Supplemental Appendix
Market opacity and liquidity: Why liquidity evaporates when it is most needed?

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A Dealers' strategies with full transparency

In this appendix, we provide a more detailed derivation of the strategies adopted by dealers in the fully transparent benchmark of Section II.

The liquidity supply that accommodates the demand of hedgers is offered by dealers who submit price contingent orders (generalized limit orders) at both rounds.

A dealer's strategy at $t = 1$ is given by (see Appendix (A.28)):

$$\begin{aligned} X_1^D(p_1) &= \frac{\gamma}{\text{Var}_1^D[p_2]} E_1^D[p_2] - \gamma \left(\frac{1}{\text{Var}_1^D[p_2]} + \frac{1}{\text{Var}[v]} \right) p_1 \\ &= -\gamma \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1^D[p_2]} u_1 - \gamma \tau_v p_1. \end{aligned} \tag{1}$$

According to (1), $X_1^D(p_1)$ reflects two trading motives: short-term return speculation (captured by the component $-u_1\gamma(\Lambda_{21} - \Lambda_1)/\text{Var}_1[p_2]$) and liquidity supply (captured by the price dependent component in (1), $-\gamma\tau_v p_1$). We now explain in more detail these components, starting from the short-term return speculation one.

Due to their ability to infer traders' endowment shock and the fact that they know these traders split their hedging order, dealers exploit the anticipated effect the shock has on expected returns. To see this, note that at the second round dealers in aggregate hold (see (A.10))

$$\begin{aligned} X_2^D(p_1, p_2) &= \gamma \frac{E_2^D[v - p_2]}{\text{Var}_2^D[v - p_2]} = \\ &= -\gamma\tau_v p_2 \\ &= \gamma\tau_v\Lambda_{21}u_1 + \gamma\tau_v\Lambda_2 u_2, \end{aligned} \tag{2}$$

where the expression at the third line in (2) originates from substituting (1b) in dealers' second period aggregate position. Expression (2) implies that at the second round dealers hold $\gamma\tau_v\Lambda_{21}$ of the first period endowment shock. Based on (1), at the first round their position is given by

$$X_1^D(p_1) = \gamma \left(\tau_u \frac{\Lambda_1 - \Lambda_{21}}{\Lambda_2^2} + \tau_v \Lambda_1 \right) u_1.$$

Hence, if $u_1 > 0$, at the first round dealers provide liquidity by absorbing part of first period traders' endowment shock ($\Lambda_1 > 0$). Additionally, they *consume* liquidity by taking a *short position* in the risky security ($\Lambda_1 - \Lambda_{21} < 0$).

At the second round, based on what said above, they provide liquidity to the incremental hedging trade of first period traders: their trade with respect to the latter is given by

$$\gamma\tau_v\Lambda_{21}u_1 - X_1^D(p_1) = \gamma \frac{\tau_u + \Lambda_2^2\tau_v}{\Lambda_2^2} (\Lambda_{21} - \Lambda_1) u_1,$$

i.e., a buy order (if $u_1 > 0$). Thus, because of their ability to anticipate returns, dealers gain from short term speculation at the first round (moderating their buying at the first round and deferring it since in expectation the price at the second round is lower: $E_1^D[p_2 - p_1] < 0$ if $u_1 > 0$).

Due to risk aversion, dealers have a limited capacity to bear risk. This implies that the price coefficients for in Table 1 capture the risk-tolerance weighted risk compensation dealers require to absorb the aggregate liquidity demand.

We now derive Λ_1. At the first round a_1 reflects the marginal position of liquidity traders:

$$a_1 = \frac{\partial x_{11}}{\partial u_1} = -\gamma_H \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]}. \tag{3}$$

As observed above, dealers also demand liquidity, since they speculate on the price impact of u_1 and their aggregate liquidity demand is given by

$$-\gamma \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} = \gamma \frac{a_1}{\gamma_H}.$$

Aggregating across liquidity traders' and dealers' demands yields the aggregate liquidity demand at the first round:

$$a_1 + \gamma \frac{a_1}{\gamma_H} = \frac{\gamma + \gamma_H}{\gamma_H} a_1.$$

At equilibrium, replacing dealers and liquidity traders' equilibrium strategies (respectively, (1) and the expression for a_1 in Table 1) in the first period equilibrium condition (2), we have:

$$\begin{aligned} x_1^D + x_{11} = 0 &\iff \gamma \frac{a_1}{\gamma_H} u_1 - \gamma \tau_v p_1 + a_1 u_1 = 0 \\ &\iff \frac{\gamma + \gamma_H}{\gamma_H} a_1 u_1 = \gamma \tau_v p_1. \end{aligned} \tag{4}$$

At a linear equilibrium the price is proportional to the aggregate endowment shock u_1: $p_1 = -\Lambda_1 u_1$. Identifying $-\Lambda_1$ in the latter expression yields:

$$\underbrace{\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_H}{\gamma_H} a_1}_{-\Lambda_1} u_1 = p_1. \tag{5}$$

Thus, $-\Lambda_1$ measures the price impact of a marginal increase in the endowment shock hitting the first period cohort. Since this covers a "cost" incurred to supply immediacy, we interpret (somewhat loosely) Λ_1 as the first period "liquidity supply" function.

