute to over-investment in information acquisition and to schedules that over-respond to private information.

These results call for policy interventions. We show that the optimal scheme is progressive when the pecuniary externalities prevail over the learning ones, regressive otherwise. To incentivize the traders to acquire information in society's best interest, it is however essential to also subsidize/tax the acquisition of information when the latter is verifiable, and to condition marginal tax/subsidy rates on the aggregate volume of trade when it is not.

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[22] See Dávila and Walther [2025] for a study of corrective taxation in environments where the Government's interventions are limited because some agents cannot be taxed, or certain activities cannot be regulated.

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6 Appendix

Proof of Proposition 1. As explained in the main text, when the traders submit affine schedules with parameters $(a, \hat{b}, \hat{c})$, the market-clearing price can be expressed as follows

$$p = \frac{\alpha + \beta\hat{b}}{1 + \beta\hat{c}} + \frac{\beta a}{1 + \beta\hat{c}} z,$$

where $z \equiv \theta + \omega$ is the endogenous public signal contained in p, with noise $\omega \equiv f(y)\eta - u/(\beta a)$ of precision $\tau_\omega(a) \equiv (\beta^2 a^2 \tau_u \tau_\eta y) \,/\, (\beta^2 a^2 \tau_u + \tau_\eta y)$.[23] In turn, this implies that the equilibrium trades $x_i = as_i + \hat{b} - \hat{c}p$ can be expressed as affine functions $x_i = as_i + b + cz$ of the traders' exogenous private information s_i and the endogenous public information z contained in the market-clearing price, with

$$b = \hat{b} - \hat{c}\frac{\alpha + \beta\hat{b}}{1 + \beta\hat{c}} \tag{16}$$

[23]To derive $\tau_\omega(a)$ we use the fact that $f(y) = 1/\sqrt{y}$.

and

$$c = -\frac{\beta a \hat{c}}{1 + \beta \hat{c}}. \tag{17}$$

For each vector $(a, \hat{b}, \hat{c})$ describing the schedules, there exists a unique vector (a, b, c) describing the equilibrium trades and vice versa. Hereafter, we find it more convenient to characterize the equilibrium use of information in terms of the vector (a, b, c) describing the equilibrium trades. Replacing $\tilde{x} = \int x_i di = (a+c)z + u/\beta + b$ into the expression for the inverse aggregate supply function $p = \alpha - u + \beta \tilde{x}$, we then have that the equilibrium price can be expressed as follows:

$$p = \alpha + \beta b + \beta(a + c)z. \tag{18}$$

Using standard projection formulas, we then have that $\mathbb{E}[\theta|I_i, p] = \mathbb{E}[\theta|s_i, z] = \gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z$, where, for any τ_ω,

$$\gamma_1(\tau_\omega) \equiv \frac{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\omega\right)}{y^2 \tau_\eta^2 \left(\tau_\omega + \tau_\epsilon + \tau_\theta\right) - \tau_\omega \tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right)} \tag{19}$$

and

$$\gamma_2(\tau_\omega) \quad \equiv \quad \left(1 - \gamma_1(\tau_\omega)\tfrac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right)\tfrac{\tau_\omega}{\tau_\omega + \tau_\theta}. \tag{20}$$

Optimality requires that the equilibrium trades satisfy $x_i = \left(\mathbb{E}[\theta|s_i, z] - p\right)/\lambda$, which, together with the results above, is equivalent to

$$x_i \quad = \quad \frac{1}{\lambda}\left[\gamma_1(\tau_\omega(a))s_i - (\alpha + \beta b) + \left(\gamma_2(\tau_\omega(a)) - \beta(a + c)\right)z\right].$$

The sensitivity a^* of the equilibrium schedules to the traders' private information must thus satisfy $a = \gamma_1(\tau_\omega(a))/\lambda$, which is equivalent to equation (9) in the main text.
The sensitivity of the equilibrium trades to the endogenous public signal must satisfy

$$c = \frac{1}{\beta + \lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right]. \tag{21}$$

The constant b in the equilibrium trades must satisfy

$$b = -\frac{\alpha}{\beta + \lambda}. \tag{22}$$

Inverting the relationship between b and $\hat{b}$ and c and $\hat{c}$ using (16) and (17), and using the formula for $\tau_\omega(a)$, we conclude that, given a^*, the values of $\hat{c}^*$ and $\hat{b}^*$ are given by the functions (7) and (8), as claimed in the proposition.

To complete the proof, it thus suffices to show that equation (9) admits a unique solution and that such a solution satisfies $0 < a^* < 1/\lambda$. To see this, use the fact that $\tau_\epsilon(y) = y\tau_e\tau_\eta/(\tau_e+\tau_\eta)$, along with the formula for $\tau_\omega(a)$, to observe that this equation is equivalent to

$$\lambda\beta^2\tau_u\left(y\tau_\eta + \tau_\theta\right)a^3 + \lambda y\left[y\tau_e\tau_\eta + \tau_\theta(\tau_e + \tau_\eta)\right]a - \tau_e\tau_\eta y^2 = 0. \tag{23}$$

Clearly, because the left-hand side is strictly increasing in a, the above cubic equation has a unique real root, which is strictly positive. Furthermore, when $a = 1/\lambda$, the left-hand side is equal to

$$\frac{\beta^2\tau_u}{\lambda^2}\left(y\tau_\eta + \tau_\theta\right) + \tau_\theta(\tau_e + \tau_\eta)y > 0.$$

We conclude that $a^* \in (0, 1/\lambda)$. Q.E.D.