At the second round, liquidity demand comes from first and second period traders coefficients a_{21} and a_2:

$$\begin{aligned} x_{21} &= \gamma_H \frac{E_1[v - p_2]}{\mathrm{Var}_1[v - p_2]} - \frac{\mathrm{Cov}_1[v, v - p_2]}{\mathrm{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{(\gamma_H \tau_v \Lambda_{21} - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}}_{= a_{21}} u_1. \end{aligned}$$

and

$$\begin{aligned} x_2 &= \gamma_H \frac{E_2[v - p_2]}{\mathrm{Var}_2[v - p_2]} - \frac{\mathrm{Cov}_2[v, v - p_2]}{\mathrm{Var}_2[v - p_2]} u_2 \\ &= \underbrace{(\gamma_H \tau_v \Lambda_2 - 1)}_{= a_2} u_2 + \underbrace{\gamma_H \tau_v \Lambda_{21}}_{= b} u_1. \end{aligned}$$

We can interpret the expressions for a_{21} and a_2 in the following way. A liquidity trader hedges a larger fraction of his shock (demands more liquidity), the lower is the impact the endowment shock has on p_2 (as a larger price impact reduces a trader's expected return from hedging), and the lower is the return uncertainty he faces (as a higher return variance dents his utility since he is risk averse). Consider now the second period market clearing condition:

$$\begin{aligned}(x_2^D - x_1^D) + x_{21} - x_{11} + x_2 = 0 &\iff x_2^D + x_{21} + x_2 = 0 \\ &\iff -\gamma\tau_v p_2 + a_2 u_2 + (a_{21} + b)u_1 = 0 \\ &\iff p_2 = \underbrace{\frac{a_2}{\gamma\tau_v}}_{= -\Lambda_2} u_2 + \underbrace{\frac{a_{21} + b}{\gamma\tau_v}}_{= -\Lambda_{21}} u_1. \end{aligned} \tag{6}$$

At the second line of the above expression we make use of the first period market clearing equation: $x_1^D + x_{11} = 0$. We then replace strategies with their equilibrium expressions and finally solve for p_2, identifying the price coefficients.

B The effects of a partially informative signal

In this appendix, we illustrate the effect of increasing the precision of second period hedgers' information on price impacts (Figure 1) and strategy coefficients (Figure 2).