Proof of Lemma 1. As explained in the proof of Proposition 1, when the traders submit schedules of the form $x_i = as_i + \hat{b} - \hat{c}p$, for some $(a, \hat{b}, \hat{c})$, the trades induced by market clearing can be expressed as $x_i = as_i + b + cz$, with the values of b and c given by (16) and (17). Using the fact that the aggregate volume of trade is $\tilde{x} = a(\theta + f(y)\eta) + b + cz$, we thus have that ex-ante welfare is equal to

$$\begin{aligned}\mathbb{E}[W] \quad &= \quad \mathbb{E}\left[(\theta - \alpha + u)\left(a(\theta + f(y)\eta) + b + cz\right) - \tfrac{\beta}{2}\left(a(\theta + f(y)\eta) + b + cz\right)^2\right] \\ &\quad -\mathbb{E}\left[\tfrac{\lambda}{2}\int_0^1 \left(as_i + b + cz\right)^2 di\right].\end{aligned}$$

Note that, given a, $\mathbb{E}[W]$ is concave in b and c . For any a, the optimal values of b and c are thus given by the FOCs $\partial\mathbb{E}[W]/\partial b = 0$ and $\partial\mathbb{E}[W]/\partial c = 0$. The values given by (21) and (22) solve these equations. Using (16) and (17) to go from the optimal trades to the schedules that implement them, we thus conclude that, for any choice of a, the optimal values of $\hat{c}^T$ and $\hat{b}^T$ are given by the functions (7) and (8), as claimed. Q.E.D.

Proof of Proposition 2. Using Lemma 1, one can show that the welfare losses can be expressed as a function

$$\begin{aligned}WL(a,\tau_\omega(a)) &= \frac{\left[\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\omega(a)}+\frac{\lambda^2a^2+2\lambda a\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{2(\beta+\lambda)y\tau_\eta}\\ &+\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\theta}+\frac{\lambda a^2}{2y\tau_e} \qquad (24)\end{aligned}$$

of a and $\tau_\omega(a)$. The socially optimal level of a must solve $dWL(a,\tau_\omega(a))/da=0$ which, using the formula for $\tau_\omega(a)$, yields the condition in the proposition. One can also verify that, at $a=1/\lambda$, $dWL(a,\tau_\omega(a))/da>0$, whereas, at $a=0$, $dWL(a,\tau_\omega(a))/da<0$. Hence a^T satisfies $0<a^T<1/\lambda$, as claimed. Q.E.D.

Derivation of Conditions (14) and (15). In the cursed economy, each trader receives a private signal $s_i=\theta+\underbrace{f(y)\eta+f(y)e_i}_{\equiv\epsilon_i}$ and a public signal $z=\theta+\underbrace{f(y)\eta+\chi}_{\equiv\zeta}$, and believes p to be orthogonal to (θ,η). Following steps similar to those leading to Proposition 1, we have that $\mathbb{E}[\theta|s_i,z]=\bar{\gamma}_1 s_i+\bar{\gamma}_2 z$, where

$$\bar{\gamma}_1\equiv\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\zeta)}{y^2\tau_\eta^2(\tau_\zeta+\tau_\epsilon+\tau_\theta)-\tau_\zeta\tau_\epsilon(\tau_\theta+2y\tau_\eta)} \text{ and } \bar{\gamma}_2\equiv\left(1-\bar{\gamma}_1\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}.$$

Because the cursed-equilibrium schedules $x_i=a^*_{exo}s_i+\hat{b}^*_{exo}-\hat{c}^*_{exo}p+\hat{d}^*_{exo}z$ must satisfy $x_i=(\mathbb{E}[\theta|s_i,z]-p)/\lambda$, we have that $a^*_{exo}=\bar{\gamma}_1/\lambda$, $\hat{b}^*_{exo}=0$, $\hat{c}^*_{exo}=1/\lambda$, and $\hat{d}^*_{exo}=\bar{\gamma}_2/\lambda$. Using the formula for $\bar{\gamma}_1$ above, we have that the formula for a^*_{exo} is equivalent to the one in (14) in the main text.

Now suppose that, given a, the planner is constrained to choose $(\hat{b},\hat{c},\hat{d})$ to maintain the same relationship between a and $(\hat{b},\hat{c},\hat{d})$ as between a^*_{exo} and $(\hat{b}^*_{exo},\hat{c}^*_{exo},\hat{d}^*_{exo})$ in the cursed equilibrium. Following steps similar to those in the proof of Proposition 2, we then have that the value of a that minimizes the welfare losses must satisfy Condition (15) in the main text. Q.E.D.

Proof of Proposition 3. Observe that the function F given, for all a, by

$$\mathrm{F}(a)=a-\frac{1}{\lambda\Lambda(\tau_\omega(a))}$$

is strictly increasing. Next, let F^T be the function given, for any a, by

$$\mathrm{F}^T(a)=a-\frac{1}{\lambda}\frac{1}{\Lambda(\tau_\omega(a))+\Delta(a)+\Xi(a)}.$$

Because Δ and Ξ are both increasing, F^T is also strictly increasing.

The first two equalities follow from the above monotonicities along with the fact that a^* solves $\mathrm{F}(a^*) = 0$ whereas a^T solves $\mathrm{F}^T(a^T) = 0$.

Next, consider the last two equalities. In the proof of Lemma 1, we established that, for any sensitivity a of the trades to private information, the sensitivity of the efficient trades to the endogenous signal z contained in the market-clearing price is given by

$$c = \frac{1}{\beta + \lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right]$$

and coincides with the sensitivity of the equilibrium trades to z when the sensitivity of the trades to private information is a. Using the formula for $\tau_\omega(a)$, we then have that $a + c > 0$. Now use Condition (17) to observe that

$$\hat{c} = -\frac{c}{\beta(a+c)}. \tag{25}$$

Because $a + c > 0$, we conclude that $\hat{c} \overset{sgn}{=} -c$. Combining this property with Condition (21), we conclude that

$$\hat{c} \overset{sgn}{=} \beta a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}.$$

Next observe that

$$\Delta(a) + \Xi(a) = \frac{\beta\tau_\eta}{\lambda\left(y\tau_\eta - \tau_\omega(a)\right)}\left(\frac{\tau_\omega(a) + \tau_\theta}{\tau_e} - \frac{\beta y^3 \tau_\eta^2 \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda\left(\beta^2 a^2 \tau_u + y\tau_\eta\right)^2\left(\tau_\omega(a) + \tau_\theta\right)}\right). \tag{26}$$

Because $y\tau_\eta - \tau_\omega(a) > 0$,

$$\Delta(a) + \Xi(a) \overset{sgn}{=} \lambda\left(\beta^2 a^2 \tau_u + y\tau_\eta\right)^2\left(\tau_\omega(a) + \tau_\theta\right)^2 - \tau_e \beta y^3 \tau_\eta^2 \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2.$$

It is then easy to see that $\Delta(a) + \Xi(a) \overset{sgn}{=} \hat{c}$. The above derivations hold no matter whether a is the sensitivity of the equilibrium schedules (equivalently, trades) to private information, or the sensitivity of the efficient schedules (equivalently, trades) to private information. Hence, $\hat{c}^* \overset{sgn}{=} \Xi(a^*) + \Delta(a^*)$ and $\hat{c}^T \overset{sgn}{=} \Xi(a^T) + \Delta(a^T)$. Because $\Xi(a^*) + \Delta(a^*) \overset{sgn}{=} \Xi(a^T) + \Delta(a^T)$, the above results also imply that $\hat{c}^* \overset{sgn}{=} \hat{c}^T$. Q.E.D.

Proof of Proposition 4. Following the same steps as in the proof of Proposition 1, we have that the equilibrium value of a under the proposed policy is the unique solution to $a = 1/[(\lambda+\delta)\Lambda(\tau_\omega(a))]$, whereas the values of b and c describing the equilibrium trades $x_i = as_i + b + cz$ are given by

$$b = \frac{t_0 - (1+t_p)\alpha}{\lambda+\delta+(1+t_p)\beta} \qquad \text{and} \qquad c = \frac{\gamma_2(\tau_\omega(a)) - (1+t_p)\beta a}{\lambda+\delta+(1+t_p)\beta},$$

where γ_2 is the function defined in the proof of Proposition 1. Hence, the equilibrium trades under the proposed policy coincide with the efficient trades $x_i = a^T s_i + b^T + c^T z$ if and only if

$$\delta = \frac{\lambda\left[\Xi(a^T) + \Delta(a^T)\right]}{\Lambda(\tau_\omega(a^T))},$$

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda}\left[\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] - \beta a^T}{\beta\left\{\frac{1}{\beta+\lambda}\left[\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] + a^T\right\}},$$

and

$$t_0 = (1+t_p)\alpha - \frac{\alpha\left[\lambda+\delta+(1+t_p)\beta\right]}{\beta+\lambda} = \alpha\frac{\lambda t_p - \delta}{\beta+\lambda}.$$

Q.E.D.

Proof of Proposition 5. The proof is in four steps. Step 1 shows that, for any $y \in [0, +\infty)$, when all other agents acquire information of quality y and submit the equilibrium schedules for information of quality y, each agent's net private marginal benefit $N(y)$ of increasing the quality of his information at $y_i = y$ (and then trade optimally) is a strictly decreasing function of y. Step 2 uses the result in step 1 to show that, when $\mathcal{C}'(0)$ is small, there is one, and only one, value of y for which $N(y) = 0$. Step 3 shows that, when $\mathcal{C}$ is sufficiently convex, if all other agents acquire information of quality y^* (where y^* is the unique solution to $N(y) = 0$) and then submit the equilibrium schedules for information of quality y^*, the payoff $V^{\#}(y^*, y_i)$ that each agent obtains by acquiring information of quality y_i and then trading optimally is strictly quasi-concave in y_i. Jointly, the above properties establish the claim in the proposition.

Step 1. First observe that, when all other agents acquire information of quality y and then submit the equilibrium schedules for information of quality y, the maximal payoff that agent i can obtain by acquiring information of quality y_i and then trading optimally is given by

$$V^{\#}(y, y_i) \equiv \sup_{g(\cdot)} \left\{ \mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))] - \mathcal{C}(y_i) \right\}$$

with

$$\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))] \equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha + \beta b + \beta(a+c)z)\, g(s_i, z) - \frac{\lambda}{2}\left(g(s_i, z)\right)^2; y_i\right],$$

where g is an arbitrary (measurable) function of the agent's private signal s_i and the public signal $z \equiv \theta + f(y)\eta - u/(\beta a)$ contained in the equilibrium price, with noise $\omega \equiv f(y)\eta - u/\beta a$ of precision $\tau_\omega(a) \equiv \beta^2(a)^2 y\tau_u\tau_\eta / \left(\beta^2(a)^2\tau_u + y\tau_\eta\right)$. The function g describes the amount of the good traded by agent i under the schedule he submits. Note that, in writing $\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))]$, we used the fact that the relationship between z and the equilibrium price is given by $p = \alpha + \beta b + \beta(a+c)z$, where (a, b, c) are the coefficients describing the equilibrium trades when the quality of information is y and all agents submit the equilibrium schedules for information of quality y – these coefficients are given by Conditions (9), (22), and (21) above. Also note that the dependence of $\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))]$ on y_i is through the fact that the agent's private signal is given by $s_i = \theta + f(y_i)(\eta + e_i)$. Using the envelope theorem, we then have that[24]

$$N(y) \equiv \left.\frac{\partial V^{\#}(y, y_i)}{\partial y_i}\right|_{y_i = y} = \frac{(\beta + \lambda)(a + c)\, a}{2\tau_\eta y^2} + \frac{\lambda a^2}{2y^2\tau_e} - \mathcal{C}'(y). \tag{27}$$

Next, use Conditions (5) and (21) to verify that $N(y) = F(a, y) - \mathcal{C}'(y)$, where

$$F(a, y) \equiv \frac{1}{2}a^2 \frac{a^2\beta^2\lambda\tau_u\tau_\theta + y\left[\lambda a^2\beta^2\tau_u\tau_\eta + \lambda(\tau_e + \tau_\eta)\tau_\theta + \beta^2\tau_e\tau_u a\right]}{y^2\tau_e\left[y\tau_\theta\tau_\eta + a^2\beta^2\tau_u\left(\tau_\theta + y\tau_\eta\right)\right]}. \tag{28}$$

As shown in the proof of Proposition 1, the equilibrium value of a (given y) is given by the unique real root to the cubic equation in Equation (23). Equivalently, let $Z \equiv a/y$ and, for any (Z, y), let

$$R(Z, y) \equiv Z^3 y\beta^2\lambda\tau_u\left(\tau_\theta + y\tau_\eta\right) + Z\lambda\left(\tau_e\tau_\theta + \tau_\theta\tau_\eta + y\tau_e\tau_\eta\right) - \tau_e\tau_\eta.$$

For any y, the equilibrium level of Z is given by the unique positive real solution to the equation $R(Z, y) = 0$, and is such that $Z < \tau_e/\lambda\tau_y$. Furthermore,

$$\frac{\partial}{\partial y}R(Z, y) = Z\lambda\left(\tau_e\tau_\eta + Z^2\beta^2\tau_u\tau_\theta + 2yZ^2\beta^2\tau_u\tau_\eta\right) > 0.$$

[24] For the steps leading to the formula in (27), see the proof of Proposition 9 below, where we establish the result for an economy in which transactions are subject to an ad valorem tax with rate t_p – the formula for the laissez-faire economy in (27) corresponds to the case in which $t_p = 0$.