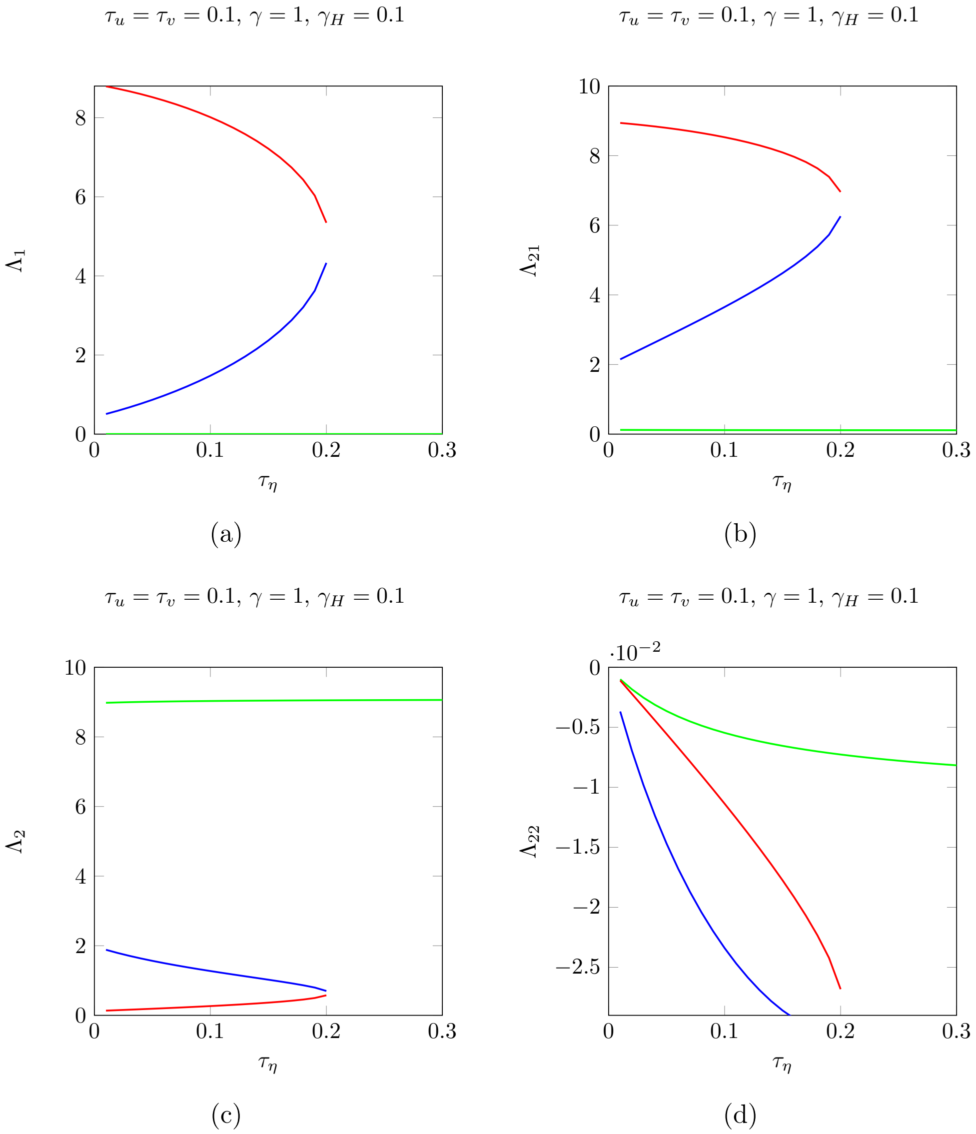
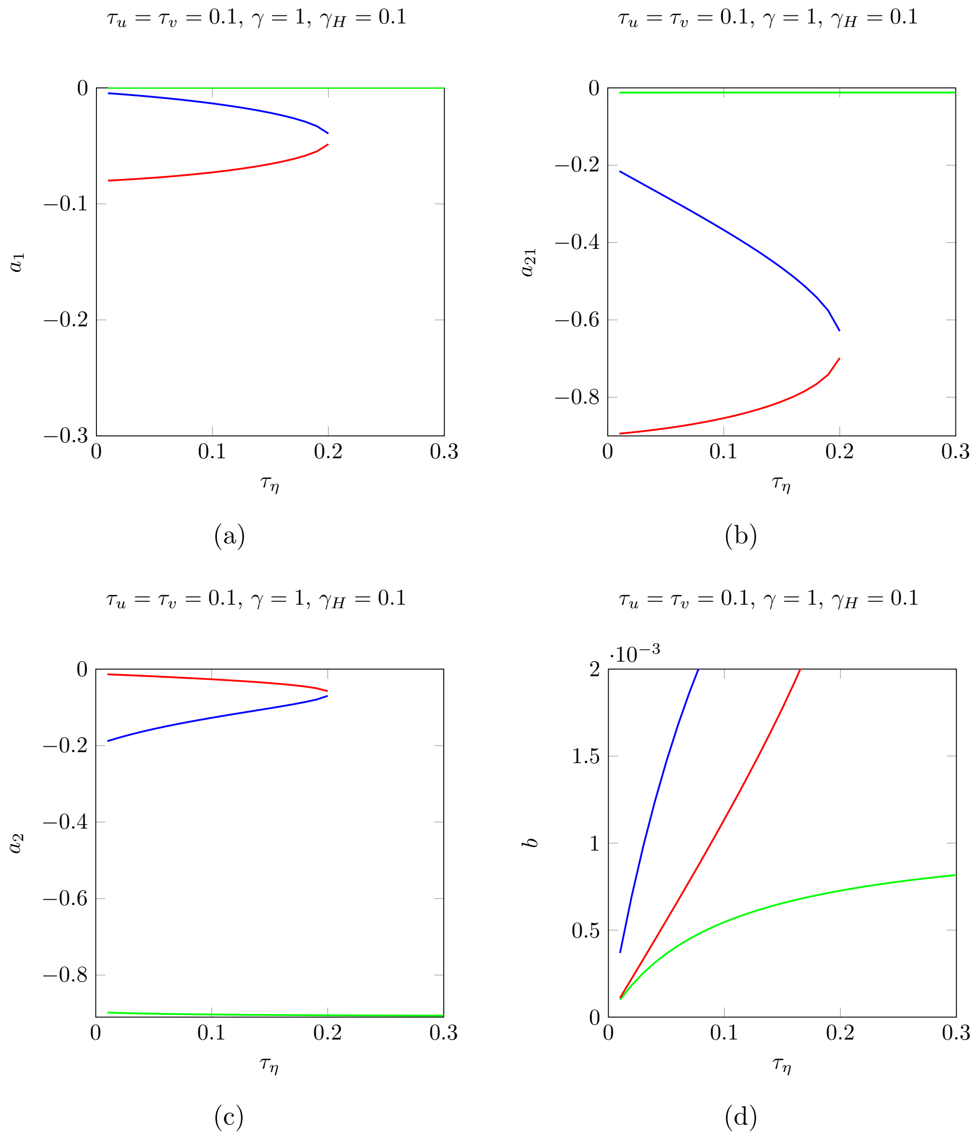


Figure 1: Price impact coefficients with a partially informative signal ($\tau_\eta < \infty$). As shown in the figure, for τ_η small, three equilibria arise. We plot them using the colors green, blue and red to indicate the equilibrium that corresponds to the two extreme, stable price impacts (respectively in green and red) and the unstable one (in blue). Parameter values are as in Figure 5 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$: we analyze the increase in transparency letting τ_η increase by 0.01.