Now let $Z^*(y)$ be the equilibrium value of Z, given y. From the Implicit Function Theorem, we thus have that $Z^*(y)$ is decreasing in y.

Next, let $G(y) \equiv F(Z^*(y)y, y)$, where $F(a, y)$ is the function defined in Condition (28) above, and where we used the fact $a = Z^*(y)y$. After some algebra, one can show that

$$G(y) = \frac{1}{2} Z^*(y) \frac{\tau_e + yZ^*(y)\lambda\tau_\eta}{\tau_e(\tau_\theta + y\tau_\eta)}.$$

Note that

$$\frac{dG(y)}{dy} = \frac{1}{2} Z^*(y)\tau_\eta \frac{-\tau_e + Z^*(y)\lambda\tau_\theta}{\tau_e(\tau_\theta + y\tau_\eta)^2} + \frac{1}{2}\frac{\tau_e + 2yZ^*(y)\lambda\tau_\eta}{\tau_e(\tau_\theta + y\tau_\eta)} \frac{dZ^*(y)}{dy} < 0,$$

where the inequality follows from the fact that $Z^*(y) < \tau_e/\lambda\tau_y$ and $dZ^*(y)/dy < 0$. Because $N(y) = G(y) - \mathcal{C}'(y)$, we conclude that $N(y)$ is a strictly decreasing function of y.

Step 2. Next, consider the limit properties of $N(y)$. We have that

$$\lim_{y \to 0} N(y) = \frac{1}{2} \frac{\tau_e \tau_\eta}{\lambda \tau_\theta^2 (\tau_e + \tau_\eta)} - \mathcal{C}'(0) \qquad \text{and} \qquad \lim_{y \to \infty} N(y) = -\lim_{y \to \infty} \mathcal{C}'(y) < 0.$$

Letting $L \equiv \tau_e\tau_\eta/[2\lambda\tau_\theta^2(\tau_e + \tau_\eta)]$, we conclude that, when $\mathcal{C}'(0) < L$, $\lim_{y\to 0} N(y) > 0$, and hence there exists one, and only one, value of y for which $N(y) = 0$.

Step 3. Assume $\mathcal{C}'(0) < L$ and let y^* be the unique solution to $N(y) = 0$. Suppose that all other agents acquire information of quality y^* and then submit the equilibrium schedules for information of quality y^*. Let (a^*, b^*, c^*) denote the coefficients describing the equilibrium trades under the equilibrium schedules for information of quality y^* (these coefficients are given by Conditions (9), (22), and (21), applied to $y = y^*$). Let $\tau_\omega^* = \tau_\omega(a^*)$ denote the precision of the endogenous signal $z \equiv \theta + f(y^*)\eta - u/(\beta a^*)$ contained in the equilibrium price when all other agents acquire information of quality y^* and then submit the equilibrium schedules for information of quality y^*.

We show that, when $\mathcal{C}$ is sufficiently convex, $V^{\#}(y^*, y_i)$ is strictly quasi-concave in y_i. To see this, first recall that optimality requires that, for any y_i, any (s_i, p), the trades that the agent induces through his schedule given (s_i, p) are equal to $x_i = (\mathbb{E}[\theta|s_i, p; y_i] - p)/\lambda$. Equivalently, for any y_i, the function $g^*(\cdot; y_i)$ that maximizes the agent's payoff $\mathbb{E}[\pi_i^{\#}(y^*, y_i; g(\cdot))] - \mathcal{C}(y_i)$ is such that, for any (s_i, z), $g^*(s_i, z; y_i) = [\mathbb{E}[\theta|s_i, z; y_i] - (\alpha + \beta b^*) - \beta(a^* + c^*)z]/\lambda$, where $\mathbb{E}[\theta|s_i, z; y_i] = \tilde{\gamma}_1(y_i)s_i + \tilde{\gamma}_2(y_i)z$, with

$$\tilde{\gamma}_1(y_i) \equiv \frac{\tau_e\tau_\eta\sqrt{y^*y_i}\left(\tau_\eta\sqrt{y^*y_i} - \tau_\omega^*\right)}{\tau_\eta y^*\left[\tau_e\tau_\eta y_i + (\tau_\omega^* + \tau_\theta)(\tau_e + \tau_\eta)\right] - \tau_\omega^*\tau_e\left(2\tau_\eta\sqrt{y^*y_i} + \tau_\theta\right)} \tag{29}$$

and

$$\tilde{\gamma}_2(y_i) \equiv \frac{\tau_\omega^* \tau_\eta \left[\left(\tau_e + \tau_\eta\right) y^* - \tau_e \sqrt{y^* y_i}\right]}{\tau_\eta y^* \left[\tau_e \tau_\eta y_i + \left(\tau_\omega^* + \tau_\theta\right)\left(\tau_e + \tau_\eta\right)\right] - \tau_\omega^* \tau_e \left(2\tau_\eta \sqrt{y^* y_i} + \tau_\theta\right)}. \tag{30}$$

In other words, for any y_i, the function $g^*(\cdot; y_i)$ is given by $g^*(s_i, z; y_i) = \tilde{a}(y_i)s_i + \tilde{b}(y_i) + \tilde{c}(y_i)z$, with $\tilde{a}(y_i) \equiv \tilde{\gamma}_1(y_i)/\lambda$, $\tilde{b}(y_i) \equiv -\left(\alpha + \beta b^*\right)/\lambda$, and $\tilde{c}(y_i) \equiv \left[\tilde{\gamma}_2(y_i) - \beta(a^* + c^*)\right]/\lambda$. Using the Envelope Theorem, we then have that

$$\begin{aligned} \tfrac{\partial V^\#(y^*, y_i)}{\partial y_i} &= \tfrac{\partial \mathbb{E}[\pi_i^\#(y^*, y_i; g^*(\cdot; y_i))]}{\partial y_i} - \mathcal{C}'(y_i) \\ &= -\tilde{a}(y_i)\tilde{\gamma}_2(y_i) f'(y_i) f(y^*) \tfrac{1}{\tau_\eta} - \lambda \tilde{a}(y_i)^2 f(y_i) f'(y_i) \left(\tfrac{1}{\tau_\eta} + \tfrac{1}{\tau_e}\right) - \mathcal{C}'(y_i). \end{aligned}$$

We then have that

$$\begin{aligned} \frac{\partial^2 V^\#(y^*, y_i)}{\partial y_i^2} = \; & -\tilde{a}'(y_i)\tilde{\gamma}_2(y_i) f'(y_i) f(y^*) \frac{1}{\tau_\eta} \\ & -\tilde{a}(y_i)\frac{d\tilde{\gamma}_2(y_i)}{dy_i} f'(y_i) f(y^*) \frac{1}{\tau_\eta} - \tilde{a}(y_i)\tilde{\gamma}_2(y_i) f'(y_i) f(y^*) \frac{1}{\tau_\eta} \frac{f''(y_i)}{f'(y_i)} \\ & -2\lambda \tilde{a}(y_i)\tilde{a}'(y_i) f(y_i) f'(y_i) \left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right) \\ & -\lambda \tilde{a}(y_i)^2 \left(f'(y_i)\right)^2 \left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right) - \lambda \tilde{a}(y_i)^2 f(y_i) f'(y_i) \left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right) \frac{f''(y_i)}{f'(y_i)} - \mathcal{C}''(y_i). \end{aligned}$$

Using the fact that $f(y) = 1/\sqrt{y}$ and $\tilde{a}(y_i) \equiv \tilde{\gamma}_1(y_i)/\lambda$ and letting $J : \mathbb{R}_+ \to \mathbb{R}$ be the function defined by

$$J(y_i) \equiv \frac{1}{\lambda}\left(\frac{1}{2y_i\sqrt{y_i y^*}} \frac{1}{\tau_\eta}\right) \frac{d}{dy_i}\left\{\tilde{\gamma}_1(y_i)\tilde{\gamma}_2(y_i)\right\} + \frac{1}{\lambda}\left[\frac{1}{2y_i\sqrt{y_i}}\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right)\right] \frac{d}{dy_i}\left\{\left(\tilde{\gamma}_1(y_i)\right)^2 \frac{1}{\sqrt{y_i}}\right\},$$

we have that, at any y_i at which $\partial V^\#(y^*, y_i)/\partial y_i = 0$,

$$\frac{\partial^2 V^\#(y^*, y_i)}{\partial y_i^2} = J(y_i) - \frac{3}{2y_i}\mathcal{C}'(y_i) - \mathcal{C}''(y_i).$$

Computing all the derivatives, we can show that

$$\begin{aligned} J(y_i) &= \tfrac{\tau_e \tau_\eta \sqrt{y^* y_i}}{4\lambda y_i^2 D^2(y_i)} \left\{\left(\tau_e + \tau_\eta\right) y^* \tau_\eta \left[3\tau_\eta \sqrt{y_i y^*} - 2\tau_\omega^*\right] - 3\tau_\omega^* \tau_e \tau_\eta \sqrt{y^* y_i} + 2\tau_e \left(\tau_\omega^*\right)^2\right\} \\ & \quad - \tfrac{\tau_e^2 \tau_\eta^2 y^* \left(\tau_\eta \sqrt{y^* y_i} - \tau_\omega^*\right)^2 \left[(\tau_e + \tau_\eta) y^* \tau_\eta - \tau_\omega^* \tau_e\right]}{\lambda y_i D^3(y_i)} \end{aligned}$$

where, for any y_i, $D(y_i) \equiv \tau_\eta y^* \left[\tau_e \tau_\eta y_i + (\tau_\omega^* + \tau_\theta)(\tau_e + \tau_\eta)\right] - \tau_\omega^* \tau_e \left(2\tau_\eta \sqrt{y^* y_i} + \tau_\theta\right)$. One can verify that $\lim_{y_i \to 0} J(y_i) = -\infty$, $\lim_{y_i \to +\infty} J(y_i) = 0$ and that $J(y_i)$ does not have vertical asymptotes. Hence it is bounded from above by a constant $M > 0$. Hence, when $\frac{3}{2y_i}\mathcal{C}'(y_i) + \mathcal{C}''(y_i) > M$ for all $y_i \geq 0$, the payoff is quasi-concave. Note that, when $\mathcal{C}(y) = \frac{B}{2}y^2$, the above condition becomes $B > \frac{2}{5}M$, which holds for B large enough, as claimed in the main text.

The above results imply that, under the conditions in the proposition, choosing quality of information $y_i = y^*$ and then submitting the schedule defined by the coefficients $(a^*, \hat{b}^*, \hat{c}^*)$ in Proposition 1 (for quality of information y^*) is a symmetric equilibrium in the full game. That there are no other symmetric equilibria in affine strategies follows from the uniqueness of the solution to $N(y) = 0$ established in Step 2. Q.E.D.

Proof of Proposition 6. Assume that all traders other than i acquire information of quality y^T and then submit the efficient schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ for quality of information y^T). Given any policy $T(x_i, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his schedule optimally is equal to $V(y^T, y_i) \equiv \sup_{g(\cdot)} \left\{\mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] - \mathcal{C}(y_i)\right\}$, where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and the endogenous signal $z = \theta + f(y^T)\eta - u/\beta a^T$ contained in the market-clearing price, with

$$\begin{aligned}\mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] \;\equiv\; & \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta\tilde{x})g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2\right] \\ & -\mathbb{E}\left[T\left(g(s_i, z), \alpha - u + \beta\tilde{x}\right)\right].\end{aligned}$$

Note that, in writing V, we used the fact that p and z are related by $p = \alpha + \beta b^T + \beta(a^T + c^T)z$, where b^T and c^T are obtained from $\hat{b}^T$ and $\hat{c}^T$ using (16) and (17).