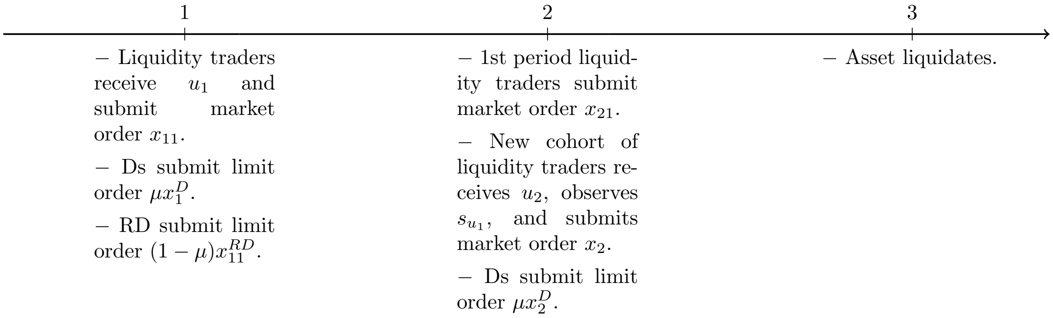
(a) (b)

(c) (d)

Figure 2: Strategy coefficients with a partially informative signal ($\tau_\eta < \infty$). As shown in the figure, for τ_η small, three equilibria arise. We plot them using the colors green, blue and red to indicate the equilibrium that corresponds to the two extreme, stable price impacts (respectively in green and red) and the unstable one (in blue). Parameter values are as in Figure 5 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$: we analyze the increase in transparency letting τ_η increase by 0.01.

C Comparative statics with restricted dealers

In this appendix we present the full set of comparative statics results about the effects on liquidity fragility of traders' risk aversion (γ_H), payoff volatility (τ_v^{-1}) and endowment shock dispersion (τ_u^{-1}) in the model with restricted dealers (see Figure 3 for a timeline of events).



– Liquidity traders receive u_1 and submit market order x_{11}.

– Ds submit limit order μx_1^D.

– RD submit limit order $(1-\mu)x_{11}^{RD}$.

– 1st period liquidity traders submit market order x_{21}.

– New cohort of liquidity traders receives u_2, observes s_{u_1}, and submits market order x_2.

– Ds submit limit order μx_2^D.

– Asset liquidates.

Figure 3: The timeline with restricted dealers.

In Figure 4, we partition the space $\mu \in (0,1]$, $\tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium (three equilibria).

Consistently with what we have found in Proposition 4, an increase in γ_H or τ_v tends to reduce the chances of liquidity fragility (compare the areas below the blue curve in panel (a) and panels (c) and (d)). The effect of an increase in τ_u is more complicated. Recall that when the signal is not perfect second period traders (1) may speculate in the "wrong" direction and (2) use p_2 and the signal to predict u_1. With a higher τ_u there is less noise in the price, which reduces second period traders speculative intensity. When μ is close to 0, almost only second period traders provide liquidity at the second round, and the reduction in speculation by these traders has a large impact on overall risk sharing. Conversely, when μ is close to 1, almost only (full) dealers provide liquidity at 2 and the reduction in speculation by 2nd period traders means that dealers have less liquidity traders to share risk with. In either case this increases liquidity fragility. For intermediate values of μ, (full) dealers have a smaller exposure to the risky security, and the additional risk sharing provided by 2nd period traders is less important. In this case, the reduction in these traders' speculation rids the market of the "wrong" trades with a positive impact on liquidity fragility.