For the policy $T(x_i, p)$ to implement the efficient acquisition and usage of information, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that $\mathbb{E}[\theta | s_i, z] = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z$, where γ_1 and γ_2 are the functions defined in the proof of Proposition 1, we thus have that, for the policy T to implement the efficient trades, it must be that T is differentiable in x_i and satisfy

$$\begin{aligned}&\tfrac{\partial}{\partial x_i}T\left(a^T s_i + b^T + c^T z, \alpha + \beta b^T + \beta(a^T + c^T)z\right) = \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right]\tfrac{x - b^T}{a^T} \\ &+ \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right)\tfrac{c^T}{a^T}\right]\tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)} - \left(\alpha + \beta b^T + \lambda b^T\right)\end{aligned}$$

for all (s_i, z), where $\frac{\partial}{\partial x_i}T(x_i, p)$ is the partial derivative of the tax bill with respect to the individual volume of trade. This means that $T(x_i, p)$ is a polynomial of second order of the form

$$T(x_i, p) = \frac{\delta}{2} x_i^2 + (t_p p - t_0)\, x_i + \tilde{K}(p), \tag{31}$$

for some vector (δ, t_p, t_0) and some function $\tilde{K}(p)$ which plays no role for incentives and which therefore we can disregard. In the proof of Proposition 4, we showed that there exists a unique vector (δ, t_p, t_0) that induces the traders to submit the efficient schedules when the precision of their private information is y^T (the vector in Proposition 4 applied to $y = y^T$). Thus, if a policy T induces efficiency in both information acquisition and information usage, it must be of the form in (31) with (δ, t_p, t_0) as in Proposition 4 applied to $y = y^T$. When the policy takes this form, for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = as_i + b + cz$, for some (a, b, c). This implies that

$$\mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] = \tilde{M} - \beta(1+t_p)(a^T + c^T)a\frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda+\delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_e},$$

where $\tilde{M}$ is a function of all terms that do not interact with y_i. The optimal g when $y_i = y^T$ is $g(x_i, z) = a^T s_i + b^T + c^T z$. Using the Envelope Theorem, we then have that

$$\left.\frac{\partial}{\partial y_i} V(y^T, y_i)\right|_{y_i = y^T} = \frac{[\beta(1+t_p) + \lambda + \delta]\,(a^T + c^T)a^T}{2\tau_\eta\left(y^T\right)^2} + \frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2} \quad - \quad \mathcal{C}'(y^T).$$

Because the efficient y^T solves

$$\frac{(\beta+\lambda)(a^T + c^T)^2}{2\tau_\eta\left(y^T\right)^2} + \frac{\lambda\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2} \quad = \quad \mathcal{C}'(y^T), \tag{32}$$

we have that, for the policy to implement the efficient acquisition of private information, it must be that $(a^T + c^T)\tau_e\left[(\beta+\lambda)c^T - (\beta t_p + \delta)a^T\right] = \delta\left(a^T\right)^2\tau_\eta$. One can verify that the values of δ and t_p from Proposition 4 do not solve the above equation except for a non-generic set of parameters. Q.E.D.

Proof of Proposition 7. When all other traders acquire information of quality y^T and submit the efficient schedules for information of quality y^T, the maximal payoff that trader i can obtain by acquiring information of quality y_i is equal to

$$\hat{V}(y^T, y_i) \equiv \sup_{a,b,c}\left\{\mathbb{E}[\hat{\pi}_i; y^T, y_i, a, b, c] - \mathcal{C}(y_i) + Ay_i\right\},$$

where

$$\mathbb{E}[\hat{\pi}_i; y^T, y_i, a, b, c] \equiv \bar{M} - \beta(1+t_p)(a+c)a\frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda+\delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_\eta} - \frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_e},$$

where $\bar{M}$ collects all variables that do not interact with y_i. Note that, in writing $\hat{V}(y^T, y_i)$, we use the fact that, for any y_i, the trader's payoff is maximized by submitting an affine schedule which induces trades $x_i = as_i + b + cz$ that are affine in (s_i, z), where $z = \theta + f(y^T)\eta - u/\beta a^T$ is the endogenous signal contained in the price. Using the Envelope Theorem, we have that

$$\left.\frac{\partial}{\partial y_i}\hat{V}(y^T, y_i)\right|_{y_i=y^T} = \frac{[\beta(1+t_p)+\lambda+\delta]\,(a^T+c^T)a^T}{2\tau_\eta\,(y^T)^2} + \frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e\,(y^T)^2} \quad - \quad \mathcal{C}'(y^T) + A,$$

where we use the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to $a^T s_i + b^T + c^T z$. Using the fact that y^T satisfies Condition (32) along with Condition (17) to express c^T as a function of $\hat{c}^T$, we thus have that the proposed policy induces the efficient acquisition of private information only if

$$A = -\frac{\left(a^T\right)^2}{2\tau_\eta\,(y^T)^2}\left[\frac{\beta(\beta+\lambda)\hat{c}^T}{(1+\beta\hat{c}^T)^2} + \frac{\beta t_p + \delta}{1+\beta\hat{c}^T}\right] - \frac{\delta\left(a^T\right)^2}{2\tau_e\,(y^T)^2}.$$

Next note that arguments similar to those in the proof of Proposition 5 imply that there exist constants $\tilde{L}, \tilde{M} \in \mathbb{R}_{++}$ such that, when $\mathcal{C}'(0) \leq \tilde{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \tilde{M}$, the function $\hat{V}(y^T, y_i)$ is globally quasi-concave in y_i. We conclude that, when $\mathcal{C}$ satisfies these conditions, the proposed policy implements the efficient acquisition and usage of information. Q.E.D.

Proof of Proposition 8. Assume that all traders other than i acquire information of quality y^T and then submit the efficient schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ for quality of information y^T). Given any policy $T(x_i, \tilde{x}, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to $\tilde{V}(y^T, y_i) \equiv \sup_{g(\cdot)}\left\{\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] - \mathcal{C}(y_i)\right\}$ where $g: \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of the good $x_i = g(s_i, z)$ that the trader trades as a function of s_i and z, with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$, and where

$$\begin{aligned}\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] \;\; &\equiv \;\; \mathbb{E}\left[\theta g(s_i,z) - (\alpha - u + \beta\tilde{x})g(s_i,z) - \tfrac{\lambda}{2}\left(g(s_i,z)\right)^2\right] \\ &\quad -\mathbb{E}\left[T\left(g(s_i,z), \tilde{x}, \alpha - u + \beta\tilde{x}\right)\right].\end{aligned}$$

For the policy $T(x_i, \tilde{x}, p)$ to induce efficiency in both information acquisition and usage, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that $\mathbb{E}\left[\theta | s_i, z; y_i, y^T\right]\big|_{y_i=y^T} = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z$, with the functions γ_1 and γ_2 as in Proposition 1, we thus have that T must be differentiable in x_i and satisfy

$$\begin{gathered}\tfrac{\partial}{\partial x_i}\mathbb{E}\left[T\left(a^T s_i + b^T + c^T z, \tilde{x}, \alpha - u + \beta\tilde{x}\right)|s_i, z; y_i, y^T\right]\Big|_{y_i=y^T} \\ = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right)\end{gathered}$$

for all (s_i, z), where $\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$, with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$. Next recall from the proof of Proposition 6 that, when the individual trades efficiently,

$$\begin{gathered}\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right) \\ = \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right]\tfrac{x - b^T}{a^T} + \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right)\tfrac{c^T}{a^T}\right]\tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)} \\ - \left(\alpha + \beta b^T + \lambda b^T\right).\end{gathered}$$

This means that $T(x_i, \tilde{x}, p)$ must be a polynomial of second order of the form

$$T(x_i, \tilde{x}, p) = \frac{\delta'}{2}x_i^2 + \left(pt'_p - t'_0 + t_{\tilde{x}}\tilde{x}\right)x_i + L'(\tilde{x}, p), \tag{33}$$

for some vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$, where $L'(\tilde{x}, p)$ is a function that does not depend on x_i, plays no role for incentives, and hence can be disregarded. Furthermore, under any such a policy,

$$\begin{gathered}\tfrac{\partial}{\partial x_i}\mathbb{E}\left[T\left(x_i, \tilde{x}, p\right)|s_i, p; y_i, y^T\right] \\ = \delta' x_i + pt'_p - t'_0 + \tfrac{t_{\tilde{x}}}{\beta}(p - \alpha) + \tfrac{t_{\tilde{x}}}{\beta}A^{\#}(y_i, y^T)s_i + \tfrac{t_{\tilde{x}}}{\beta}B^{\#}(y_i, y^T)p + \tfrac{t_{\tilde{x}}}{\beta}C^{\#}(y_i, y^T),\end{gathered}$$

where $A^{\#}(y_i, y^T)$, $B^{\#}(y_i, y^T)$, and $C^{\#}(y_i, y^T)$ are the coefficients of the projection

$$\mathbb{E}\left[u|s_i, p; y_i, y^T\right] = A^{\#}(y_i, y^T)s_i + B^{\#}(y_i, y^T)p + C^{\#}(y_i, y^T)$$

of u on (s_i, p). When trader i acquires information of quality $y_i = y^T$ and trades efficiently,

$$\frac{\partial}{\partial x_i}\mathbb{E}\left[T\left(x_i, \tilde{x}, p\right)|s_i, p; y^T, y^T\right] = \delta x_i + t_p p - t_0$$

where

$$\delta = \delta' + \frac{t_{\tilde{x}}}{\beta}\hat{A}^{\#}, \tag{34}$$

$$t_p = t'_p + t_{\tilde{x}} \frac{1 + \hat{B}^{\#}}{\beta}, \tag{35}$$

and

$$t_0 = t'_0 + t_{\tilde{x}} \frac{\alpha}{\beta} - \frac{t_{\tilde{x}}}{\beta} \hat{C}^{\#}, \tag{36}$$

with $\hat{A}^{\#} \equiv A^{\#}(y^T, y^T)/a^T$,

$$\hat{B}^{\#} \equiv \left[B^{\#}(y^T, y^T) - \frac{A^{\#}(y^T, y^T) c^T}{a^T \beta (a^T + c^T)} \right],$$

and

$$\hat{C}^{\#} \equiv C^{\#}(y^T, y^T) - \frac{A^{\#}(y^T, y^T) b^T}{a^T} + \frac{A^{\#}(y^T, y^T) c^T (\alpha + \beta b^T)}{a^T \beta (a^T + c^T)}.$$

In the proof of Proposition 4, we showed that, when agents acquire information of quality y^T, for them to trade efficiently, the values of (δ, t_p, t_0) must coincide with those in Proposition 4 (applied to $y = y^T$). Thus, for the above policy to induce efficiency in both information acquisition and information usage, it must be that the vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$ satisfies Conditions (34)-(36) with (δ, t_p, t_0) given by the values determined in Proposition 4 applied to $y = y^T$. Note that, for any $t_{\tilde{x}}$, there exist unique values of (δ', t'_p, t'_0) that solve the above three conditions. Abusing notation, denote these values by $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$.

Next, note that, when the policy takes the form in (33), for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = a s_i + b + cz$, for some (a, b, c). This implies that

$$\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] = \hat{M} - \left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}}))\right] \frac{a(a^T + c^T)}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda+\delta) ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda+\delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda+\delta}{2} \frac{a^2}{y_i \tau_e},$$

where $\hat{M}$ is a function of all variables that do not interact with y_i. Using the Envelope Theorem, we then have that

$$\frac{\partial}{\partial y_i} \tilde{V}(y^T, y_i) \Big|_{y_i = y^T} = \frac{\left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}})) + \lambda + \delta\right](a^T + c^T) a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e (y^T)^2} - \mathcal{C}'(y^T).$$

Once again, in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades $a^T s_i + b^T + c^T z$. Finally, recall that y^T is defined by Condition (32). Hence, for the above policy to induce efficiency in information acquisition, it must be that

$$\frac{(\beta+\lambda)(a^T + c^T)^2}{\tau_\eta} + \frac{\lambda\left(a^T\right)^2}{\tau_e} = \frac{\left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}})) + \lambda + \delta\right](a^T + c^T) a^T}{\tau_\eta} + \frac{(\lambda+\delta)\left(a^T\right)^2}{\tau_e}. \tag{37}$$

Using (35), we have that $t'_p(t_{\tilde{x}}) = t_p - t_{\tilde{x}}\left(1+\hat{B}^{\#}\right)/\beta$, with t_p given by the unique value determined in Proposition 4 applied to $y = y^T$. Because the function $\tilde{H} : \mathbb{R} \to \mathbb{R}$ given by $\tilde{H}(t_{\tilde{x}}) \equiv t_{\tilde{x}} + \beta t'_p(t_{\tilde{x}}) = \beta t_p - t_{\tilde{x}}\hat{B}^{\#}$ is linear, there exists a (unique) value of $t_{\tilde{x}}$ that solves (37).