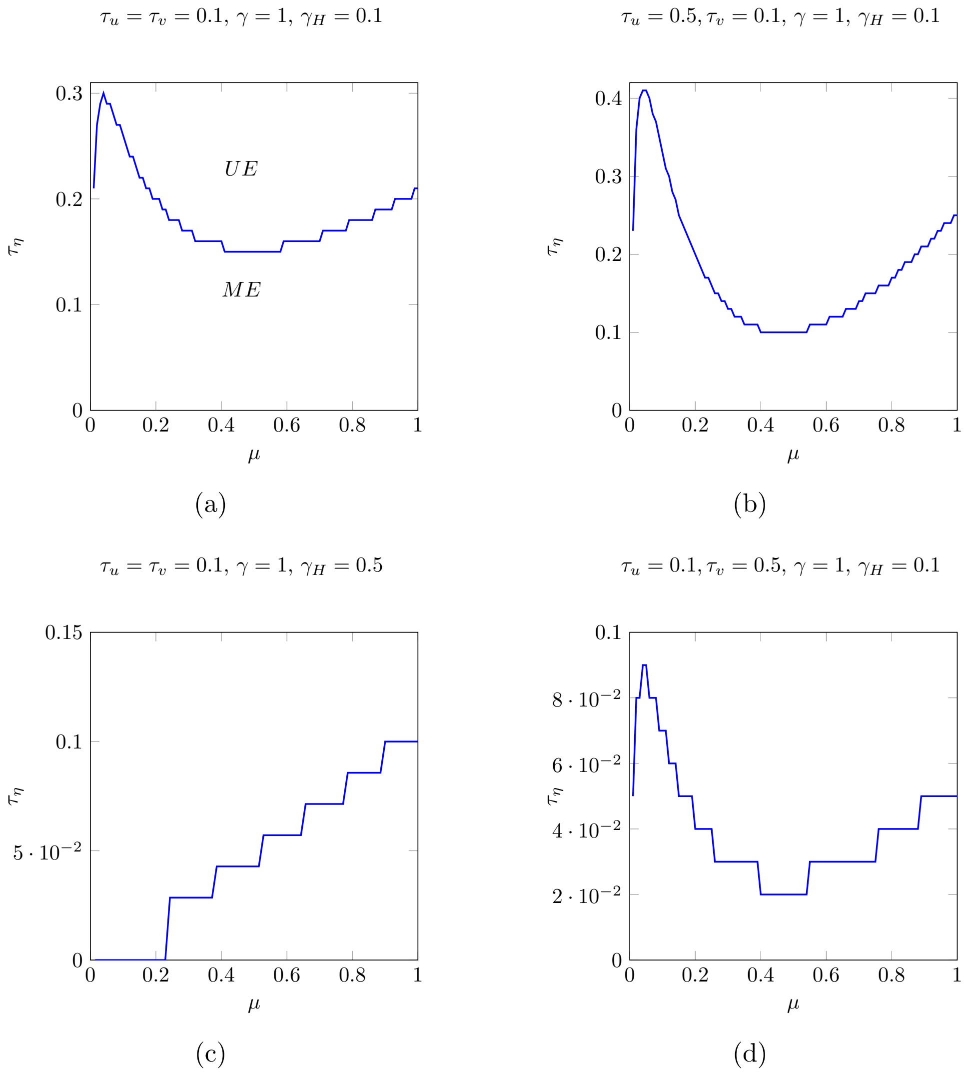


Figure 4: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.

Figure 5 displays the effect of a small unanticipated change in μ on the equilibrium when parameters are such that a unique equilibrium is obtained.

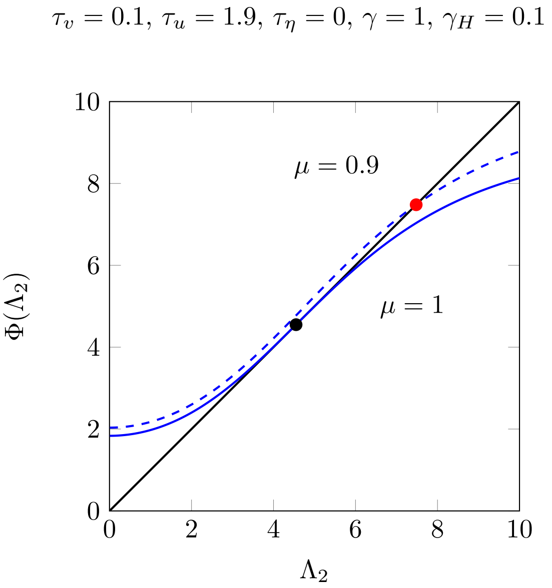

$\tau_v = 0.1,\ \tau_u = 1.9,\ \tau_\eta = 0,\ \gamma = 1,\ \gamma_H = 0.1$

Figure 5: The effect of a small reduction in μ when τ_η is low. Total illiquidity increases.

Figure 6 illustrates the effect of increasing transparency and μ on total welfare and $WAPI$.

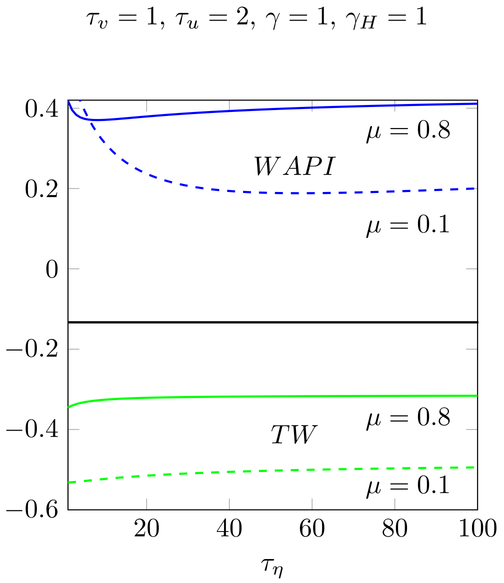

$\tau_v = 1,\ \tau_u = 2,\ \gamma = 1,\ \gamma_H = 1$

Figure 6: TW and $WAPI$ as a function of transparency.

D Liquidity trading and noise trading

In this appendix, we consider the implications of our analysis for the time series properties of noise trading and returns. First, note that, based on Proposition 2 we can say that with transparency, at the second round dealers absorb a smaller portion of the first period endowment shock (compared to the second period one), and the noise process is stable: $\beta \equiv \Lambda_{21}/\Lambda_2 < 1$. That is, the second period endowment shock impacts p_2 more than u_1.

Second, the first and second period returns are *positively* serially correlated. That is, the model displays momentum, in the absence of any fundamentals information:

$$\begin{aligned} \mathrm{Cov}[p_2 - p_1, p_1] &= \mathrm{Cov}[-(\Lambda_2 u_2 + (\Lambda_{21} - \Lambda_1)u_1), -\Lambda_1 u_1] \\ &= (\Lambda_{21} - \Lambda_1)\Lambda_1 \tau_u^{-1} > 0, \end{aligned} \tag{7}$$

due to Proposition 2. At the first round hedgers can count on the additional liquidity supplied by second period traders, which implies that they will increase their first period hedging position, inducing $\Lambda_{21} > \Lambda_1$.