Following steps similar to those in the proof of Proposition 5, one can show that there exist scalars $\hat{L}, \hat{M} \in \mathbb{R}_{++}$ such that, when when $\mathcal{C}'(0) \leq \hat{L}$, and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \hat{M}$, the function $\tilde{V}(y^T, y_i)$ is globally quasi-concave in y_i. We conclude that, when $\mathcal{C}$ satisfies the above properties, the policy in (33), with $t_{\tilde{x}}$ given by the unique solution to (37) and with (δ', t'_p, t'_0) given by the unique solution $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$ to Conditions (34)-(36), induces efficiency in both information acquisition and information usage. Q.E.D.

Proof of Proposition 9. Following the same steps as in the proof of Proposition 1, one can show that the equilibrium trades are given by $x_i = as_i + b + cz$ where a is given by the same value as in Proposition 1 whereas

$$b = -(1+t_p)\frac{\alpha}{(1+t_p)\beta + \lambda} \tag{38}$$

and

$$c = \frac{1}{\beta(1+t_p)+\lambda}\left[\left(1 - \lambda a\frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - (1+t_p)\beta a\right]. \tag{39}$$

Hence, any ad valorem tax $t_p \neq 0$ induces the same sensitivity of the equilibrium trades to private information as in the laissez-faire equilibrium but different values of b and c.

Next, we show that the equilibrium value of y is also invariant in t_p. To see this, fix t_p, and, for any (y, y_i), let

$$V^{\#}(y, y_i) \equiv \sup_{g(\cdot)}\left\{\mathbb{E}[\pi_i^{\#}; y, y_i, g(\cdot))] - \mathcal{C}(y_i)\right\}$$

denote the maximal payoff that trader i can obtain by selecting information of quality y_i when all other agents acquire information of quality y and then submit the equilibrium schedules for information of quality y when the tax rate is t_p. The function $g : \mathbb{R}^2 \to \mathbb{R}$ specifies the amount of the good $x_i = g(s_i, z)$ traded as a function of s_i and the endogenous public signal z contained in the equilibrium price. Let (a, b, c) be the parameters defining the equilibrium trades when information is of quality y and the tax rate is t_p. We then have that[25]

$$\mathbb{E}[\pi_i^{\#}; y, y_i, g(\cdot))] \equiv \mathbb{E}\left[\theta g(s_i, z) - (1+t_p)\left(\alpha + \beta b + \beta(a+c)z\right) g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2 |y_i\right].$$

Note that in writing $\mathbb{E}[\pi_i^{\#}; y, y_i, g(\cdot))]$ we used the fact that the equilibrium price is given by $p = \alpha + \beta b + \beta(a+c)z$ with $z = \theta + f(y)\eta - u/(\beta a)$. By the definition of equilibrium, if agent i acquires information of quality $y_i = y$, the schedule that maximizes his payoff is the equilibrium one (that is, the one corresponding to the coefficients (a, b, c)). The Envelope Theorem then implies that

$$\tilde{N}(y) \equiv \left.\frac{\partial V^{\#}(y, y_i)}{\partial y_i}\right|_{y_i = y} = \frac{\beta(1+t_p)(a+c)a}{2\tau_\eta y^2} + \frac{\lambda a(a+c)}{2\tau_\eta y^2} + \frac{\lambda a^2}{2y^2\tau_e} - \mathcal{C}'(y). \tag{40}$$

Hence, the equilibrium value of y, must satisfy $\tilde{N}(y) = 0$. Let $M^{\#}(t_p, a, c, y)$ denote the function defined by the right-hand-side of (40). Next, use the derivations above to observe that, given (t_p, y), the equilibrium values of (a, b, c) are given by (9), (38), and (39). From the Implicit Function Theorem, we then have that

$$\frac{dy}{dt_p} = -\frac{\frac{\partial M^{\#}(t_p,a,c,y)}{\partial t_p} + \frac{\partial M^{\#}(t_p,a,c,y)}{\partial c}\frac{\partial c}{\partial t_p}}{\frac{\partial M^{\#}(t_p,a,c,y)}{\partial y} + \frac{\partial M^{\#}(t_p,a,c,y)}{\partial a}\frac{\partial a}{\partial y} + \frac{\partial M^{\#}(t_p,a,c,y)}{\partial c}\frac{\partial c}{\partial y}},$$

where we used the fact that, given any y, the equilibrium level of a is invariant in t_p. Note that $\partial c/\partial t_p$ is the partial derivative of the equilibrium level of c with respect to t_p, holding y constant, whereas $\partial a/\partial y$ and $\partial c/\partial y$ are the partial derivatives of the equilibrium levels of a and c with respect to y, holding t_p fixed. Because, for any y, $\frac{\partial}{\partial t_p}M^{\#}(t_p, a, c, y) = \left(\beta(a+c)a\right)/2\tau_\eta y^2$, $\frac{\partial}{\partial c}M^{\#}(t_p, a, c, y) = \left[\beta(1+t_p) + \lambda\right] a/2\tau_\eta y^2$, and $\partial c/\partial t_p = -\beta(a+c)/\left[\beta(1+t_p) + \lambda\right]$, we conclude that $dy/dt_p = 0$.

Hence both the equilibrium value of y and the equilibrium value of a are invariant in t_p. Because, given (y, a), the equilibrium values of b and c (equivalently, of $\hat{b}$ and $\hat{c}$) in the laissez-faire economy maximize welfare, as shown in Lemma (1), we conclude that the optimal value of t_p is $t_p = 0$. Q.E.D.

[25]Given (a, b, c), the sensitivity of the equilibrium schedules to the price, $\hat{c}$, and the constant $\hat{b}$ in the equilibrium schedules are obtained from (a, b, c) using (16) and (17).