We collect these results in the following corollary:

Corollary 1. *When the market is transparent: (1) liquidity trading behaves as a stable AR(1) process; (2) first and second period returns are positively serially correlated.*

The following result states the implications for the time series properties of noise trades and returns autocovariance when the market is fully opaque:

Corollary 2. *With multiple equilibria, the autocovariance of first and second period returns increases in Λ_{21} and also increases compared to the case with full transparency at both equilibria; (3) at the intermediate equilibrium or when we have a unique equilibrium we have that $\beta = 1$ since $\Lambda_{21} = \Lambda_2$.*

We evaluate the expression for returns autocovariance at the equilibrium with full transparency (and $\mu = 1$):

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\gamma \tau_u^2 \tau_v (\gamma + \gamma_H)^5}{\left(\tau_u \tau_v \left(2\gamma^2 + 4\gamma\gamma_H + \gamma_H^2\right)(\gamma + \gamma_H) + \tau_u^2 \tau_v^2 (\gamma + 2\gamma_H)(\gamma + \gamma_H)^4 + \gamma\right)^2} \tag{8}$$

and at both the equilibria that obtain under the parameter restriction ensuring multiplicity, when the market is fully opaque, when $\Lambda_{21} = \Lambda_{21}^{*}$ we have

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)}\right)^3 \left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)} + \gamma\right)}{16\gamma^4 \tau_u \tau_v^2 (\gamma + \gamma_H)^2}, \tag{9}$$

and when $\Lambda_{21} = \Lambda_{21}^{***}$ we have instead

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)}\right) \left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma + \gamma_H)^3\right)} + \gamma\right)^3}{16\gamma^4 \tau_u \tau_v^2 (\gamma + \gamma_H)^2}. \tag{10}$$

Comparing the two latter expressions shows that return autocovariance is higher when $\Lambda_{21} = \Lambda_{21}^{*}$. While comparing the latter expression above with (8) shows that it increases with respect to the case with full transparency.

E First period hedgers observing u_2

Suppose that at the second round first period traders perfectly observe u_2, while second period traders do not know u_1.

Assume that prices are linear in the endowment shocks:

$$
\begin{aligned}
p_2 &= -\Lambda_2 u_2 - \Lambda_{21} u_1 && (11)\\
p_1 &= -\Lambda_1 u_1. && (12)
\end{aligned}
$$

To characterize the equilibrium, we start from second period traders whose position is given by:

$$x_2 = \gamma_H \frac{E_2[v - p_2]}{\mathrm{Var}_2[v - p_2]} - \frac{\mathrm{Cov}_2[v, v - p_2]}{\mathrm{Var}_2[v - p_2]} u_2, \tag{13}$$

where

$$
\begin{aligned}
E_2[v - p_2] &= \Lambda_2 u_2 && (14)\\
\mathrm{Var}_2[v - p_2] &= (\tau_u + \Lambda_{21}^2 \tau_v)\tau_u^{-1}\tau_v^{-1} && (15)\\
\mathrm{Cov}_2[v, v - p_2] &= \tau_v^{-1}. && (16)
\end{aligned}
$$

Replacing the latter expressions into (13) and rearranging yields

$$x_2 = \underbrace{\frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_u + \Lambda_{21}^2 \tau_v} \tau_u}_{a_2} u_2. \tag{17}$$

First period traders, when they re-trade at the second round have a position given by:

$$x_{21} = \gamma_H \frac{E_{21}[v - p_2]}{\mathrm{Var}_{21}[v - p_2]} - \frac{\mathrm{Cov}_{21}[v, v - p_2]}{\mathrm{Var}_{21}[v - p_2]} u_1, \tag{18}$$

where

$$
\begin{aligned}
E_{21}[v - p_2] &= \Lambda_2 u_2 + \Lambda_{21} u_1 && (19)\\
\mathrm{Var}_{21}[v - p_2] &= \tau_v^{-1} && (20)\\
\mathrm{Cov}_{21}[v, v - p_2] &= \tau_v^{-1}. && (21)
\end{aligned}
$$

Replacing the latter expressions into (18) and rearranging yields:

$$
\begin{aligned}
x_{21} &= \underbrace{(\gamma_H \tau_v \Lambda_{21} - 1)}_{a_{21}} u_1 + \underbrace{\gamma_L \tau_v \Lambda_2}_{b} u_2 && (22)\\
&= -\gamma_H \tau_v p_2 - u_1.
\end{aligned}
$$

Because dealers observe u_1 and u_2, and submit limit orders, at the second round their position is given by

$$x_2^D = -\gamma \tau_v p_2. \tag{23}$$

Replacing (13), (22) and (23) in the second period market clearing condition yields

$$x_2^D + x_{21} + x_2 = 0 \iff -\gamma \tau_v p_2 + (\gamma_H \tau_u \Lambda_{21} - 1) u_1 + \gamma_H \tau_v \Lambda_2 u_2 + \frac{\gamma_H \tau_v \Lambda_2 - 1}{\tau_u + \Lambda_{21}^2 \tau_v} \tau_u u_2 = 0. \tag{24}$$

Solving for p_2 and identifying the price coefficients we obtain (11) with:

$$\Lambda_2 = \frac{(\gamma+\gamma_H)\tau_u}{1+(\gamma+2\gamma_H)(\gamma+\gamma_H)\tau_u\tau_v} \tag{25}$$

$$\Lambda_{21} = \frac{1}{(\gamma+\gamma_H)\tau_v}. \tag{26}$$

Based on (17), (22), and the expressions for the price coefficients above, at the second round second period traders hedge their endowment shock (selling the risky security if $u_2 > 0$ and buying it otherwise), while first period traders hedge and speculate on the imbalance due to second period traders' order. Therefore, the fact that information on order imbalances is observed by first period traders implies that the additional source of risk sharing dealers rely upon comes from them.

At the first round, the strategy of a dealer is like in the current benchmark of the paper, that is:

$$x_1^D = -\gamma\tau_u\frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2}u_1 - \gamma\tau_v p_1. \tag{27}$$

Denoting by $\pi_1 = (p_2-p_1)x_1 + (v-p_2)x_{21} + u_1 v$, first period traders' profit, we pin down their strategy maximizing the following value function, obtained by substituting first period traders' equilibrium strategy into the second period objective function and rearranging:

$$-E[\exp\{-\pi_1/\gamma_H\}|u_1] = -E\left[\exp\{-((p_2-p_1)x_1 + \frac{1}{2\gamma_L\tau_v}(x_{21}^2 - u_1^2))/\gamma_H\}|u_1\right]. \tag{28}$$

Applying the usual transformation to the expression at the exponent of dealers' objective function yields:

$$\begin{aligned} &-E\left[\exp\{-((p_2-p_1)x_1 + \frac{1}{2\gamma_L\tau_v}(x_{21}^2 - u_1^2))/\gamma_H\}|u_1\right] \qquad (29)\\ &\quad = -\exp\left\{-\left((\Lambda_1-\Lambda_{21})u_1x_1 + \frac{(a_{21}^2-1)}{2\gamma_L\tau_v}u_1^2 - \frac{1}{2}\left(\frac{a_{21}b}{\gamma_L\tau_v}u_1 - \Lambda_2 x_1\right)^2\left(\tau_u^{-1} + b^2/\gamma_H\tau_v\right)\right)/\gamma_H\right\}. \end{aligned}$$

Differentiating the argument of the objective function and equating the result to zero, we solve for first period traders' optimal strategy at the first round obtaining:

$$\begin{aligned} x_1 &= \left(\frac{a_{21}b}{\gamma_H\Lambda_2\tau_v} + \frac{\gamma_H(\Lambda_1-\Lambda_{21})\tau_u\tau_v}{(b^2\tau_u+\gamma_H\tau_v)\Lambda_2^2}\right)u_1 \qquad (30)\\ &= \underbrace{\left(\gamma_H\Lambda_{21}\tau_v - 1 + \frac{(\Lambda_1-\Lambda_{21})\tau_u}{(1+\gamma_H\Lambda_2^2\tau_u\tau_v)\Lambda_2^2}\right)}_{a_1}u_1. \end{aligned}$$

Finally, we replace (27) and (30) in the first period market clearing condition:

$$-\gamma\tau_u\frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2}u_1 - \gamma\tau_v p_1 + a_1u_1 = 0, \tag{31}$$

solve for p_1 and identify the first period price coefficient Λ_1:

$$\Lambda_1 = \frac{\Lambda_2^2\left(\gamma_H\Lambda_{21}\tau_v\left(\gamma\tau_u^2-1\right)+1\right) + (1+\gamma)\Lambda_{21}\tau_u + \gamma_H\Lambda_2^4\tau_u\tau_v(1-\gamma_H\Lambda_{21}\tau_v)}{\gamma\gamma_H\Lambda_2^4\tau_u\tau_v^2 + \gamma\Lambda_2^2\tau_v\left(1+\gamma_H\tau_u^2\right) + (1+\gamma)\tau_u}. \tag{32}$$

Substituting (25) and (26) in the above expression and simplifying yields:

$$\Lambda_1 = \Lambda_{21} = \frac{1}{(\gamma+\gamma_H)\tau_v}. \tag{33}$$

Therefore, when first period traders observe u_2:

1. Again, the case with transparency has a unique equilibrium.

2. However, now it's the 1st period traders who, at the second round, "speculate" on u_2, posting a contrarian market order which represents the only change in their position. That is, first period traders' exposure to their endowment shock does not change across trading rounds. The reason for this effect is that according to (30) first period traders at the first round hedge the same fraction they will hedge at the second round modified to take advantage of differences in their price impact across rounds. However, the only reason why Λ_{21} may differ from Λ_1 is a change in liquidity providers' exposure to u_1, which depends on traders' liquidity demand at the second round. But liquidity traders' have no reason to change their position, since market conditions have not changed compared to the first trading round: they are not learning anything new about v, and they can fully control the execution risk due to second period traders' order. The consequence of this is that $\Lambda_{21} = \Lambda_1$ (dealers' exposure to u_1 does not change across trading rounds).

3. In turn, this implies that the autocovariance of 1st and 2nd period returns is null:
$$\text{Cov}[p_2 - p_1, p_1] = 0.$$

4. Noise trading persistence. It is still the case that $p_2 = -\Lambda_2\theta_2$, $p_1 = -\Lambda_1\theta_1$, with $\theta_1 \equiv u_1$ and $\theta_2 \equiv u_2 + \beta\theta_1$,
$$\beta \equiv \frac{\Lambda_{21}}{\Lambda_2} > 1.$$

F Hedgers' aggressiveness under opacity and transparency

In this appendix, we analytically compare Λ_2, Λ_{21} and a_2, a_{21} across the two regimes of Section II and Section III.A. We denote by a_2^T (a_{21}^T) and Λ_2^T (Λ_{21}^T), and a_2^O (a_{21}^O) and Λ_2^O (a_{21}^O), respectively, the hedging intensity of second (first) period traders and second (first) period endowment shock price impact with transparency, and opacity.

Corollary 3. *With transparency:*

1. *Second-period liquidity traders hedge more and Λ_2 is higher than with opacity:*

$$|a_2^O| < |a_2^T|, \;\; \Lambda_2^O < \Lambda_2^T.$$

2. *When there are multiple equilibria, first-period liquidity traders hedge more and Λ_{21} is higher with opacity:*

$$|a_{21}^O| > |a_{21}^T|, \;\; \Lambda_{21}^O > \Lambda_{21}^T.$$

When a unique equilibrium arises, the result is ambiguous.

Proof. It is immediate to check that $\Lambda_2^{***} < \Lambda_2^T \equiv \gamma/(\gamma+\gamma_H)\tau_v$. Recall that with opacity $-1 < a_2^{***} < a_2^{**} < a_2^{*} < 0$. Thus, it is enough to check that

$$a_2^T \equiv -\frac{\gamma}{\gamma+\gamma_H} < a_2^{***} \equiv -\frac{\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)},$$

for $0 < 4\tau_u\tau_v \leq \gamma/(\gamma+\gamma_H)^3$. Next, for $4\tau_u\tau_v > \gamma/(\gamma+\gamma_H)^3$, with opacity, the unique equilibrium obtains as the unique real root of the cubic (18), which is strictly increasing in Λ_2 and negative at $\Lambda_2 = 0$. Evaluating it at Λ_2^T yields

$$\varphi(\Lambda_2^T) \equiv \frac{\gamma}{(\gamma+\gamma_H)^3\tau_v} > 0,$$

which proves our result for Λ_2. To see that $|a_2^O|$ is lower than $|a_2^T|$ in this case too, recall that, independently of the information regime, at equilibrium $\Lambda_2 = -a_2/\gamma\tau_v$.

Turning to a_{21}, recall that in either regime, we have

$$a_{21} = \frac{\gamma_H\tau_v\Lambda_{21} - 1}{\tau_u + \Lambda_2^2\tau_v}\tau_u.$$

Substituting the values for the price impact coefficients under transparency in the above expression yields:

$$a_{21}^T = -\frac{(\gamma+\gamma_H)^2\tau_u\tau_v}{1 + (\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v}. \tag{34}$$

With opacity, we need to distinguish between the two parameter regions identified in Proposition 4. When multiple equilibria arise, at the equilibrium with high Λ_2, we have

$$a_{21}^O = -\frac{\gamma + \sqrt{(\gamma - 4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)}. \tag{35}$$

Comparing (35) with (34), yields that

$$|a_{21}^O| > |a_{21}^T|,$$

for $0 < \tau_u \tau_v \leq (\gamma/(4(\gamma+\gamma_H)^3)$ (because of the ranking we established in Corollary 5, this is sufficient). Additionally, when multiple equilibria obtain with opacity, we have

$$\Lambda_{21}^{T} \equiv \frac{(\gamma+\gamma_H)\tau_u}{1+(\gamma+\gamma_H)(\gamma+2\gamma_H)\tau_u\tau_v} < \Lambda_{21}^{***} \equiv \frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_H)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_H)\gamma\tau_v}.$$

When a unique equilibrium obtains with opacity our numerical simulations show that the relationship between a_{21}^{T} and a_{21}^{O} is ambiguous. See Figure 7 where for parameter values such that we are well into the region with a unique equilibrium under opacity, hedging aggressiveness is larger with transparency. For the illiquidity ranking, we evaluate $\varphi(\cdot)$ at Λ_{21}^{T} and obtain $\varphi(\Lambda_{21}^{T}) < 0$, implying that with opacity even when a unique equilibrium obtains $\Lambda_{21}^{O} > \Lambda_{21}^{T}$. $\square$

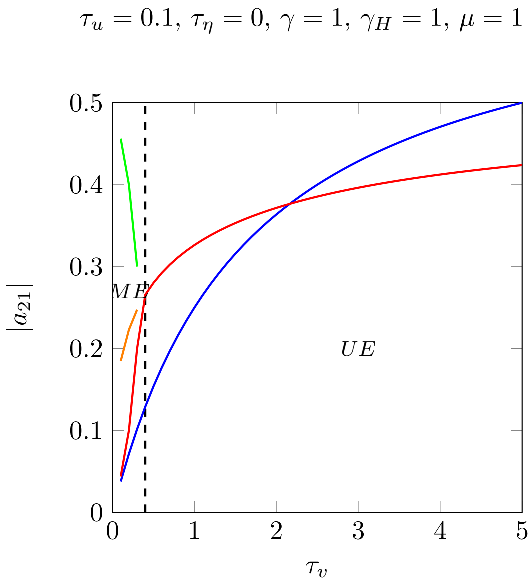


Figure 7: Comparing hedging aggressiveness across the two regimes. The blue curve illustrates $|a_{21}^{T}|$, while the green, orange and red curves $|a_{21}^{O}|$; we denote by ME and UE respectively the parameter region for which with opacity, multiple equilibria or a unique equilibrium obtain.